



ANALYTICAL APPROACH TO DETERMINING THE CONDITIONS FOR MAXIMUM DISCHARGE THROUGH A CHIMNEY

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Abstract

Flue gas channels have been utilized to exchange heat with the air inside a chimney and the ambient. This heat which is usually given to the environment is termed thermal pollution. The thermal efficiency of the system is the ratio of useful energy against given heat energy. Adopting the analytical approach and modeling, this paper investigates the conditions for maximum discharge through a chimney. The net pressure difference causing the flow through the combustion chamber, Δp , was found to be dependent on the pressure due to the column of hot gas of height, $\rho_g \cdot gH$, inside the chimney. The mass of the gases flowing through any cross-section of the chimney, \dot{m}_g , is given by $\rho_g \cdot A \cdot C$ (kg/s). Assuming no loss, the velocity of the gases passing through the chimney is given by; $C = \sqrt{2gH_1}$. Thus, we see that the absolute temperature of the chimney gases bears a certain ratio to the absolute temperature of the outside air, as eqn. (13) portrays. Following this analysis, it could be concluded that the efficiency of a chimney is proportional to the height; but even for a very tall chimney, will be less than 1 percent.

Keywords: Analytical Approach, Modeling, Experimental Conditions, Maximum Discharge, Chimney

1.1 Introduction

A chimney is an architectural ventilation structure made of masonry, clay, or metal, that isolates hot toxic exhaust gases or smoke produced by a boiler, stove, furnace, incinerator, or fireplace,

from human living areas. Chimneys are typically vertical, or as near as possible to vertical, to ensure that the gases flow smoothly, drawing air into the combustion in what is known as the stack, or chimney effect [1, 2]. The space inside a chimney is called the flue. Chimneys are adjacent to large industrial refineries, fossil fuel combustion facilities or part of buildings, steam locomotives and ships. The height of a chimney influences its ability to transfer flue gases to the external environment via stack effect [3]. Additionally, the dispersion of pollutants at higher altitudes can reduce their impact on immediate surroundings. The dispersion of pollutants over a greater area can reduce their concentrations and facilitate compliance with regulatory limits [4]. Stack effect or chimney effect is the movement of air into and out of buildings, chimneys, flue-gas stacks, or other containers, resulting from air buoyancy. Buoyancy occurs due to a difference in indoor-to-outdoor air density resulting from temperature and moisture differences. The result is either a positive or negative buoyancy force. The greater the thermal difference and the height of the structure, the greater the buoyancy force, and thus the stack effect. The stack effect helps drive natural ventilation, air infiltration, and fires. In the case of fire, the stack effect needs to be controlled to prevent the spread of smoke and fire, and to maintain tenable conditions for occupants and firefighters [5]. While natural ventilation methods may be effective, such as air outlets being installed closer to the ground, mechanical ventilation is often preferred for taller structures or in buildings with limited space. Smoke extraction is a key consideration in new constructions, and must be evaluated in design stages [6].

A current of cool air in a room or other confined spaces is, in mechanical engineering parlance, more appropriately referred to as draught. Similarly, a small pressure difference which causes a flow of gas to take place, whether in open or closed systems, is termed draught. The force needed to effect the draw is the draught. This force, as already indicated, may be due to pressure differential in the stream or the current of gas or air which causes the flow to take place. Usually, draught can be naturally actuated or mechanically induced. In boilers, to support combustion, it is necessary to supply a quantity of air, and to remove the products of combustion, by means of draught [7, 8]. The function of the draught, in case of a boiler, is to force air to the fire and to carry away the gaseous products of combustion. In a boiler furnace, proper combustion takes place only when sufficient quantity of air is supplied to the burning fuel. Draught may be classified as shown in Fig. 1.

