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A NUMERICAL METHOD FOR SOLVING LINEAR INTEGRAL EQUATIONS

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Abstract

Integral equations find special applicability within scientific and mathematical disciplines. A powerful and efficient modified Adomian Decomposition methodology in solving linear integral equations is presented. To check the numerical method, it is applied to solve different test problems with known exact solutions and the numerical solutions obtained confirm the validity of the numerical method and suggest that it is an interesting and viable alternative to existing numerical methods for solving the problem under consideration. It also converges to the exact solution.

Keywords:
Modified Adomian Decomposition Method, Integral Equations, Numerical Method.

1. Introduction

The decomposition method has been shown to solve effectively, easily, and accurately a large class of linear and nonlinear, ordinary, partial, deterministic or stochastic differential equations and integro-differential equations with approximate solutions which converge rapidly to accurate solutions, Cherrualt (1989), Abbaoui and cherruault (1994). The basic motivation of this work is to apply the MADM to the transformed volterra integral equation. This algorithm provides the solution in a rapidly convergent series solution Wazwaz (2011).

Some numerical series solutions obtained results suggest that the newly improvement technique introduces a promising tool and powerful improvement for solving integral equations.

2. Description of the Method

The decomposition method is similar to the Picard's successive approximation method except for the fact that in the Picard's method the number of terms subjected to the operator increases by one term at each iteration whereas in the decomposition method each term involves only one term subjected to the operator. In the decomposition method, we usually express the solution $y(x)$ of the problem in a series form defined by

$$y(x) = \sum_{n=0}^{\infty} y_n(x) \qquad \dots (1)$$

For instance, consider the Volterra integral equation of the second kind

$$y(x) = f(x) + \lambda \int_a^x k(x,t) y(t) dt \qquad \dots (2)$$

Where, $f(x) = f_1(x) + f_2(x)$ ... (3)

Substituting the decomposition equation (1) into both sides of (2) yields

$$\sum_{n=0}^{\infty} y_n(x) = f(x) + \lambda \int_a^x k(x,t) \left(\sum_{n=0}^{\infty} y_n(t) \right) dt \qquad \dots (4)$$

The components $y_0(x)$, $y_1(x)$, $y_2(x)$, ... of the unknown function $y(x)$ are completely determined in a recurrence manner if we set

$y_0(x) = f_1(x)$

$$y_1(x) = f_2(x) + \lambda \int_a^x k(x,t) y_0(t) dt$$

$$y_2(x) = \lambda \int_a^x k(x,t)\, y_1(t) dt$$

$$y_3(x) = \lambda \int_a^x k(x,t)\, y_2(t) dt$$

and so on. The above decomposition scheme for determination of the components $y_0(x), y_1(x), y_2(x), y_3(x), \ldots$ of the solution $y(x)$ of equation (2) can be written in a recurrence form by

$$y_0(x) = f_1(x)$$

$$y_1(x) = f_2(x) + \lambda \int_a^x k(x,t) y_0(t) dt$$

$$\vdots$$

$$y_{n+1}(x) = \int_0^x k(x,t)\, y_n(t) dt, n \geq 0$$

So that the equation is assume the form

$$y(x) = y_0(x) + y_1(x) + y_2(x) + \cdots + y_n(x) \qquad \ldots (5)$$

3. Numerical Results and Discussion

The MADM provides an analytical solution in terms of an infinite series. However, there is a practical need to evaluate this solution. The consequent series truncation, and the practical procedure conducted to accomplish this task, together transforms the analytical results into an exact solution, which is evaluated to a finite degree of accuracy. In order to investigate the accuracy of the MADM solution with a finite number of terms, two examples were solved.

Example 1: Consider the Voltera integral equation of the second kind

$$y(x) = 1 + x + \int_0^x (x-t) y(t)\, dt$$

The Exact solution of the above equation is given by

$$y(x) = e^x$$

To apply the method, we write

$$\sum_{n=0}^{\infty} y_n(x) = 1 + x + \int_0^x (x-t) \sum_{n=0}^{\infty} y_{n-1}(t) dt$$

Where we take

$$y_0(x) = 1$$

$$y_1(x) = x + \int_0^x (x-t)\, y_0(t) dt = x + \frac{1}{2}x^2$$

$$y_2(x) = \int_0^x (x-t)\, y_1(t) dt = \frac{1}{6}x^3 + \frac{1}{24}x^4$$

$$y_3(x) = \int_0^x (x-t)\, y_2(t) dt = \frac{1}{120}x^5 + \frac{1}{720}x^6$$

The approximate solution involving three terms is

$$y(x) = y_0(x) + y_1(x) + y_2(x) \ldots = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 = e^x$$

