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APPLICATION OF MATHEMATICAL MODELING ON POPULATION CARRY CAPACITY A CASE STUDY OF SOKOTO SOURTH LOCAL GOVERNMENT.

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ABSTRACT

Methods and ways of predicting the future population as well as carrying capacity of the Sokoto Source Local Government are being considered and taken care up using Malthusian law of Population. A model is generated that will be used to estimate the future population for Sokoto Source Local Government. It was found out that the population of the Sokoto Sourth Local Government will attain equilibrium at P (2292) = 1,858432481 i.e. when t =282 in the year 2292. The leadership of the Sokoto Sourth Local Government can now understand their population growth better have been established for them a model that will help provide them with the needed information of their population growth to aid proper planning and distribution resources.

1.0 INTRODUCTION

Mathematical modeling is the art of translating problems an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight, for the original application. This Research focuses mainly on population models. Although the level of mathematics required for this research work is not difficult, setting up and manipulating models requires through effort and usually discussion after problem have many answers due to different models used can illuminate different fact of the problem. The objective is to provide an approach to problems using some Mathematical methodology.

Two models-exponential growth model and logistics growth model are popular in research of the population growth. The exponential growth model was proposed by Malthus in 1798, and it is also called the Malthusian growth model.

However, when it comes to human population, whether this constant growth rate can be observed is in doubt. This is mainly because we can hardly define the conditions. Human reproduce

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sexually and have consciousness. We cannot be sure under what condition will an individual be reproduced. Therefore, it is in doubt whether models based on constant growth rate can explain human population growth. Taking carrying capacity into regards, logistic growth model improves the preceding exponential growth model. However, whether it can describe human population growth is in dispute.

The first important exercise in every modeling endeavor is to choose the facts of the investigated system that we wish to capture in our model. As I mentioned earlier before, this research work is restricted to Sokoto South local government population, there are only two methods used in this research namely exponential growth model and logistic model.

1.1 EXPONENTIAL GROWTH;

Exponential growth is exhibited when the growth rate of the function current value, resulting in its growth with time being an exponential in a function in which the same way when the growth rate is negative. In case of a discrete domain of definition with equal intervals, it is also called geometric growth or geometric decay, the function values forming a geometric progression. In either exponential decay, the ratio of the rate of change of the quantity to its current size remains constant over time (Malthus T.R. 1798).

1.2 LOGISTIC MODEL

Logistic Model is a typical application of the logistic equation is a common model of population growth originality due to Pierre Francios Verhulst in 1838 where the rate of reproduction is proportional to both the existing population and the amount of available resources, all else being equal (Verhulst P.F. 1838).

1.3. AIMS AND OBJECTIVE OF THE STUDY.

A mathematical model achieving the twin objectives of simplicity and accuracy for the simulation of immobilized packed bed fomenters.

The main aim and objectives of this work is

i. to introduce a very simple model in order to interpret data and make demographic projection and know its carrying capacity.
ii. to produce the most common quantitative approaches to population dynamics, emphasizing the different theoretical foundation and assumption.
iii. to predict future population of the study area, using the modeling approach on the position of population dynamics.

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2.0 REVIEW OF RELATED LITERATURE

Dingyadi M. A. in her project (2012) titled the mathematical modeling on population model, a case study of Bodinga Local Government; use of Malthusian law of population model to find out the percentage that the population of Bodinga Local Government is increasing. Using the census obtained in 2006.

According to Malthus (1798-1826), nature has built in systems of checks and balances that limit the amount of damage that can be caused by a large group by reducing the population of that group once it increase a certain point. Similarly, Malthus put forth an exponential growth model for human population and finally concluded that eventually, the population would exceed the capacity to grow and adequate food supply.

Similar studies have been carried out more frequently at a national level. Ulrich (1998), however, argued that immigration can only slow an inevitable decline of the population of Germany. He applied different fertility assumptions for natives and foreigners and different immigration levels by group of immigrants, and estimated the population size of Germany and its structure in 2030. Wanner (2000), Elbardt. L.L and Bereiric J.M (2002) in their study also showed that in the absence of future migration the total population of the country would start declining much earlier and would be 5.6 million in 2050, about 1.5 million less than what is currently projected .