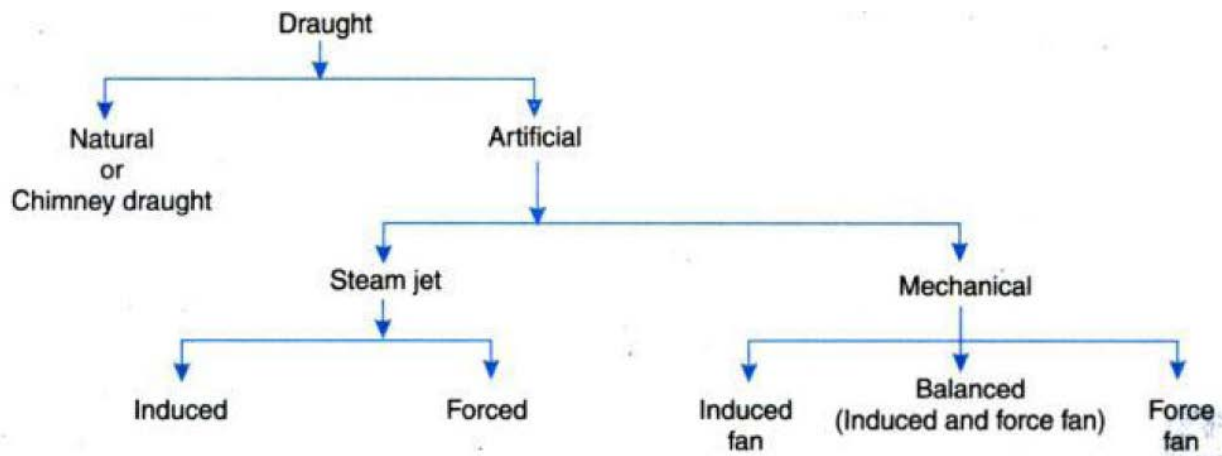


Fig. 1: Classification of Draught

1.2 Natural Draught

Natural draught is obtained by the use of a chimney. The chimney in a boiler installation performs one or more of the following functions:

- It produces the draught whereby the air and gas are forced through the fuel bed, furnace, boiler passes and settings.
- It carries the product of combustion to such a height before discharging them, that they will not be objectionable or injurious to surroundings.

A chimney is vertical tubular structure built either of masonry, concrete, or steel. The draught produced by the chimney is due to the density difference between the column of hot gases inside the chimney and the cold air outside. Fig. 2 shows a diagrammatic arrangement of a chimney of height, H metres, above the grate.

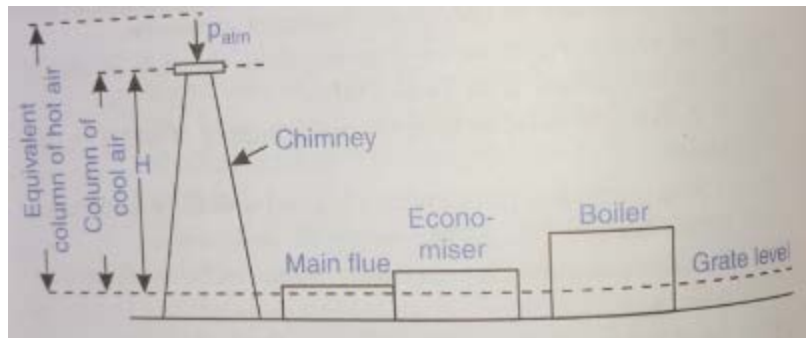


Fig. 2: Arrangement of a Chimney

We have, $P_1 = P_a + \rho_g \cdot gH$

Where, $P_1 =$ Pressure at the grate level (Chimney side),

$P_a =$ Atmospheric pressure at chimney top,

$\rho_g \cdot gH =$ Pressure due to the column of hot gas of height H metres, and

$\rho_g =$ Average mass density of hot gas

Similarly, $P_2 = P_a + \rho_a \cdot gH$

Where, $P_2 =$ Pressure acting on the grate on the open side,

$\rho_g \cdot gH =$ Pressure exerted by the column of cold air outside the chimney of height H metres, and

$\rho_a =$ Mass density of air outside the chimney

Therefore, Net pressure difference causing the flow through the combustion chamber, Δp , equals;

$$\Delta p = P_2 - P_1 = (\rho_a - \rho_g) gH \text{ ----- (1)}$$

This difference of pressure causing the flow of gases is known as **static draught**. Its value is small and is generally measured by a water manometer. It may be noted that this pressure difference in chimney is generally less than 12 mm of water.

