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APPLICATION OF DIFFERENTIAL EQUATIONS IN PHYSICS

Alfred H. Mishi, Arigu I. Sabari, Deborah A. Amos, Chinedu F. Egbogu, Charity A. Kuje, John O. Ojosipe

Alfred H. Mishi : Physics Department, University of Jos, Nigeria. E-mail:fredsteingenius@gmail.com
Arigu I. Sabari : Physics Department, University of Jos, Nigeria. E-mail:arigusabari@gmail.com
Deborah A. Amos : Physics Department, University of Jos, Nigeria. E-mail:debidigun@gmail.com
Chinedu F. Egbogu : Physics Department, University of Jos, Nigeria. E-mail:egboguc509@gmail.com
Charity A. Kuje : Physics Department, University of Jos, Nigeria. E-mail:charitykuje01@gmail.com
John O. Ojosipe : Physics Department, University of Jos, Nigeria. E-mail:johnojosipe@gmail.com

KeyWords

Differential Equations, Mechanics, Electronics, Nuclear Physics, Modern Physics, Grad-Shafranov Equation, Lagrange's Formulation

ABSTRACT

In this paper, we discuss about some applications of differential equations in physics. We look at lagrangian mechanics. Lagragian mechanics is widely used to solve mechanical problems in physics and when Newton's formulation of classical mechanics is not convenient. Lagragian mechanics applies to the dynamics of particles, while fields are described using a Lagragian density. We also look at simple electric circuit problems. Finally we look at the application of differential equations in Modern and Nuclear physics. Nuclear fusion is a thermonuclear reaction in which two or more light nuclei collide together to form a larger nucleus, releasing a great amount of binding energy the in the process. Fusion and fission are natural processes that occur in stars. Fission is the process in which an unstable nucleus splits into two nuclei over a period of time or by induced fission of a neutron bombarding a radioactive atomic nucleus. In stars, it is understood that the fusion-fission process provides a near constant source of energy from proton-proton chain reactions. We look at how the Grad-Shafranov equation plays a pivotale role in the operation of tokamak and stellar reactors.

INTRODUCTION

We may trace the origin of differential equations back to Newton in 1687 and his treatise on the gravitational force and what is known to us as Newton's second law in dynamics. Newton had most of the relations for his laws ready 22 years earlier, when according to legend he was contemplating falling apples. However, it took more than two decades before he published his theories, chiefly because he was lacking an essential mathematical tool, differential calculus. Needless to say, differential equations pervade the sciences and are to us the tools by which we attempt to express in a concise mathematical language the laws of motion of nature. We uncover these laws via the dialectics between theories, simulations and experiments, and we use them on a daily basis which spans from applications in engineering or financial engineering to basic research in for example biology, chemistry, mechanics, physics, ecological models or medicine.

DEFINITION

A differential equation is an equation which contains one or more terms which involve the derivatives of one variable (dependable variable) with respect to the other variable (independable variable)

$$\frac{dx}{dt} = v(x, t)$$

Here "t" is an independable variable and "x" is a dependable variable.

A differential equation that contains derivatives which are either partial derivatives or ordinary derivatives. The derivatives represent a rate of change, and the differential equation describes a relationship between the quantity that is continuously varying and the speed of change.

TYPES OF DIFFERENTIAL EQUATION

- Ordinary Differential Equations
  An ordinary differential equation (or ODE) is a relation that contains functions of only one independent variable, and one or more of its derivatives with respect to that variable. A simple example is Newton's second law of motion, which leads to the differential equation
  Newton's second law, the time-independent Schro'dinger equation, and the equations governing the generation of spherically symmetric electromagnetic and gravitational fields, are a few examples of ordinary differential equation.
  Ordinary differential equations arise in many different contexts including geometry, mechanics, astronomy and population modelling. Many famous mathematicians have studied differential equations and contributed to the field, including Newton, Leibniz, the Bernoullis, Riccati, Clairaut, D'Alembert and Euler.

  $$m\frac{d^2x}{dt^2} = -kx$$

- Partial Differential Equations
  A partial differential equation (or briefly a PDE) is a mathematical equation that involves two or more independent variables, an unknown function (dependent on those variables), and partial derivatives of the unknown function with respect to the independent variables. The order of a partial differential equation is the order of the highest derivative involved.
  Partial differential equations are used to mathematically formulate, and thus aid the solution of, physical and other problems involving functions of several variables, such as the propagation of heat or sound, fluid flow, elasticity, electrostatics, electrodynamics, wave equation, e.t.c

$$ih\frac{\partial\psi(x,t)}{\partial t}=\frac{h^2}{2m}\left(\frac{\partial^2\psi(r,t)}{\partial x^2}+\frac{\partial^2\psi(r,t)}{\partial y^2}+\frac{\partial^2\psi(r,t)}{\partial z^2}\right)+V(x)\psi(x,t)$$

- Linear Differential Equations
  A differential equation is called linear if there are no multiplications among dependent variables and their derivatives. In other words, all coefficients are functions of independent variables.

$$\frac{dy}{dt}=g^3(t)y(t)$$

- Non – Linear Differential Equations
  Differential equations that do not satisfy the definition of linear are non-linear.

$$\frac{dy}{dt}=g^3(t)y(t)-g(t)y^2(t)$$

- Homogeneous Differential Equations
  A differential equation is homogeneous if every single term contains the dependent variables or their derivatives.

$$\frac{dy}{dx}=f\left(\frac{y}{x}\right)$$

- Non – homogenous Differential Equations
  Differential equations which do not satisfy the definition of homogeneous are considered to be non-homogeneous.

$$\frac{d^2y}{dx^2}+p(x)\frac{dy}{dx}+q(x)y=g(x)$$

APPLICATION OF DIFFERENTIAL EQUATION IN PHYSICS

This section describes the applications of Differential Equation in the area of Physics. Differential Equation is widely used in following:

a. Electronics:

Electronics comprises of the physics, engineering, technology and applications that deal with the emission, flow, and control of electrons in vacuum and matter. It can also be said to be a branch of physics and technology concerned with the design of circuits using transistors and microchips, and with the behaviour and movement of electrons in semiconductor, conductor, vacuum, or gas.