The Lokta – Voltera predator-prey equations are first-order and nonlinear differential equation defined as;

$$\frac{dv}{dt} = av - bvp, \qquad \frac{dp}{dt} = -cp + dvp$$

Where the classic model a,b,c,d are all positive constants

A Malthusian growth model, sometimes called a simple exponential growth model, is essentially exponential growth loosed on a constant rate. The model is named after Thomas Robert Malthus (1798), who wrote an essay on the principal of population, Malthusian models have the following form $P(t) = P_0e^{rt}$ where $P(t) = P_0$ is the initial population size, r= the population growth rate (Malthusian parameter), and t = time.

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3.0 METHODOLOGY

Let $p(1)$ be the population of Sokoto South Local Government at anytime t. Now considering $p(t)$ being continuous there is need to determine the two ratios, namely, growth and death ratio.

But in this case we assume the death rate to be zero and growth rate to be proportional to the population present; hence this assumption is consistent with observation of growth provided.

Thus, from the assumption that growth rate is proportional to the population present for time t. we have (Chase J.M et. Al 2002):

⇨ $\frac{dp}{dt} \propto kp$, $\frac{dp}{dt} = kp$……………………(1)

Now, from equation (1) we have
by separating variables

$$\frac{dp}{p} = kdt$$

We integrate both sides

$\int \frac{dp}{p} = k\int dt$, $\ln p = kt + c$

Take the exponential of both sides

⇨ $p = e^{kt+c}$, $p = e^{kt}.e^{c}$, $p = me^{kt}$

But the population is at time t. hence $p(t) = me^{kt}$............................(2)

Initially if we let $t = 0$, we have

$$P(0) = me^{k(0)}$$

$P(t) = P_0 e^{kt}$…………………………(3) , Where $m = P_0$

Equation (3) is called the Malthusian law of population growth model and the so called equation predicts that population always grows exponentially with time.

Now we have to apply the equation to find out whether the population of Sokoto South Local Government is increasing or decreasing by solving some numbers of problems, using the projection of census of 2010 as compiled by National Population Commission Sokoto South Local Government of Sokoto State.

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Problem 1.

The population of Sokoto South Local Government in the year 2010 and 2011 are 224681 and 231987 respectively. Using Malthusian model we can estimate that population of Sokoto South Local Government as a function of time.

Solution

Let $t = 0$ to be the year in 2010 and P_0 =224681 then from equation (3)

$P(t) = 224681e^{kt}$……………………………(4)

Now, we observe that in the year 2011 $t = 1$ (i.e 2011- 2010), then equation (4) becomes; $(P)(1) = 224681e^{k(1)} = 231987$

$$224681e^{k} = 231987$$

To solve for k, we take the natural in of both sides. Thus,

$$k = 0.032$$

Now, equation (4) becomes , $P(t) = 224681e^{0.032t}$………………………(a)

From the result obtained in 1 to 6 above, we observed that, the population is increasing in three closed percentage which disclosed the increasing rate to be 3.2%.

Thus, using the percentage above, we can find the true increase in the population of Sokoto South Local Government.

We now, consider other problems that gives us the reality of the percentage that the population of Sokoto South is increasing.

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Problem 2.

The population of Sokoto South Local Government in 2010 and 2012 are 224681 and 239530 respectively. We can estimate the population as a function of time t.