1.3 Chimney Height and Diameter

1.3.1 Chimney Height

Let us assume that the volume of the products of combustion is equal to the volume of air supplied, both reduced to the same temperature and pressure conditions.

Let, m_a = Mass of air supplied per kg of fuel

$m_a + 1$ = Mass of chimney gases

T_a = Absolute temperature of atmosphere, and

T_g = Average absolute temperature of chimney gases.

Also, $\frac{\text{Mass of hot gases}}{\text{Mass of air}} = \frac{m_a + 1}{m_a}$, temperature and pressure being the same.

Now,
$$\rho_a = \frac{P}{RT_a} = \frac{1.01325 \times 10^5}{287} \cdot \frac{1}{T_a} = 353 \cdot \frac{1}{T_a} \quad (2)$$

And,
$$\begin{aligned} \rho_g &= \frac{P}{RT_g} \cdot \frac{m_a + 1}{m_a} \\ &= \frac{1.01325 \times 10^5}{287} \cdot \frac{1}{T_g} \cdot \frac{m_a + 1}{m_a} \\ &= 353 \cdot \frac{1}{T_g} \cdot \frac{m_a + 1}{m_a} \quad (3) \end{aligned}$$

Inserting the values of ρ_a and ρ_g into Eqn. 1, we get;

$$\Delta p = 353 \text{ gH} \left[\frac{1}{T_g} - \frac{1}{T_a} \cdot \left(\frac{m_a + 1}{m_a} \right) \right] \text{ (N/m}^2\text{)} \quad (4)$$

Assuming that at the draught pressure, Δp , produced is equivalent to H_1 metres height of burnt gases, we have:

$$\begin{aligned} \Delta p &= \rho_g \cdot gH_1 \\ &= 353 \left(\frac{m_a + 1}{m_a} \right) \cdot \frac{1}{T_g} \text{ gH}_1 \quad (5) \end{aligned}$$

Equating eqns. (4) and (5), we get:

$$353 \left(\frac{m_a+1}{m_a} \right) \cdot \frac{1}{T_a} gH_1 = 353 gH \left[\frac{1}{T_g} - \frac{1}{T_g} \cdot \left(\frac{m_a+1}{m_a} \right) \right]$$

$$\frac{353}{T_a} \left(\frac{m_a+1}{m_a} \right) \cdot gH_1 = \frac{353}{T_g} gH - \frac{353}{T_g} gH \left(\frac{m_a+1}{m_a} \right) \text{ or}$$

$$\frac{353}{T_a} \left(\frac{m_a+1}{m_a} \right) \cdot gH_1 = \frac{353}{T_g} \left(\frac{m_a+1}{m_a} \right) gH \left[\left(\frac{m_a}{m_a+1} \right) \frac{T_g}{T_a} - 1 \right]$$

Therefore, $H_1 = H \left[\left(\frac{m_a}{m_a+1} \right) \frac{T_g}{T_a} - 1 \right]$ _____ (6)

Due to losses at various sections along the path of the flue gas, the actual draught available is always less than that given by the eqn. (4). If h_w is the height, in mm of a column of water which will produce the pressure Δp , then;

$$h_w = 353 H \left[\frac{1}{T_a} - \frac{1}{T_g} \cdot \left(\frac{m_a+1}{m_a} \right) \right]$$
 _____ (7)

(1 mm of water = 9.81 Pa)

The height, h_w , would be shown by the use of a U-tube. The formula as expressed by eqn. (7) is used for numerical calculation work only.