Differential Equation is widely used by physicists to solve quickly problems occurring in the analysis of electronic circuits.

b. Mechanics:

Mechanics is the area of physics concerned with the motions of macroscopic objects. Forces applied to objects results in displacements, or changes of an object's position relative to its environ. It is also a branch of classical mechanics that deals with particles that are either at rest or are moving with velocities significantly less than the speed of light. Mechanics has especially often been viewed as a model for other exact sciences. Essential in this aspect is the extensive use of mathematics in theories, as well as the decisive role played by experiment in generating and testing them.

Differential Equation is used to simplify calculations in Rectilinear Motion, Vertical Motion, Elastic String, Simple Harmonic Motion, Pendulum, and Projectile and so on. Other famous differential equations are Lagrange's Formulation.

c. Modern and Nuclear Physics:

Nuclear physics is a branch of physics that deals with the structure of the atomic nucleus and the radiation from unstable nuclei. Quantum theory is needed for the understanding of nuclear structure. Nuclear fusion is a thermonuclear reaction in which two or more light nuclei collide together to form a larger nucleus, releasing a great amount of binding energy the in the process. Fusion and fission are natural processes that occur in stars. Fission is the process in which an unstable nucleus splits into two nuclei over a period of time or by induced fission of a neutron bombarding a radioactive atomic nucleus. In stars, it is understood that the fusion-fission process provides a near constant source of energy from proton-proton chain reactions. Although a fusion reaction generates more energy than a fission reaction, modern nuclear power plants utilize fission processes due to the stability of the fission reaction, convenience, and cost of production. If nuclear fusion could be produced in a commercial setting, it could provide 3-4 times the energy a fission reaction generates. Fusion material such as deuterium, a key component in thermonuclear reactions, can be distilled from seawater providing a virtually infinite and promising source of energy in the future. The understanding and development of thermonuclear reactions and reactors is accomplished by the aid of differential equations. Engineers and scientists are able to observe behaviours, such as mass conservation, hydrostatic equilibrium states, and energy generation, of induced nuclear reactions from differential equations when developing fusion reactors. Nuclear fusion reactors are undergoing development to replace obsolete nuclear fission reactors. Two potential types of fusion reactors in development are laser ignition and magnetic confinement.

Modern physics is a branch of physics that helps to understand the underlying processes of the interactions with matter, utilizing the tools of science and engineering. It consists of classical physics, the standard model of physics and theoretical physics including quantum physics, relativity and more.

At the end of the nineteenth century, many scientists believed that they had learned most of what there was to know about physics. Newton's laws of motion and his theory of universal gravitation, Maxwell's theoretical work in unifying electricity and magnetism, the laws of thermodynamics and kinetic theory, and the principles of optics were highly successful in explaining a variety of phenomena.

As the nineteenth century turned to the twentieth, however, a major revolution shook the world of physics. In 1900 Planck provided the basic ideas that led to the formulation of the quantum theory, and in 1905 Einstein formulated his brilliant special theory of relativity. The excitement of the times is captured in Einstein's own words: "It was a marvelous time to be alive." Both ideas were to have a profound effect on our understanding of nature. Within a few decades, these two theories inspired new developments and theories in the fields of atomic physics, nuclear physics, and condensed-matter physics.

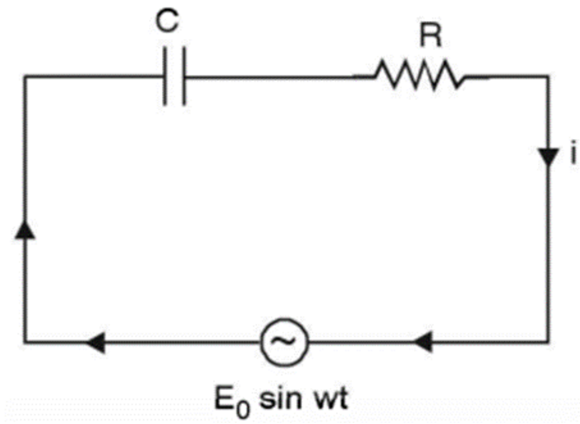
Some of the examples of problems in physics in which differential equations are used to solve are presented below. The following examples highlights the importance of differential equations in different fields of physics.

Differential Equations in Simple Electric Circuits:

1. The equation of the E.M.F for an electric circuit with a current i , resistance R, and a condenser of capacity C, arranged in series, is given by

$$E = Ri + \int \frac{i}{C} dt$$

Find the current at any time t, when $E = E_0 \sin wt$.