Solution

Let t = o be the year 2010 and P_o = 224681 then from equation (3)

$p(t) = 224681e^{kt}$…………………………….(v)

We observe that in the year 2012 t = 2 i.e. (2012 – 2010)

Thus, $P(2) = 224681e^{k(2)} = 239530$

$224681e^{2k} = 239530$

Take thenatural In of both sides to find the value of k

$\text{In}\,(224681e^{2k}) = \text{In}\ 239530$, $\text{In}\ 224681 + \text{In}e^{2k} = \text{In}239530$, $2k = \text{In}239530 - \text{In}224681$

$2k = 12.3864 - 12.3224$, $K = 0.032$

Substituting k into equation (5) we have; $p(t) = 224681e^{0.032t}$………………………(b)

Problem 3

In 2010 the population of Sokoto South Local Government was found to be 224681 and in 2013 it was 247319, we estimate the population of Sokoto South Local Government as a function of time t using equation (3)

Solution

Let t = o be the year 2010, then equation (3) becomes:

$p(t) = 224681e^{kt}$……………………………….(vi)

To determine k we observe that in 2013 when t =3, we have, $P(3) = 224681e^{3k} = 247319$

$224681e^{3k} = 247319$, Take the natural In of both sides, $\text{In}\,(224681e^{3k}) = \text{In}247319$

$\text{In}\ 224681 + \text{In}e^{3k} = \text{In}247319$, $3k = \text{In}\ 247319 - \text{In}\ 247319$

$3k = 12.4184 - 12.3224$, $3k = 0.096$, $K = 0.032$ (by dividing both sides by 3)

Now, equation (6) becomes

$p(t) = 224681e^{0.032t}$……………………………(c)

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Problem 4

If in the year 2010 and 2014 the population of Sokoto South was found to be 224681 and 255361 respectively, then we can estimate its population as function of time using Malthusian model.

Solution:

Suppose t = o is the year 2010, then we have; $P(t) = 224681e^{kt}$

But in 2014, t = 4 and P(t) = 255361, so we can determine the value of k

$p(t) = 224681e^{kt}$……………………………(vii)

$224681e^{4k} = 255361$

take the natural in of both sides, $\ln(224681e^{4k}) = \ln 255361$, $\ln 224681 + \ln e^{4k} = \ln 255361$

e^{4t} $4k = \ln 255361 - \ln 224681$, $4k = 12.4504 - 12.3224$

$4k = 0.128$, $K = 0.032$

Now, equation (7) becomes; $p(t) = 224681e^{0.032t}$…………………….(d) If we substitute the value of k.

Problem 5

In the year 2010 and 2015 the population of Sokoto South Local Government was found to be 224681 and 263665 respectively. We estimate its population using Malthusian model as a function of t.

Solution

We let t = o in the year 2006. To determine the value of k from equation (3) we have t = 5 (i.e 2015 2010) in year 2015

$p(t) = 224681e^{kt}$…………………………………….(viii)

$P(5) = 224681e^{5k} = 263665$, Where $P(t) = 263665$

$224681e^{5k} = 263665$, take the natural ln of both sides, $\ln(224681e^{5k} = 263665$

take the natural ln of both sides, $\ln(224681e^{5k}) = \ln 263665$, $\ln 224681 + \ln e^{5k} = \ln 263665$, $5k = \ln 263665 - \ln 224681$

$5k = 12.4824 - 12.3224$, $K = 0.032$

Substituting k into equation (8) we have, $p(t) = 224681e^{o.o32t}$…………………………(e)

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Problem 6

In 2010 the population of Sokoto South was 224681 and 272239 in 2016. Then we estimate the population of Sokoto South as a function of time using same model.

Solution

If t = 0 be the yeat 2010, the by equation (3),

$p(t) = 224681e^{kt}$………………..(9) , But in the year 2016 t = 6

$P(6) = 224681e^{6k} = 272239$, $224681e^{6k} = 272239$

Take the natural ln of both sides, $\ln(224681e^{6k}) = \ln 272239$

$\ln 224681 + \ln e^{6k} = \ln 272239$, $6k = \ln 272239 - \ln 224681$

$6k = 12.5144 - 12.3224$, $K = 0.032$

Put k in equation (9)

As a result of the above solved problem from 1 to 6, we found out that population is increasing in three closed percentage which show that the estimation was 3.2%.

Thus, using the percentage above one can find the truth increase of Sokoto South Local Government population.