1.3.2 Chimney Diameter

Assuming no loss, the velocity of the gases passing through the chimney is given by;

$$C = \sqrt{2gH_1}$$

If the pressure loss in the chimney is equivalent to a hot-gas column of h' metres, then;

$$C = \sqrt{2g(H_1 - h')} = 4.43\sqrt{H_1 - h'}$$

$$= 4.43\sqrt{H_1} \sqrt{1 - \frac{h'}{H_1}} = K\sqrt{H_1}$$
 _____ (8)

Where, $K = 4.43\sqrt{1 - \frac{h'}{H_1}}$

The value of K : 0.825 ----- For brick chimneys, and

1.1 ----- For steel chimneys

The mass of the gases flowing through any cross-section of the chimney is given by;

$$\dot{m}_g = \rho_g \cdot A \cdot C. \text{ (kg/s)}$$

Or
$$\dot{m}_g = \rho_g \cdot \frac{\pi}{4} D^2 \cdot C.$$

Therefore,
$$D^2 = \frac{\dot{m}_g}{\rho_g \cdot C} \times \frac{4}{\pi}$$

$$D = 1.128 \sqrt{\frac{\dot{m}_g}{\rho_g \cdot C}} \text{-----} \quad (9)$$

1.4 Conditions for Maximum Discharge through a Chimney

The chimney draught is most effective when the maximum weight of hot gases is discharged in a given time; and it will be shown that this occurs when the absolute temperature of the chimney gases bears a certain relation to the absolute temperature of the outside air. We know that the velocity of gas through the chimney, assuming the losses to be negligible, is given by;

$$C = \sqrt{2gH_1}, \text{ where } h' = 0$$

Inserting the value of H_1 , from eqn. (6)

$$C = \sqrt{2gH \left[\left(\frac{m_a}{m_a + 1} \right) \frac{T_g}{T_a} - 1 \right]} \text{-----} \quad (10)$$

The density of the hot gas is given by

$$\rho_g = \frac{P}{RT_g} \text{-----} \quad (11)$$

The mass of gas discharged per second,

$$m_g = A \times C \times \rho_g$$

Inserting the values of C and ρ_g from eqns (10) and (11), we obtain

$$m_g = A \sqrt{2gH \left[\left(\frac{m_a}{m_a+1} \right) \frac{T_g}{T_a} - 1 \right]} \frac{P}{RT_g}$$

$$m_g = \frac{K}{T_g} \sqrt{\left[\left(\frac{m_a}{m_a+1} \right) \frac{T_g}{T_a} - 1 \right]} \quad (12)$$

Where constant, $K = \frac{A \times p \times \sqrt{2gH}}{R}$

The value of m_g will be maximum, if

$\frac{dm_g}{dT_g} = 0$, as T_a and m_a are fixed quantities

$$\therefore \frac{d}{dT_g} \left[\frac{K}{T_g} \sqrt{\left[\left(\frac{m_a}{m_a+1} \right) \frac{T_g}{T_a} - 1 \right]} \right] = 0$$

$$\text{or, } \frac{d}{dT_g} \left[\frac{1}{T_g} \sqrt{\left[\left(\frac{m_a}{m_a+1} \right) \frac{T_g}{T_a} - 1 \right]} \right] = 0$$

$$\text{or, } \frac{d}{dT_g} \left[\frac{(ZT_g - 1)^{1/2}}{T_g} \right] = 0$$

$$\text{where, } Z = \left(\frac{m_a}{m_a+1} \right) \cdot \frac{1}{T_a}$$

$$\text{or, } \frac{d}{dT_g} [(ZT_g - 1)^{1/2} \times T_g^{-1}]$$

$$\text{or, } (ZT_g - 1)^{1/2} \cdot (-1)(T_g)^{-2} + T_g^{-1} \times \frac{1}{2} (ZT_g - 1)^{-1/2} \times Z = 0$$

$$\begin{aligned}
 \text{or} \quad & \frac{-(ZT_g - 1)^{1/2}}{T_g^2} + \frac{Z}{2T_g(ZT_g - 1)^{1/2}} = 0 \\
 \text{or} \quad & \frac{-2(ZT_g - 1) + ZT_g}{2(T_g)^3(ZT_g - 1)^{1/2}} = 0 \\
 \text{or} \quad & -2(ZT_g - 1) + ZT_g = 0 \\
 \text{or} \quad & -2ZT_g + 2 + ZT_g = 0 \\
 \text{or} \quad & ZT_g = 2 \\
 \text{or} \quad & \frac{m_a}{m_a + 1} \cdot \frac{T_g}{T_a} = 2
 \end{aligned}$$

i.e.,

$$\frac{T_g}{T_a} = 2 \left(\frac{m_a + 1}{m_a} \right) \quad \text{----- (13)}$$

Thus, we see that the absolute temperature of the chimney gases bears a certain ratio to the absolute temperature of the outside air.