Example 2: Consider the linear Fredholm integral equation of the second kind

$$y(x) = \cos x + \frac{1}{2}\int_0^{\frac{\pi}{2}} \sin x\, y(t)\, dt$$

The Exact solution of the above equation is given by

$$y(x) = \cos x + \sin x$$

To apply the method, we write

$$\sum_{n=0}^{\infty} y_n(x) = 0 + \cos x + \frac{1}{2}\int_0^{\frac{\pi}{2}} \sin x \sum_{n=0}^{\infty} y_{n-1}(t) dt$$

Where we take

$$y_0(x) = 0$$

$$y_1(x) = \cos x$$

$$y_2(x) = \frac{1}{2}\int_0^x \sin x\, y_1(t) dt = \frac{1}{2}\sin x$$

$$y_3(x) = \frac{1}{2}\int_0^x \sin x\, y_2(t) dt = \frac{1}{4}\sin x$$

$$y_4(x) = \frac{1}{2}\int_0^x \sin x\, y_3(t)dt = \frac{1}{8}\sin x$$

$$y_5(x) = \frac{1}{2}\int_0^x \sin x\, y_4(t)dt = \frac{1}{16}\sin x$$

$\vdots$

$$y(x) = \cos x + sinx \sum_{n=1}^{8} \frac{1}{2n} = \cos x + \sin x$$

Which coincides with the exact solution

Example 3: Consider the linear Volterra integral equation of the second kind

$$y(x) = \cos x - \sin x + 2\int_0^x \sin(x-t)y(t)\, dt$$

The Exact solution of the above equation is given by

$$y(x) = e^{-x}$$

To apply the method, we write

$$\sum_{n=0}^{\infty} y_n(x) = \cos x - \sin x + 2\int_0^x \sin(x-t)\sum_{n=0}^{\infty} y_{n-1}\,(t)dt$$

Where we take

$$y_0(x) = 0$$

$$y_1(x) = \cos x - \sin x + 2\int_0^x \sin(x-t)\, y_0(t)dt = \cos x - \sin x$$

$$y_2(x) = 2\int_0^x \sin(x-t)\, y_1(t)dt = x\cos x + x\sin x - \sin x$$

$\vdots$

$$y_n(x) = 2\int_0^x \sin(x-t)\, y_{n-1}(t)dt$$

The Approximate solution involving only three terms is given below as

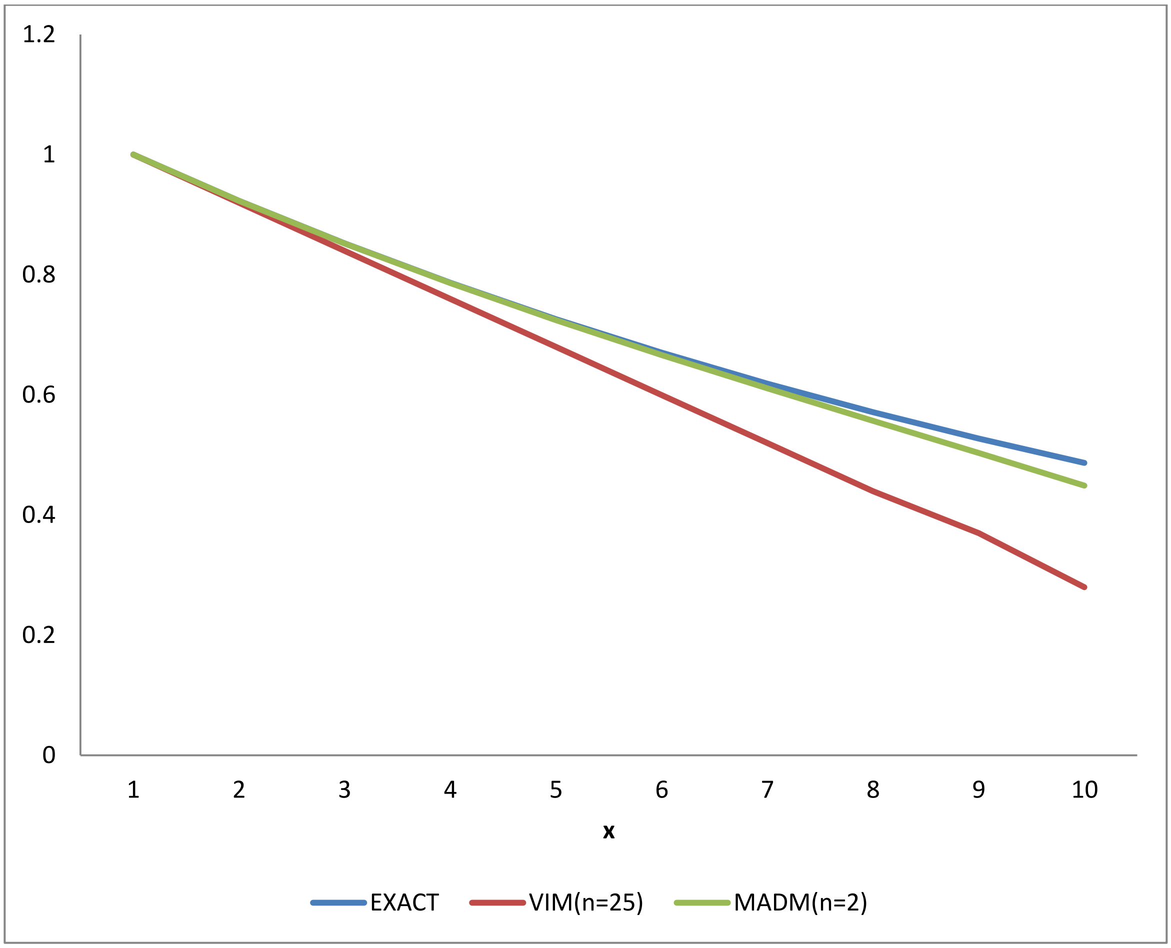
$$y(x) \cong y_0(x) + y_1(x) + y_2(x) = \cos x - 2\sin x + x\cos x + x\sin x$$