We now, consider other problems that that gives us the reality of the percentage that the population of Sokoto South is increasing.

If the population of Sokoto South Local Government was found to be 224681 in 2010 and increasing at rate of 3.2% per year when the population reach 231987.

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Problem 7

Solution

$\frac{dp(t)}{dt} = kp$, by separating the variable we have , $\frac{dp(t)}{p} = kdt$

but $k = 3.2\% = 0.032$, $\frac{dp(t)}{p} = 0.032dt$, by integrating both sides we have; $\ln p = 0.032t + c$

Take the exponential of both sides $P = e^{0.032+c}$, $P = P_0e^{0.032t}$ where $P_0 = e^c$

We are measuring time from 2010 when $P_0 = 224681$, then equation (2) gives

$P(t) = 224681e^{0.032t}$

So we calculate t when $P(t) = 231987$, $224681e^{0.032t} = 231987$, $0.032t = 0.032, t = 1$

$t = 1$ year i.e. it reaches in $(2006 + 1) = 2007$

Problem 8

If the population of Sokoto South was found to be 224681 in 2010 and is increasing at the rate of 3.2% annually, when will the population reaches 239530?

Solution

We know that $k = 0.032$, $\frac{dp(t)}{dt} = 0.032p$, Separate the variable, $\frac{dp(t)}{p} = 0.032dt$

Integrate both sides

$\ln p = 0.032t + c$ Take the exponential of both sides

$P(t) = e^{0.032t+c}$, $P(t) = P_0e^{0.032t}$

We calculate t when $P(t) = 239530$ and $P_0 = 224681$

$224681e^{0.032t} = 239530$

Take the natural ln of both sides $\ln 224681 + 0.032t = \ln 239530$, $0.032t = 0.064$

$t = 2$ years, i.e. $2010 + 2 = 2012$

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Problem 9

If the population of Sokoto South was found to be 224681 in 2010 and increases at a rate of 3.2% per year then the population will reached 247319 when:

Solution

$\frac{dp(t)}{dt} = 0.032p$ where $k = 0.032$

$\frac{dp(t)}{p} = 0.032dt$, (by separating variable)

By integrating both sides we have; $\ln P(t) = 0.032t + c$

take the exponential of both sides, $P(t) = e^{0.032t+c}$

$P(t) = P_0 e^{0.032t}$, where $P_0 = e^c$, Use $P(t) = 247319$ and $P_0 = 224681$ to find t

$224681e^{0.032} = 247319$

Take the natural ln of both sides

$\ln(224681e^{0.032t}) = \ln 247319$

$0.032t = 0.096$, $t = 3$years

Hence, the population will reach 247319 in $(2010 + 3) = 2013$

Problem 10

If the population of Sokoto South was 224681 in 2010 and was found to be increasing at 3.2% annually, when will the population reaches 255361?

Solution

$\frac{dp(t)}{dt} = kp$, where k = 0.032 $\frac{dp(t)}{p} = 0.032p$, $\frac{dp(t)}{p} = 0.032dt$,

(by separating variables)

Integrate both sides, we have, $\ln p(t) = 0.032t + c$

Take the exponential of both sides, $P(t) = e^{0.032t+c,}$ $P(t) = P_0 e^{0.032t,}$ Where $P_0 = e^c$

We are measuring time from 2010 when $P_0 = 224681$ and calculating time when $P(t) = 255361$

$224681e^{0.032t} = 255361$, Take the natural ln of both sides, $\ln(224681e^{0.032t}) = \ln 255361$

$$t = 4 \text{ years}$$

(by dividing both sides by 0.032). Hence the population will reach 255361 in (2010 + 4) = 2014

Problem 11

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In 2010 the population of Sokoto South was 224681 and is increasing at the rate of 3.2% per year, when will the population reaches 263665?

