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A Brief Overview on Explaining Prime Numbers

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Introduction

Preliminary numbers among the other numbers are interesting numbers that scientists have discussed and still have their views on .Until now, there is no clear order in which these numbers can be arranged .There are large optional gaps between consecutive primes. In addition, larger primitive numbers are used to determine the cryptographic key for security.But, unfortunately, there is no general formula that produces only prime.

Even most guesses about the initial numbers are still dumb. For example, we can see the difference between prime numbers 2 and 3are 1 and between 3 and 5 is two and between 7 and 11 is 4 and the difference between 11and 13, 19, 21 are 2.At the same time, there is a theorem that we get as many consecutive primes as we wish, and this raises a question in our minds that the primes may be finite, but there is another theorem that indicates that the prime numbers may be infinite.

Theme design

Starting the topic by mentioning some of the properties of the prime numbers that are presented in the theorem.

Theorem

There are large optional gaps between consecutive primes(1387 , شهریاری م).

Proof

The proposition is that for any positive integer n there are n sequences of n continuous integer numbersthat are not prime. In fact:

$$(n + 1)! + 2, (n + 1)! + 3, (n + 1)! + 4 \ldots\ldots\ldots\ldots, (n + 1)! + (n + 1)$$

are the sequence of n integer's number. Clearly, any number of recent sequences is not prime, because for any $2 \leq k \leq n+1$, $k/(n + 1)! + k$

Therefore, $(n+1)! +k$ has a division other than ±1 and itself.

On the other hand, the bellow theorem proof that the number of primitive numbers are infinite.

Theorem

The number of primitive number is infinite (JamesJ, 2005).

Proof

We suppose that the number of primitive number is not infinite and assume that the last primitive number is P_n.let's $P_{1,}$ P_2, P_3, ... , Pn be prime numbers. New, if $N = P_1P_2 \ldots P_n + 1$ is a number which is the product of these plus one, it is clearly seen that N is not a prime number and N > 1 is divisible by prime number. further, n is not divisible by any prime number because if the prime number P_i be the divisor of N then P_i is the divisor of $N - P_1P_2 \ldots P_n = 1$ which is a contradiction to our assumption. Hence, there is not any prime number to be divisor of 1 or it is either prime, or divisible by primes bigger than P_n.

Theorem

If n > 1 is an integer number which is not prime, then there exist a prime number p such that: p\n and p≤√n.

Proof

Since n is not prime, there are integer numbers a and b such that:

$2 \leq a \leq b < n$ and n=ab then

$n = ab \geq a^2$ or $a \leq \sqrt{n}$

Now assume that p is prime and p\a .

Then, finally we can say p\n and $p \leq a \leq \sqrt{n}$.

Theorem

If P_n is the nth prime number, then $P_n \leq 2^{2^{n-1}}$ (1382,صادق زاده د).

Proof

By using induction method, we have,

For n=1 it is clear that non equation is trough. Now assume that the non-equation is through for n > 1, then for n+1 we proof, then

$$P_{n+1} \leq P_1P_2 \ldots P_n+1 \leq 2.2^2 \ldots 2^{2^{n-1}} \leq 2^{1+2+2^2+\ldots+2^{n-1}}$$

Since $1 + 2 + 2^2 + \ldots + 2^{n-1} = 2^{n-1}$, therefore

$$P_{n+1} = 2^{2^{n-1}} + 1$$

From other hand, since for any n > 1, $1 \leq 2^{2^{n-1}}$ then we have,

$$P_{n+1} = 2^{2^{n-1}} + 2^{2^{n-1}} = 2.\ 2^{2^{n-1}} = 2^{2^n}$$

Gold Bach's Conjecture: the Gold Bach's conjecture is the oldest and the best-known unsolved problems in number theory and still is not proven but it is just stated as a conjecture. Although prime numbers are infinite, their distribution in positive integer numbers is very confusing. In the distribution of these numbers, the difference between the twoconsecutivenumbers like 11 and 13, 17 and 19 and or 1000000000061 and 1000000000063 can be 1. At the same time, there are very large intervals of integer numbers that don't have any primitive numbers.

