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ADJUSTED R_c CRITERION FOR COMPETING 2K-P RESOLUTION III FACTORIAL DESIGNS

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Abstract

Optimal designs guaranteed precise estimates of any parameter of interest. Therefore, a suitable procedure is required to identify optimal designs among countless number of competing designs available to the experimenter. This research work gave a reason why one of the popular procedures would not be effective and provided a suitable procedure.

Key words: Minimum aberration, optimality criteria, resolutions, defining contrast, precision, finite projective geometry, condition number.

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1. INTRODUCTION

Interaction assumption has popularized the usefulness of factorial designs in various field of human endeavor (Montgomery, 2001). An increase in the number of factors tends to increase the number of units needed for an experiment which leads to a very expensive experiment (Salawu et al., 2012). Partitioning the full factorial designs into problem solving fractions is called fractional factorial design. Further assumptions like sparcity of effects, sequential experimentation, and so on have made fractional factorial design a better alternative to factorial design in that, it is very cost effective and far reaching. The focus of this work is to provide a means to compare competing resolution III factorial designs from the context that $X'X$ procedure failed.

A lot of literature has shown the usefulness of fractional factorial designs which when used appropriately lead to appreciable gain in precision. Prominent among such researchers are: Cheng and Mukerjee (1998) who considered finite projective geometry method in the construction of two level fractional factorial designs. Maximum Estimation Capacity and Minimum Aberration criteria were used in the selection of optimal resolution III designs. Montgomery (2001), Rasch et al., (2011), Fontana and Sampo (2013) and Li et al., (2016) gave a lot of contributions in the use of fractional factorial designs.

A design that minimizes the minimum word length pattern in the defining relation is called minimum aberration design. Minimum Aberration Criterion is a stringent law to consider at this realm (Montgomery 2001, Jaynes et al. 2017). A fractional factorial experiment where the number of the levels of any factor of the experiment is the same is called symmetric fractional factorial experiment; otherwise it is called asymmetric fractional factorial experiment. In this study, two level symmetric fractional factorial designs are considered denoted as 2^{k-p}. There are k number of factors and p number of independent interaction effects in the experiment. The number of units required for this experiment is 2^{k-p} where $2^{k-p}-1$ effects are possible for estimation (Cheng and Mukerjee, 1998). Balanced and orthogonal properties in this experiment take effect by replicating a level of any factor the same number of times like any other level of the same factor and linearly independence is observed on pair-wise of any two factors of the experiment (Salawu et al., 2012).

Using $X'X$ procedure considering the properties of balance and orthogonality in order to compare fractional factorial designs when the same number of factors and the same number of independent effects are involved would fail. This is due to the fact that $X'X$ produces the same diagonal matrix for competing designs which would definitely lead to inability to distinguish optimal designs from non-optimal designs. This research work provides a suitable procedure using the concept of distance.

2. MATERIALS AND METHOD

A model for fractional factorial experiments can be defined generally in matrix notation as

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{e}$$

When there are two competing designs indicated as $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ then

$$\boldsymbol{y} = \boldsymbol{X_1\beta} + \boldsymbol{e} \quad \text{and} \quad \boldsymbol{y} = \boldsymbol{X_2\beta} + \boldsymbol{e}$$

Comparison is needed to obtain better information between these two designs above.

The independent interaction effects are the effects an experimenter decides to sacrifice in the experiment but there are some other interaction effects that would be sacrificed together with the chosen ones, that is why careful consideration must be given to the interaction effects an experimenter would decide upon (Montgomery, 2001). These set of interaction effects sacrificed together with the chosen ones are called generalized interaction effects.

Fraction of a full factorial design is obtained using the concept of defining contrast upon which finite projective geometry approach is based (Cheng and Mukerjee, 1998). The independent interaction effects selected are used to construct fractional factorial designs while the generalized interaction effects contribute to the precision of the effect of interest.

The condition number for design matrix X is defined as

$$\zeta(X) = ||X^{-1}|| * ||X|| \text{, such that } \zeta(X) \geq 1$$

While reciprocal of a condition number for the design is defined as

$$R_c(X) = \frac{1}{\zeta(X)}, \text{ such that } 0 \leq R_c(X) \leq 1$$

Where $||.||$ denotes Euclidean distance.

Using condition number for design matrix X may be too big and $R_c(X) \approx 0$. Adjustment are provided in order to make $R_c(X) \approx 1$ such that $0 \leq R_c(X) \leq 1$in this work

$$\zeta_{Adj}(X) = \left||(p^{n_*-p+1}+1)^{-1}X^{-1}|\right| * \left||(p^{n_*-p+1}+1)^{-1}X|\right|$$

$$\zeta_{Adj}(X) = (p^{n_*-p+1}+1)^{-1}\left||X^{-1}|\right| * (p^{n_*-p+1}+1)^{-1}\left||X|\right|$$

Where $n_* = k - p$

$$\zeta_{Adj}(X) = (p^{n_*-p+1}+1)^{-2}\left||X^{-1}|\right| * \left||X|\right|$$

$$R_{cAdj}(X) = \frac{1}{\zeta_{Adj}(X)}, \text{ such that } 0 \leq R_{cAdj}(X) \leq 1$$

But as $p \to \infty$, $p^{n_*-p+1} + 1 \to \boldsymbol{\Lambda}$, such that $0 \leq R_{cAdj}(X) \leq 1$

3. RESULTS AND DISCUSSION

R codes were used for the construction and estimation of the optimality values.

Table 3.1: Competing Resolution III Symmetric Designs for $\boldsymbol{k = 6,\ n = 16}$.

SN	$\boldsymbol{k, p}$	Design Generators	$\boldsymbol{R_{cAdj}(X)}$ Criterion	Optimality sign
1	6,2	**ABC DEF ABCDEF**	**0.7893252**	
2	6,2	**ABC BDEF ACDEF**	**0.8293465**	←

A design having arrow is the optimal.

Table 3.2: Competing Resolution III Symmetric Designs for $\boldsymbol{k = 7,\ n = 32}$.

SN	$\boldsymbol{k, p}$	Design Generators	$\boldsymbol{R_{cAdj}(X)}$ Criterion	Optimality sign
1	7,2	**ABC DEFG ABCDEFG**	**0.8291376**	
2	7,2	**ABC CDEFG ABEFG**	**0.8576896**	←

A design having arrow is the optimal.

For Table **3.1** and Table **3.2** above, the design that maximizes the minimum word length pattern in the defining relation is chosen as the optimal design. Such a design minimizes the maximum word length pattern.

A design that maximizes the minimum word length pattern in the defining relation is called minimum aberration design. Having $\boldsymbol{R_{cAdj}(X)}$ **Criterion** selecting minimum aberration design as an optimal design, shows its efficiency.

4. CONCLUSION

To have a precise estimate of any parameter of interest under fractional factorial experiment, optimal design and alias chain must receive serious attention. A good optimality criterion would give hint on how good a particular design is in respect to another design.

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