Solution:
From the equations above we have that,

$$Ri + \int \frac{i}{C} dt = E_0 \sin wt \quad 1$$

We differentiate with respect to t on both sides, we obtain that,

$$R\frac{di}{dt} + \frac{i}{C} = E_0 w \cos wt \quad 2$$

Rearranging we obtain,

$$\frac{di}{dt} + \frac{i}{RC} = \frac{E_0}{R} w \cos wt \quad 3$$

$$\frac{dy}{dx} + Qy = P \quad 4$$

Comparing (3) and (4), $Q = \frac{i}{RC}$ and $P = \frac{E_0}{R} wcoswt$

Integrating factor, $I.F = e^{\int Qdt} = e^{\int \frac{1}{RC} dt} = e^{\frac{t}{RC}}$
Multiply both sides of equation (4) by the integrating factor

$$e^{\frac{t}{RC}}\left(\frac{di}{dt} + \frac{i}{RC}\right) = e^{\frac{t}{RC}}\left(\frac{E_0}{R} wcoswt\right) \quad 5$$

Integrating both sides of equation 5

$$e^{\frac{t}{RC}}(i) = \frac{E_0 w}{R} \int \cos wt \, e^{\frac{t}{RC}} dt + A \quad 6$$

Integrating the R.H.S of equation (6) by parts we have

$$e^{\frac{t}{RC}}(i) = \frac{E_0 w}{R} \frac{e^{\frac{t}{RC}}}{\frac{1}{R^2C^2} + w^2}\left(\frac{1}{RC}\cos wt + w \sin wt\right) + A \quad 7$$

$$e^{\frac{t}{RC}}(i) = \frac{E_0 w}{R} \frac{e^{\frac{t}{RC}} R^2 C^2}{1 + R^2 C^2 w^2} \left(\frac{1}{RC} \cos wt + k \sin wt \right) + A \qquad 8$$

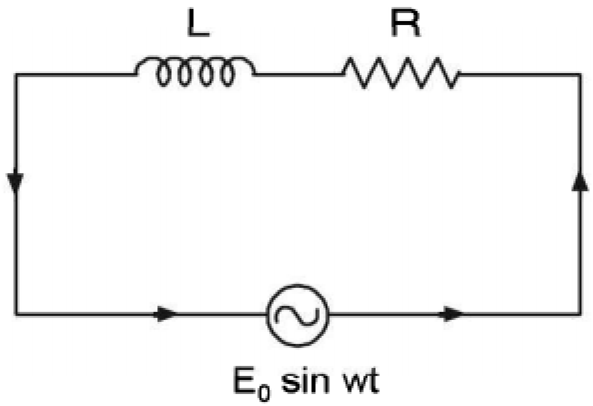
Rearranging and writing equation (8) in terms of "i" we have,

$$(i) = E_0 w \frac{C}{1 + R^2 C^2 w^2} (\cos wt + w\, RC \sin wt) + A e^{\frac{-t}{RC}} \qquad 9$$

Equation (9) shows the current at any time t.

2. Simplify the equation $L \frac{di}{dt} + Ri = E_0 \sin wt$

Taking note that $R, L\ and\ E_0$ are constants, further explain a case when t tends to infinity.



Solution:

We have from the question that,

$$L \frac{di}{dt} + Ri = E_0 \sin wt \qquad 1$$

Rearrange equation (1),

$$\frac{di}{dt} + \frac{R}{L} i = \frac{E_0}{L} \sin wt \qquad 2$$

But,

$$\frac{dy}{dx} + Qy = P \qquad 3$$

Compare equation (2) and (3), $Q = \frac{R}{L}\ and\ P = \frac{E_0}{L} \sin wt$

Integrating Factor, $I.F = e^{\int Q dt} = e^{\int \frac{R}{L} dt} = e^{\frac{R}{L} t}$

Multiplying and integrating both sides of equation (2) we obtain,

$$i\left(e^{\frac{R}{L}t}\right) = \frac{E_0}{L}\int e^{\frac{R}{L}t}\sin wt\, dt + B \qquad 4$$

From,

$$\int e^{bx}\sin ax\, dx = \frac{e^{bx}}{\sqrt{b^2+a^2}}\sin\left(ax - \cot\frac{a}{b}\right) \qquad 5$$

We rewrite equation (4) as,

$$i\left(e^{\frac{R}{L}t}\right) = \frac{E_0}{L}\frac{e^{\frac{R}{L}t}}{\sqrt{\frac{R^2}{L^2}+w^2}}\sin\left(wt - \cot\frac{Lw}{R}\right) + B \qquad 6$$

Writing equation (6) in terms of current i,

$$i = \frac{E_0}{\sqrt{R^2+w^2L^2}}\sin\left(wt - \cot\frac{Lw}{R}\right) + Be^{\frac{-R}{L}t} \qquad 7$$

When $t \to \infty$, then $Be^{\frac{-R}{L}t} \to 0$

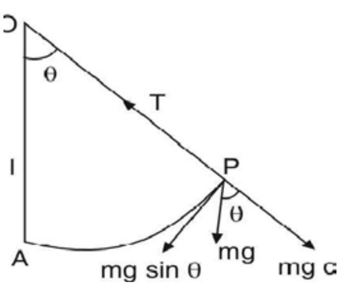
Therefore the current i is

$$i = \frac{E_0}{\sqrt{R^2+w^2L^2}}\sin\left(wt - \cot\frac{Lw}{R}\right) \qquad 8$$

Differential Equations in Mechanics:

1. An object of mass m suspended vertically by a string oscillating under the force of gravity forms a simple pendulum

We consider a string of length l, let O be the fixed point, and A be the initial position of the object having a displacement position P at any time t, then the acting forces on the object are the vertically downward weight mg and the tension T in the string.