Now, an unsolved question is created in our mind that are there so many pairs of primitive numbers which their difference is 2? These consecutive number are p and p+2 that both are prim.It follows from the above description it is resulted that primitive numbers cannot be so closed but can be arbitrary distancesfrom one another.

The Gold Bach's conjecture state that " Every even integer number of greater than two can be write in the form of sum of two primitive numbers " for instance 4=2+2, 6=3+3, 8=5+3 and 10 can be write as 10=5+5 or 10=3+7.

By proceeding this we see that there are number of ways in which even numbers can be written as sums of two prim number, 50=3+47 = 19+41 = 19 + 31. As a contract, He considered the number 1 as a prim number. In a more specific form, it is easy to see that each even number greater and equal than four can be writhe as a sum of two odd prim, as

2=1+1, 4=2+2=1+3, 6=3+3=1+5, 8=3+5=1+7,10 =3+7=5+5, 12=5+7=1+11, 14=7+7=1+13, 16=3+13=5+11. . . .

From the numerical evidence, the validity of these guesses is satisfactory, (Gold Bach's conjecture up to is right 100000) But so far, there are no proofs or examples for rejecting it. The recent results achieved so far are the result of a Russian mathematician Vinogradov which state: approximately all even numbers are the sum of two prim number. Now, if we show all even integer number m ≤ n as A(n), which is not convertible into two prim number, then we have

$$\lim_{x\to\infty} A(n)/n = 0$$

now if the Gold Bache's conjecture be through, then every odd number greater than 7 can be write in the form of three odd prim.

We consider an odd prim number n greater than 7, in this case n-3 is an even number and greater than 4.

If we can write n-3 in the form of a sum of two prim number, then n is the sum of three numbers.

A recent description by Vinogradovs's has been shown that is correct for each large odd numbers.

It follows from the result of the work of Vinogradov that every single pair of pairs can be sufficiently summed up to a sum of more than four initial numbers Also it follows from the Vinogradoff's research investigation that every large even integer can be sufficiently summed up to a sum of more than four prim number.

In fact, there is a number N, so that each even number can be written as a sum of two or more than four odd prim number. From division algorithm we have, that any positive integer is represented uniquely in the form of one of the four cases bellow:

$$4n, \qquad 4n + 1, \qquad 4n + 2, \qquad 4n + 3 \qquad , \quad (n > 0)$$

Clearly, it is visible that 4n and 4n + 2 = 2(2n + 1) are both even. So, all prim integer number is in the form of two rest mentioned above which is 4n + 1 for instance, 1,5,9,13,17,21… and the second form is 4n + 3, for instance

3, 7, 11, 15, 19, 23, …

From the above description, the following result is obtained.

Lemma: The product of multiplication of two or more prime numbers like 4n + 1 are the same (David A, 2005).

Proof: It seems appropriate to choose the product of multiplication of only two integers such as $k = 4n + 1$ and $k^{'} = 4m + 1$. Now, we would have

$$Kk^{'} = (4n + 1)(4m + 1) = 16nm + 4n + 4m +1 = 4(4nm + n + m) + 1$$

Which is the same as mentioned before.

Theorem

Infinite prim number are in the form 4n + 3.

Proof

We use indirect proof. Suppose there are finite prim number in the form 4n + 3, denoted by q_1, q_2,. . ,q_s. now consider the positive integer number bellow

$$N = 4\, q_1 q_2 . . . q_s - 1 = 4(q_1\, q_2 . . . q_s) + 3$$

And let $N = r_1 r_2 . . . r_t$ be the agent factors of N. since N is an odd integer, then for all k, $r_k \neq 0$ so r_k is in the form 4n + 1 or 4n + 3. Now, from the previous lemma we have that product of two or more prim number in the form 4n + 1 is the same as 4n + 1 form. If N is of the form 4n + 3, then at least one of r_i has the form 4n + 1. But there is no such r_i between $q_1, q_2, . . ., q_s$. So, we are faced to contradiction i.e. $1/r_i$.

Now the question arises as to whether there are infinitely many prime numbers in the form of 4n + 1? The answer to this question is positive. Bothe above cases are the executive case of the following theorem.