Putting the value of $\frac{T_g}{T_a}$ in eqn. (6), we get

$$\begin{aligned}
 (H_1)_{\max} &= H \left[\left(\frac{m_a}{m_a + 1} \right) \times 2 \left(\frac{m_a + 1}{m_a} \right) - 1 \right] \\
 &= H(2 - 1) = H
 \end{aligned}$$

i.e., $(H_1)_{\max} = H$ ----- (14)

The draught in mm of water column for maximum discharge can be evaluated by inserting the value $\frac{T_g}{T_a}$ in eqn. (7)

\therefore $(h_w)_{\max} = 353 H \left[\frac{1}{T_a} - \frac{1}{2T_a} \right]$

$$= \frac{176.5 H}{T_a} \text{ mm of water} \quad (15)$$

The loss in a draught may be due to the reasons mentioned below;

- The frictional resistance offered by the flues and the gas passages to the flow of the flue gases.
- Loss near the bends in the gas flow circuit.
- Loss due to friction head in equipments like grate, economizer, superheater, etc
- Loss due to imparting velocity to the flue gases.

The loss in draught in a chimney is 20 percent of the total draught produced by it.

1.5 Conclusion

The temperature of the flue gases leaving a chimney, in case of natural draught, is higher than that of flue gases leaving it in case of artificial draught system. This is because a certain minimum temperature is needed to produce a given draught with the given height of the chimney. As far as steam generation is concerned, in case of natural draught system, the heat carried away by the gases is more due to higher flue gas temperature. This indicates that the draught is created at the cost of thermal efficiency of the boiler plant installation, since a portion of heat carried away by flue gases to produce the required draught could have been used either in heating the air going to furnace or in heating the feed water going to boiler; thereby increasing the thermal efficiency of the installation. Let,

T' = Absolute temperature of the flue gases leaving the chimney to create the draught of h_w mm of water

T'' = Absolute temperature of the flue gases leaving the chimney in case of artificial draught of h_w mm of water

c_p = Mean specific heat of flue gases

The extra heat carried away by 1 kg of flue gas due to higher temperature required to produce the natural draught

$$= c_p (T' - T''), \text{ since } T' \text{ is greater than } T''$$

The draught pressure produced by the natural draught system in height of hot gases column,

$$H_1 = H \left[\left(\frac{m_a}{m_a + 1} \right) \times \frac{T_g}{T_a} - 1 \right] \text{ metre head}$$

The maximum energy this head would give to 1 kg of the flue gas which is at the expense of extra heat carried away from the boiler plant

$$= H \left[\left(\frac{m_a}{m_a + 1} \right) \times \frac{T_g}{T_a} - 1 \right] \times g \text{ Nm/kg (or J/kg)}$$

$$= \frac{H}{J} \left[\left(\frac{m_a}{m_a + 1} \right) \times \frac{T_g}{T_a} - 1 \right] \times g \times 10^{-3} \text{ kJ/kg}$$

∴ Efficiency of chimney;

$$\eta_{ch} = \frac{\frac{H}{J} \left[\left(\frac{m_a}{m_a + 1} \right) \times \frac{T_g}{T_a} - 1 \right] \times g \times 10^{-3} \text{ (kJ/kg)}}{c_p (T' - T'') \left(\frac{\text{kJ}}{\text{kg}} \right)} \text{----- (16)}$$

In the equation, $T' = T_g$.

Note: $J = 1$, in SI units

The efficiency of a chimney is proportional to the height; but even for a very tall chimney, will be less than 1 percent. Thus, we see that chimney is very inefficient as an instrument for creating draught.

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