From the figure above we see that,
Tension, $T = mgcos\theta$
Restoring force $= m\frac{d^2x}{dt^2} = -mgsin\theta$

$$\frac{d^2x}{dt^2} = -gsin\theta \dots \dots \dots 1$$

If $\theta \cong sin\theta$, rewriting equation (1) we get,

$$\frac{d^2x}{dt^2} = -g\theta$$

$$= -g\frac{x}{l}$$

$$\Rightarrow D^2x + g\frac{x}{l} = 0$$

$$\Rightarrow \left(D^2 + \frac{g}{l}\right)x = 0$$

$$the\ auxillary\ equation\ becomes, m^2 + \frac{g}{l} = 0$$

$$m = \pm i\sqrt{\frac{g}{l}}$$

Therefore the value of displacement x is.

$$x = a_1 cos\sqrt{\frac{g}{l}}t + a_2 sin\sqrt{\frac{g}{l}}t \dots \dots \dots 2$$

Equation (2) shows that the motion of the object is a simple harmonic. Therefore its Period of Oscillation will be $T = 2\pi\sqrt{\frac{l}{g}}$

2. Lagrange's formulation

Recall from newton's second law

$$F = \frac{dP}{dt}$$

$$F_i = \frac{dP_i}{dt} = \dot{P} \qquad \Rightarrow F_i - \dot{P}_i = 0$$

Where $F_i\ and\ \dot{P}_i$ are the applied and reversed effective force respectively

We can write the workdone as

$$W = (F_i - \dot{P}_i).\delta r_i = 0 \qquad \Rightarrow W = \sum_i (F_i^a + F_i^b - \dot{P}_i).\delta r_i = 0$$

$$\Rightarrow \sum_i (F_i^a - \dot{P}_i).\delta r_i = 0\ (D'Alembert\ Principle\ for\ a\ dynamic\ body) \quad \Rightarrow F_i^a\ is\ the\ actual\ force$$

The position vector of an ith rigid body can be given as:

$$r_i = r_i(q_1, q_2, q_3, \dots q_f, t)$$

The rigid body can be in equilibrium given by D'Alembert principle:

$$\sum_i (F_i - \dot{P}_i).\delta r_i = 0$$

$$\sum_i F_i.\delta r_i = \sum_i \dot{P}_i.\delta r_i$$

$$\sum_i F_i \delta r_i = \sum_i m_i.\ddot{r}_i.\delta r_i \dots \dots 1$$

For virtual workdone, it occurs at time $t = 0 \quad i.e$

$$\delta r_i = \frac{\partial r_i}{\partial q_1}\delta q_1 + \frac{\partial r_i}{\partial q_2}\delta q_2 + \dots + \frac{\partial r_t}{\partial q_j}\delta q_j$$

$$\delta r_i = \sum \frac{\partial r_i}{\partial q_j} \delta q_j$$

where the term δt is absent because the virtual displacement is assumed to take

place only in the co-ordinates and at the particular instant. Then we have that

$$\sum_i \sum_j F_i \frac{\partial r_i}{\partial q_j} \delta q_j = \sum_i \sum_j m_i \ddot{r}_i \frac{\partial r_i}{\partial q_j} \delta q_j$$

$$\sum_j \left(\sum_i F_i \frac{\partial r_i}{\partial q_j} \right) \delta q_j = \sum_{i,j} m_i \ddot{r}_i \frac{\partial r_i}{\partial q_j} \delta q_j$$

$$\sum_j Q_j \delta q_j = \sum_{i,j} m_i \ddot{r}_i \frac{\partial r_i}{\partial q_j} \delta q_j$$

where

$$Q = \sum F_i \frac{\partial r_i}{\partial q_j}$$

are called the components of generalized forces. We then consider

$$\frac{d}{dt}\left(\dot{r}_i \frac{\partial r_i}{\partial q_j} \right) = \ddot{r}_i \frac{\partial r_i}{\partial q_j} + \dot{r}_i \frac{d}{dt}\left(\frac{\partial r_i}{\partial q_j} \right)$$

Substituting above we have

$$\sum_j Q_j \delta q_j = \sum_{i,j} m_i \left[\frac{d}{dt}\left(\dot{r}_i \frac{\partial r_i}{\partial q_j} \right) - \dot{r}_i \frac{d}{dt}\left(\frac{\partial r_i}{\partial q_j} \right) \right] \delta q_j$$

but

$$\dot{r}_i = \sum_k \frac{\partial r_i}{\partial q_k} \dot{q}_k + \frac{\partial r_i}{\partial t}$$

Differentiating this with respect to q_j we get

$$\frac{\partial \dot{r}_i}{\partial q_j} = \sum_k \frac{\partial^2 r_i}{\partial q_k \partial q_i} \dot{q}_k + \frac{\partial^2 r_i}{\partial t \partial q_j}$$

we also have that

$$\frac{d}{dt}\left(\frac{\partial r_i}{\partial q_j}\right) = \sum_k \frac{\partial^2{}_{r_i}}{\partial q_j \partial q_k}\dot{q}_k + \frac{\partial^2{}_{r_i}}{\partial q_j \partial t}$$

From the two equations above we have that

$$\frac{\partial \dot{r}_i}{\partial q_j} = \frac{d}{dt}\left(\frac{\partial r_i}{\partial q_j}\right)$$

Therefore in general we have that

$$\frac{\partial}{\partial q_j}\left(\frac{d}{dt}\right) = \frac{d}{dt}\left(\frac{\partial}{\partial q_j}\right)$$