Solution

$$\frac{dp(t)}{dt} = kp = 0.032p$$

$\frac{dp(t)}{p} = 0.032dt$, (by separating variables)

Integrate both sides, we have

$$\ln p(t) = 0.032t + c$$

take the exponential of both sides

$P(t) = e^{0.032t+c}$, $P(t) = P_0 e^{0.032t}$

but in 2010 where $P(t) = 263665$ and $P_0 = 224681$

$$224681e^{0.032t} = 263665$$

Take the natural ln of both sides , $\ln(224681e^{0.032t}) = \ln 263665, 0.032t = 0.16$

$t = 5$ years (if 0.032 is divided to both sides). Thus, the population will reaches 263665 in 2015 (i.e. 2010 + 5)

Problem 12

If the population of Sokoto South was 224681 and increases at a rate of 3.2% per year, when will the population reach 272239?

Solution

$\frac{dp(t)}{dt} = kp = 0.032$, $\frac{dp(t)}{p} = 0.032dt$ (by separating variables)

Integrate both sides, $\ln p(t) = 0.032t + c$

Take the exponential of both sides

$P(t) = e^{0.032t+c}, P(t) = P_0 e^{0.032t}$

But we are measuring time from 2010 when $P_0 = 224681$ then we have

$P(t) = 224681e^{0.032t}$

So we calculate for t when $P(t) = 272239$ Thus,

$224681e^{0.032t} = 272239$, take the natural ln of both sides

$\ln(224681e^{0.032t}) = \ln 272239, \ln 224681 + 0.032t = \ln 272239$

$$0.032t = 0.192$$

Divide both sides by 0.032, $t = 6$ years

Therefore, the population reach 272239 in 2016 (i.e. 2010 + 6).

Moreover, for continuously increasing population without bound at t, the population reaches a point where the environment can no longer support it. We call this point “k” known as the carry capacity of the environment (i.e. the number of people that the environment can support). If ‘r’ is the growth constant then a reasonable modification of r to support k is given as

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$r = r\left(-\frac{p}{k}\right)$, Recall that $\frac{dp}{dt} = rp$ … … … … … … … … … … ..(i)

Substituting for r in equation (i) we have, $\frac{dp}{dt} = r\left((1-\frac{p}{k})\right)p$, $\frac{dp}{dt} = rp(1-\frac{p}{k})$,
Which is the logistic equation. This form is a nonlinear ordinary differential equation of the first order.

Now, we solve the differential equation, $\frac{dp}{dt} = \frac{rp((k-p))}{k}$

Separating the variables we have, $dp = \frac{r((k-p))}{k}dt$

Multiplying through by $1/(k-p)$, $\frac{dp}{p(k-p)} = \frac{r}{k}dt$

Integrate both sides, $\int dp/p(k-p)$

We express $\frac{dp}{p(k-p)}$ by partial fraction. Thus,

$\frac{1}{p(k-p)}dp = \frac{A}{p} + B/(k-p)$……………….(ii)

Multiplying through by L.C.M. we have, $1 = A(k-p) + Bp$, $1 = Ak - Ap + Bp$

$Ak = 1, A = \frac{1}{k}$, Also, $B = 1/k$

Substituting A and B into (ii) we have

$$\frac{\frac{1}{k}}{p} + \frac{\frac{1}{k}}{(k-p)} = \int \frac{1}{kp} + \int \frac{1}{k((k-p))} = \int \frac{r}{k}dt$$

$$\frac{1}{k}\ln p + \frac{1}{k}\ln(k-p) = \left(\frac{1}{k}\right)\ln p + \left(\frac{1}{k}\right)\ln((k-p)) = \frac{rt}{k} + c$$

$\frac{\ln p}{k} - \frac{\ln((p-k))}{k} = \frac{rt}{k} + c$……………….(iii)

Solving for c at $t = 0$ and $p = p_0$ we have, $\ln\frac{P_0}{k} - \ln(p_0 - k)/k = c$

Substituting for c into equation (iii) we have

$$\frac{\ln p}{k} - \frac{\ln((p-k))}{k} = rt/k + \ln P_0/k - \ln(P_0-k)/k$$

Multiply through by k we have, $\ln p - \ln((p-k)) = rt + \ln P_0 - \ln(P_0 - k)$