Which is tabulated and depicted in the figure below

Tabulated Numerical Solutions obtained from Exact, Variation Iteration method (VIM) and MADM

X	EXACT	VIM(n=25)	MADM(n=2)
0	1	1	1
0.08	0.923116	0.919882	0.92310963
0.16	0.852143	0.839856	0.852038149
0.24	0.786627	0.759834	0.786102466
0.32	0.726147	0.679815	0.72451893
0.4	0.670318	0.599799	0.666416044
0.48	0.618781	0.519786	0.610848139
0.56	0.571207	0.439775	0.556809848

Fig: Plots of Numerical Results



6. Conclusion

The method is a powerful procedure for solving linear integral equations. The examples given in this work illustrate the ability and reliability of the method. It is simple and effective in treating such problems. The solutions obtained from the first two examples converges rapidly to the exact solutions while the third example shows supremacy with only three terms considered in the series solution in comparison with variation iteration method (VIM) with 25 iterations. The rapid convergence of the method can be clearly seen in the figure above. Therefore the MADM can be an effective method for solutions of wide class of problems describing physical system, providing generally a rapidly convergence series solutions.

REFERENCES

Adawi, A., Awawdeh, F. & Jaradat, H. (2009). A numerical method for solving linear integral Equations,*int. J. Comptemp. Maths. Sciences*, **4**(10): 485-496.

Adomian, G., Rach, R. C. & and Meyers, R.E (1991). An Efficient Methodology for the Physical Sciences. *Kybernetes* **20**(7):24-34.

Adomian, G. (1991). A Review of Decomposition Method and some Recent Results for Nonlinear Equations. *Comp. Math. Appl.* **21**(5):101-127

Adomian, G. (1988). A Review of Decomposition Method in Applied Mathematics, *J. Maths. Anal.*, **135**:501-544.

Adamu M. Y, Okai J. O & Suleiman, E. (2016). On the Modified Adomian Decomposition Method Applied to a Transformed Fredholm Integro-Differential Equations of the Second Kind, *Journal of the Nigerian Association of Mathematical Physics* **35:** 413-416.

Adomian, G. (1989). Nonlinear Stochastic Systems, Theory and Application to Physics, Kluser Academic Publ.

Adomian, G. (1996). Modified Adomian Polynomials. *Maths. Comput. Modeling* **24**(11):39-46.

Ahmed A.H. & Kirtiwant P.G (2016). On the Numerical Solution of

Nonlinear Volterra-Fredholm Integral Equations by Variation Iteration method, *International journal of Advanced Scientific And Technical Research* **6**(3): 2249-9954

Abbaoui, k. and Cherruault, Y. (1994). Convergence of Adomian's method applied to Differential Equations. *Mathl. Comput. Modeling* 28(5): 103-109.

Abbaoui, k., Cherruault, Y. and Seng, V. (1195). Practical Formulae for the calculus of multivariable Adomian's polynomials. *mathl. comput. Modeling* 22(1) : 89-93.

Burton, T.A. (2005). Volterra integral and Differential Equations, 2nd ed. *Mathematics in science and Engineering*, 202. *Elsevier*, 2005.

Cherruault, Y. (1989). Convergence of Adomian's method. Kybernetes 18(2) : 31-38.

Maturi, D.A. (2014). Adomian decomposition method of Fredholm integral Equation of the second kind using Maple, journal of Advances in Mathematics, vol.9, No. 1, pp. 1868-1875.

Mirzae, F. (2010). Numerical computational solution of the linear volterra integral equations system via Rationalized Haar Functions, journal of king Saud University -science, 22(4)(2010), 265-268.

Wazwaz, A.M. (2011). Linear and Nonlinear integral Equation: Methods and Applications, Springer, 2011.