Rewriting we have that

$$\sum_j Q_j \delta q_j = \sum_{i,j}\left[\frac{d}{dt}\left(m_i v_i \frac{\partial v_i}{\partial \dot{q}_j}\right) - m_i v_i \frac{\partial v_i}{\partial q_j}\right]\delta q_j$$

$$\Rightarrow \sum_j Q_j \delta q_j = \sum_j \left[\frac{d}{dt}\left(\frac{\partial}{\partial \dot{q}_j}\left(\sum_i \frac{1}{2} m_i v_i^2\right)\right) - \frac{\partial}{\partial q_j}\left(\sum_i \frac{1}{2} m_i v_i^2\right)\right]\delta q_j$$

or

$$\sum_j Q_j \delta q_j = \sum_j \left[\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j}\right]\delta q_j$$

where

$$T = \frac{1}{2}\sum_i m_i v_i^2$$

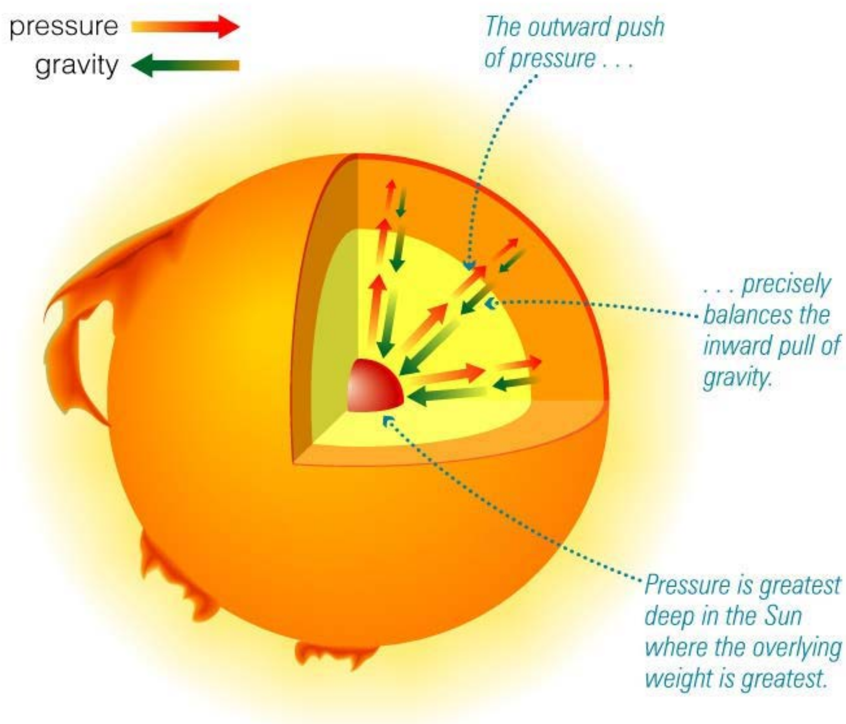
is the total kinetic energy of the system of particles

$$\Rightarrow \sum_j \left[\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} - Q_j\right]\delta q_j = 0$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j, \quad j = 1, 2, 3, \dots, n$$

Differential Equation in Modern and Nuclear Physics:

Natural Fusion in Stars



Source: http://lasp.colorado.edu/education/outerplanets/solsys_star.php

At radius r in a static, spherically symmetric star:

Mass conservation

- $\frac{dm}{dr} = 4\pi r^2 \rho$

Hydrostatic equilibrium

- $\frac{dP}{dr} = -\frac{Gm}{r^2}\rho$

Energy transport due to radiation (only)

- $\frac{dT}{dr} = -\frac{3}{4ac}\frac{k\rho}{T^3}\frac{L}{4\pi r^2}$

Energy generation

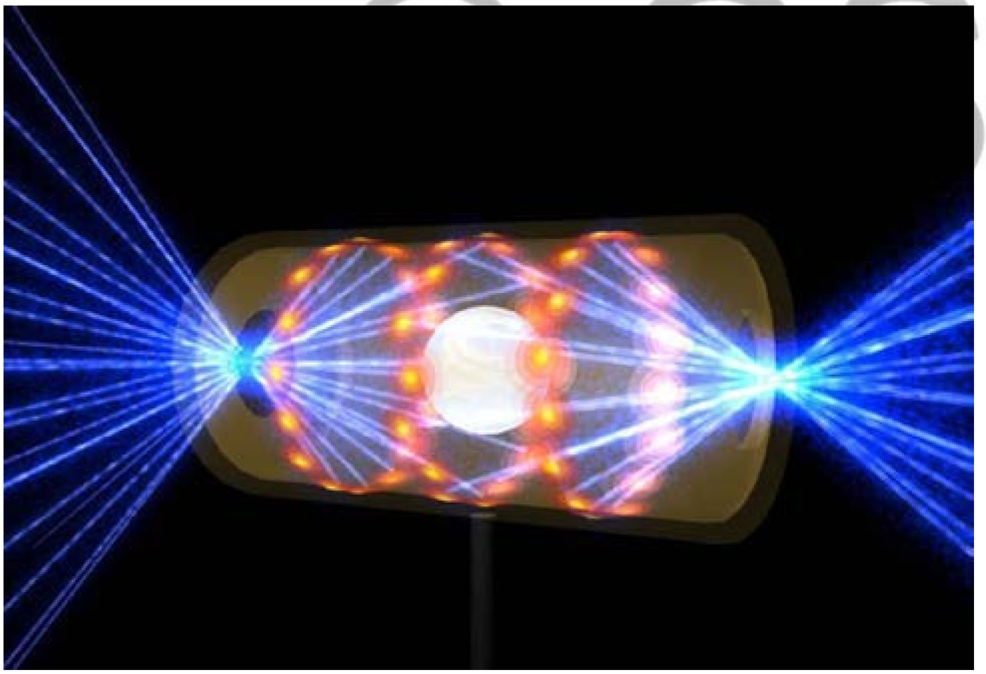
- $\frac{dL}{dr} = 4\pi r^2 \rho q$

Where: m=enclosed mass; ρ=density; P=pressure; G=gravitational constant; T=temperature; κ=opacity; L=luminosity; a=radiation constant; c=speed of light in vacuum; and q=rate of energy generation per unit mass.