By law of ln, $\ln\left(\frac{p}{p-k}\right) = rt + \ln(P_0/(P_0-k))$

Take the exponential of both sides

$\frac{p}{p-k} = e^{rt}.P_0/(P_0-k)$………………………………(iv)

Cross multiply

$P(P_0 - k) = PP_0e^{rt} - kP_0e^{rt}$, $P(P_0-k)PP_0e^{rt} = -kP_0e^{rt}$

Dividing through by $-P_0e^{rt}$, $-P(P_0-k)/P_0e^{rt} + P = k$…………………….(v)

$P = \frac{k}{\left[\frac{-P_0}{P_0e^{rt}} + \frac{k}{P_0e^{rt}}\right]+1}$ $P = \frac{k}{\left[-e^{-rt} + \frac{k}{P_0e^{rt}}\right]+1}$ $P = \frac{k}{\left[-e^{-rt} + \frac{ke^{-rt}}{P_0}\right]+1}$ $P = \frac{k}{\left[-1+\frac{k}{P_0}\right]e^{-rt}+1}$

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$$P = \frac{k}{\left[\frac{k}{P_0}-1\right]e^{-rt+1}} \text{...............................m...(vi)}$$

In the limit$\rightarrow \infty$, $P(t) \rightarrow k$ and the expression for $P(t)$ gives the initial condition $P = P_0$. Thus, this model predicts that the population increases steadily from the initial value to the population P_0 and approaches to carry capacity k as time also increases, and following the assumption made earlier that the population change $\frac{dp}{dt}$ increases positively. Without loss of generating let us state that to predict future population of Sokoto South Local Government which is the case study of this project work, we first of all estimate the carrying capacity k of the local government following its population dynamics. Thus an expression of equation for k can be obtained from equation (iv) (Barned et.al 2009) i.e.

$\frac{p}{(p-k)} = P_0e^{rt}/(P_0 - k)$, Cross multiply $P\big((P_0 - k)\big) = (P_0e^{rt})(P - k)$

We can get the parameters of the model from the population data (census of the local government of Sokoto south.

3.2 PRESENTATION OF DATA

The population of the people of Sokoto South Local Government Area (2010 -2016)

The analysis here was based on the population of Sokoto South Local Government which gives the estimate population of Sokoto South from 2010 – 2016 going by equation (iii) i.e. $P(t) = P_0e^{rt}$ where, P_0 is representing the initial population, $P(t)$ is the present or total population size K is growth rate constant and t is the time interval between the period of the population growth without loss of generality we can state that:

$P_0 = 224681$ and $P(t) = 272239$ if t is the initial time, then at 6 years interval = 6 Then k is to be determined.

$P(t) = P_0e^{kt}$

$224681e^{6k} = 272239 \ = e^{6k} = \frac{272239}{224681} \ = 6k = \ln 1.2117 \ k = 0.032$

From the previous problems i.e. (problem 1-6) and this analysis it implies that $r = 0.032$ is the growth rate per year, hence the percentage rate of growth in Sokoto South Local Government is $0.032 * 100 = 3.2\%$. Based on the census of the population of Sokoto South Local Government, the maximum number of individuals in which the local government can carry will be calculated from equation (vii).Hence,

$$K = \frac{PP_0e^{rt} - PP_0}{P_0e^{rt} - P}$$

Where P(t)= 272239 = P, P_0 = 224681, r = 0.032 and t = 6.

K = <u>272239x224681e$^{0.032x6}$- 272239 x 224681</u>

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$224681e^{0.032x6} \quad - \quad 272239$

$\Rightarrow \quad k = 1858432481$

This implies that, the realistic resources are exhausted when the population attains the equilibrium value i.e. when $P(t) = k$ and the population become more than the local government can carry resulting to competition for space, subsequent land dispute, disease e.g. epidemic and finally over use of basic infrastructure.