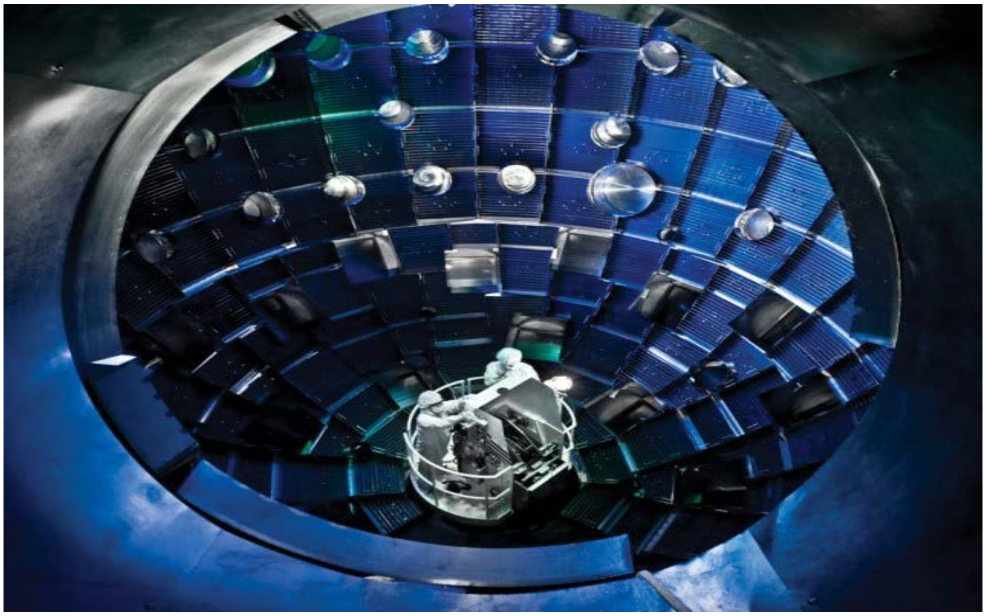
Laser Ignition Reactors

Lasers are fired at a pellet of deuterium-tritium, fusing it together.

Source: https://www.llnl.gov/str/pdfs/07_99.1.pdf



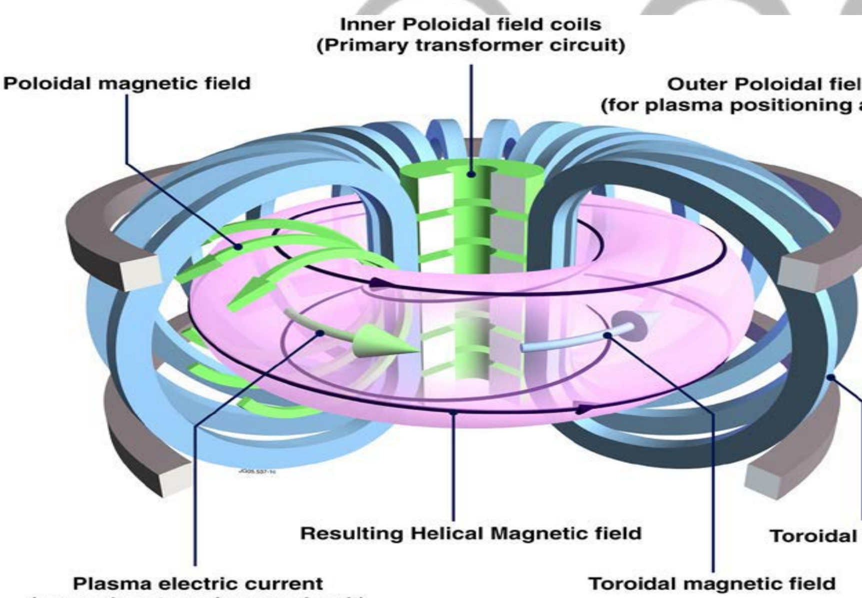
Source:http://monimega.com/blog/2013/10/10/us-fusion-lab-almost-breaks-even-takes-a-big-step-towards-clean-limitless-power/



Source:http://monimega.com/blog/2013/10/10/us-fusion-lab-almost-breaks-even-takes-a-big-step-towards-clean-limitless-power/

<u>Magnetic Confinement Reactors</u>

Uses magnetic fields to confine hot fusion materials in the form of a plasma toroid. Tokamak



Source: http://www.efda.org/glossary/poloidal-field-coils/

Stellarator

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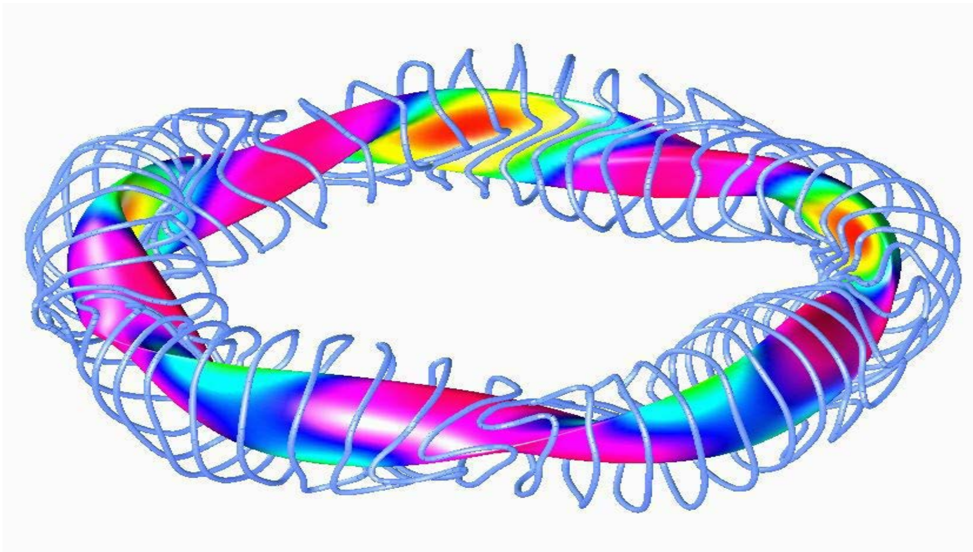
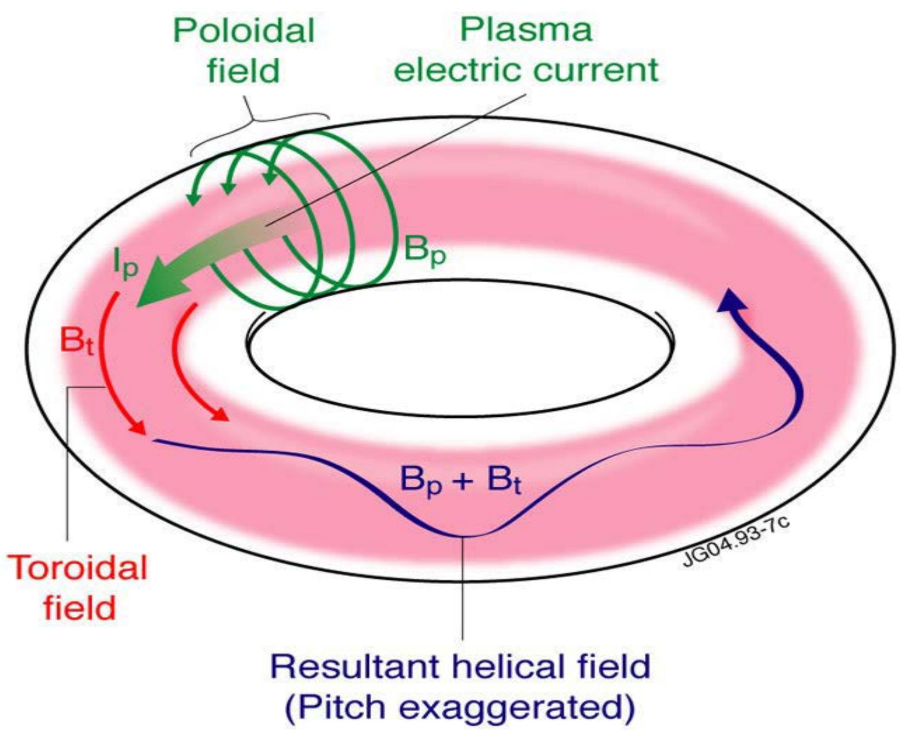


Source: http://www.physics.ucla.edu/icnsp/Html/spong/spong.htm



Source: http://www.efda.org/fusion/focus-on/plasma-heating-current-drive/ohmic-heating/

The Grad-Shafranov Equation

$$\Delta * \psi = -\mu_0 R^2 \frac{dp}{d\psi} - \frac{1}{2}\frac{dF^2}{d\psi}$$

$$\Delta * \psi = R^2\, \vec{\nabla}.\left(\frac{1}{R^2}\vec{\nabla}\psi\right) = R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial \psi}{\partial R}\right) + \frac{\partial^2 \psi}{\partial Z^2}$$

$$F(\psi) = RB_{\phi}$$

$$\vec{B} = \frac{1}{R}\nabla\psi \times \hat{e}_{\phi} + \frac{F}{R}\hat{e}_{\phi}$$

$$\mu_0\vec{J} = \frac{1}{R}\frac{dF}{d\psi}\nabla\psi \times \vec{e}_{\phi} - \frac{1}{R}\Delta * \psi\vec{e}_{\phi}$$