ANALYSIS OF THE FUTURE POPULATION OF SOKOTO SOUTH LOCAL GOVERNMENT AREA

As we have seen how the population is increasing by the percentage per year in problem 7-12, we can now make the analysis for future population of Sokoto South Local Government and make assumptions about predicted population. We can now predict and project the population of Sokoto South Local Government and relate it with the carrying capacity of the local government.

Comparing with the environment, we have the required equation to forecast the future population using equation (vi) as follows:

$$P((t)) = \frac{k}{((p_0 - 1))e^{-rt} + 1}$$

Now, we test the reality of this model by investigating whether the required population in 2016 will correspond with the one compiled by National Population Commission (NPC).

Thus, $P_0 = 224681$

$P = 272239$, $r = 0.032$, $k = 1858432481$, $t = 6$

$$\Rightarrow P(2016) = \frac{1858432481}{\left(\frac{1858432481}{224681} - 1\right) e^{-0.032x6} + 1}$$

P (2016) = 272239

Hence, the model is reliable to predict the future population of Sokoto South Local Government from 2019 to 2030.

By applying equation (vi)

$$P(t) = \frac{k}{\left[\frac{k}{P_0} - 1\right] e^{-rt} + 1}$$

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4.2 ANALYSIS OF WHEN THE POPULATION OF SOKOTO SOUTH LOCAL GOVERNMENT REACH IT CARRYING CAPACITY.

Now, let analyze when the population will reach its carrying capacity i.e. k using equation (iii) as follows: $P(t) = P_0e^{rt}$

When $P(t) = 1858432481$

$$P_0 = 224681$$

and $r = 0.032$, then we can find the value of t

$$1858432481 = 224681e^{0.032t}$$

$$\ln 224681 + 0.032t = \ln 1858432481$$

$$0.032t = \ln 185432481 - \ln 224681$$

$$0.032t = 21.3430 - 12.3224$$

$$t = \frac{9.0206}{0.032}$$

t = 281.9 = 282, t = 282 years

Hence, the population 1858432481 (i.e. carrying capacity k) reach in (2010+282) = 2292. This shows that in the year 2292 Sokoto South Local Government reach a population that is more than it can carry, which result to competition for spaces and land dispute and many more. The table below summarized the estimated population of Sokoto South Local Government from 2019 to 2035.

Table 4.1

Year	Total Population
2019	299,643
2020	309,349
2021	309,460
2022	329,846
2023	340,559
2024	351,642
2025	363,074
2026	374,878
2027	387,066
2028	299,649
2029	412,642
2030	426,057
2031	439,650
2032	454,209
2033	468,975
2034	484,221
2035	499,963

Predicted Population of Sokoto South Local Government.

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SUMMARY

In this research we considered methods and way of predicting the future population as well as the carrying capacity of the Study area using the formula of Malthusian law of population. In the research also some derivations of the formulae was achieved.

The future population of the Sokoto South Local Government in the next eighteen years (18yrs) was accomplished, this will really help the Leadership of the Local Government in planning and the decision making, thereby providing clear picture of the expected population size for the next 18 years and hence the needed security, Healthcare, infra-structure and human capital development will be adequately taken care up for the wellbeing of the people under the study area.

CONCLUSION

Based on the results obtained from the data analysis National Population Commission for Sokoto South Local Government Sokoto State, it was found that the Sokoto South Local Government when it attains the equilibrium i.e. $P(t) = k$ (or in other word when it reaches the population 'k' i.e. the carry capacity of the Local Government), in essence the resources are exhausted and the population become larger than the local government can contain. And this will bring about competition for space, land dispute, diseases out break and many other survival challenges.

This carrying capacity is $k = 1{,}858{,}432{,}481$ (i.e. one billion, eight hundred and fifty eight million, four hundred and thirty two thousand, four hundred and eighty one).

This population will be reached based on the prediction been made and formula used in the year when t=282 (i.e. in the year 2292).

This shows when $P\big((2292)\big) = 1{,}858{,}432{,}481$

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