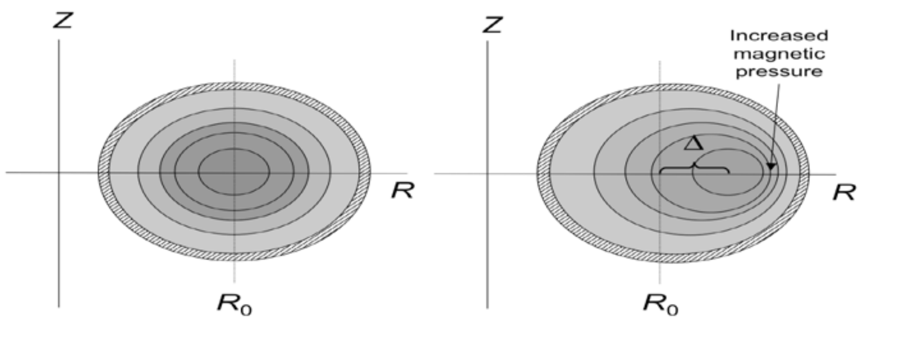
$$\psi = flux$$

$$\vec{B} = magnectic\ field$$

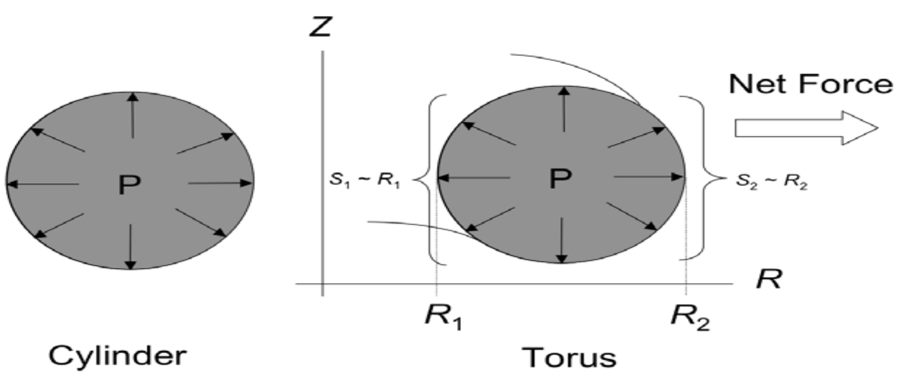
$$\mu_0\vec{J} = current$$

$$\mu_0 = magnetic\ permeability\ constant$$

$$p(\psi) = pressure$$



Source: http://www.physics.wisc.edu/grads/courses/726-f07/files/Section_18_Toroidal_Eq_02.pdf



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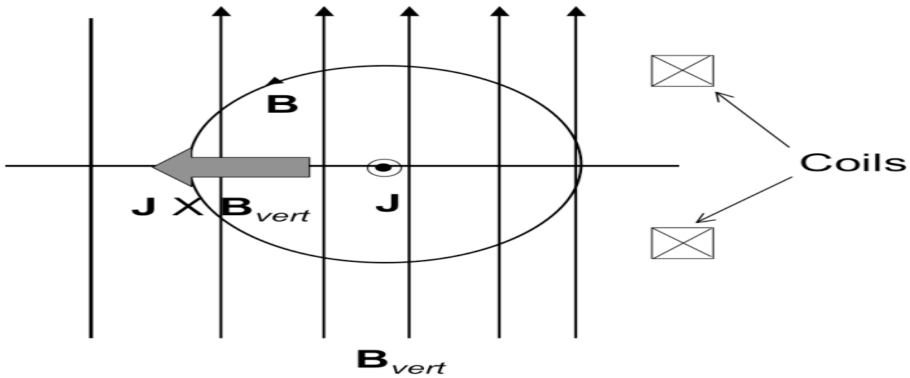
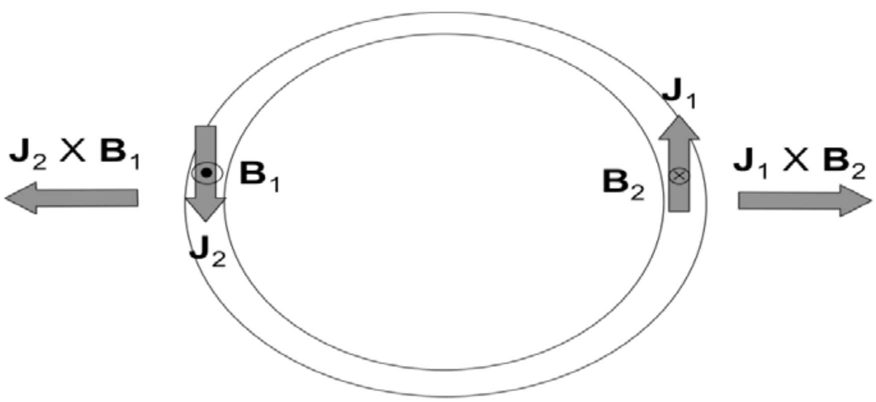


Source: http://www.physics.wisc.edu/grads/courses/726-f07/files/Section_18_Toroidal_Eq_02.pdf



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Conclusion

Through this paper we present the application of Differential Equations in some fields of physics, like Electronics, Mechanics, and Modern and Nuclear Physics. Besides these, Differential Equation play an important role in modelling virtually every physical, technical, or biological process, from celestia motion, to bridge design, to interactions between neurons. Differential Equations such as those used to solve real-life problems may not necessarily be directly solvable, i.e. do not have closed form solutions. Instead, solutions can be approximated using numerical methods.

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References

[1] B.V. Ramanna, Higher Engineering Mathematics, Tata Mc-Graw Hill Publication

[2] H.K.Dass and Er. Rajnish Verma, Higher Engineering Mathematics, S. Chand Publication, New Delhi.

[3] M. Peer Mohamed, Department of Mathematics, Marudupandiyar College, Thanjavar, IJSRP, Vol 6, Aug 2016, ISSN 2250-3553.

[4] http://www.astro.caltech.edu/~george/ay20/Ay20-Lec7x.pdf

[5] http://www-naweb.iaea.org/napc/physics/2ndgenconf/data/Proceedings%201958/papers%20Vol31/Paper33_Vol31.pdf

[6] http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=6266984

[7] http://community.vcsu.edu/facultypages/takayuki.yamauchi/My_Webpage/hpb_images/on_the_regularity_of_magnetic_flux_function_in_nuclear_fusionjournalrevised.p_neev26.pdf

[8] http://www.physics.wisc.edu/grads/courses/726-f07/files/Section_18_Toroidal_Eq_02.pdf

[9] http://en.m.wikipedia.org/wiki/Differential_equation

[10] http://en.m.wikipedia.org/wiki/Lagrangian_mechanics

[11] https://www.efunda.com/math/ode/generalterms.cfm

[12] https://www.efunda.com/math/ode/generalterms.cfm

[13] https://www.cs.mcgill.ca/~rwest/wikispeedia/wpcd/wp/o/Ordinary_differential_equation.htm

[14] http://www.personal.psu.edu/djs75/files/poster_student_2014c.pdf