Theorem

If a and b are positive integer relative prim to each other, then the following arith-metic sequence

$$a,\ a + b, a + 2b, . . .$$

Has an infinite number of prim numbers (Harvey C, 1992).

Result: There is no arithmetic sequence of numbers a,a + b, a + 2b, . . . which has only a finite number of prime numbers.

Proof: Suppose p = a +nb such that p is prim. If $n_k = n + k_p$ is substituted for k = 1, 2, 3, . . . then n_k is below sequence.

$$a + n_k b = a + (n + kp)b = (a + nb) + kpb = p + kpb.$$

Since any number on the right hand side of the current relation is divisible by p,then $a + n_k b$ is also divisible by p.It is a conjecture that there is an arithmetic sequence of finite length such as above that has prime consecutive numbers.

For example, 41, 47, 53 and 251, 257, 263, 269 are the arithmetic sequence that has three and four prime consecutive number. But so far there is no function or relation to provide an arithmetic sequence that has seven primary consecutive numbers. But recently we introduce a function whose domain is positive integers and generates prime numbers. i.e. $f(n) = n^2 + n + 41$ (David M.B, 2011).

We see that this function for n = 0, 1, 2, . . . ,39 the all co-domain of the function is prime or the generate prime number, but for n = 40 and 41 it is not correct, because

$$F(40) = 40^2 + 40 + 41 = 41^2$$

$$F(41) = 41^2 + 41 + 41 = 41.43$$

It can be seen that both of these functions have a common factor of 41. Likewise if we look the next number we can see that by setting the number 42 at the domain of the function again it generate the prime number. Finally, we cannot argue that the above function produces all numbers to be primitive.

Mersenne primes

The number in the form $M_n = 2^n - 1$ is called Mersenne primes for some integer n. Mersenne offered an incorrecttheory but motivating comment on primes.In fact, Mersenne stated that $M_p$is prime for p = 2, 3, 5, 7, 13, 17, 19, 31, 67, 127 and otherwise for p < 257 is not prime or is composite.Mersenne could not pass the any proof of exam for every number he had declared to be prime, But Euler stated in his technique, shown below, that M_{31} is prime and M_{67} , M_{127} and M_{257} are composite i.e. $2^{30}(2^{31}-1)$ = 2, 305, 843, 008, 139, 952,128).Now we know that Mersenne made a few mistakes: He mistakenly concluded that the numbers $M_{67,}$ andM_{257} are prime and removed the numbers M_{61}, M_{89}, M_{107} from the prime list. Then Edouard Lucas showed that M_{67} is composite.But he could not identify the multiplicative factors.Later by American mathematician Fried Rick Nelson, a computer programmer programmed it into larger multiplication factor (Kenneth H, 2011).

There are many methods to indicate that a Mersenne number is a composite number or a prime number.One of them is described below:

Theorem

Suppose that p and q = 2p + 1 are prime number. Then either $q\backslash M_p$ or $q\backslash M_p + 2$ but not both of them.

Proof

From the Fermat's theorem we have (1381 , گولدنشتین ج).

$2^{q-1} - 1 \equiv 0(\text{mod} \quad q)$and by factorization of left hand side we achieved that

$$(2^{(q-1)/2} - 1)(2^{(q-1)/2} + 1) = (2^p -1)(2^p +1) \equiv 0(\text{mod} \quad q)$$

Then Mp $(M_p + 2)\equiv 0(\text{mod} \quad q)$, from this it is clear that both Mp and M_p+2are not divisible by q. Because then it must be 2 divided by q.

Conclusion

In fact, Prime numbers are very important and essential in our daily life and in calculating the arithmetic sums and problems. Moreover, prime number are the main branches of number theory and the most fundamental discipline of mathematics. If we really think deeply about prime numbers, it can be seen that these are prime numbers that determine all multipliers of others number. So, every number has its own unique set of prime factors.Prime factorization means to express a number using only its prime factors.

Prime numbers are a set of numbers that do not yet have an exact way to identifying them, but what we have obtained from various sources enable us to point out that how such integers are generated.By studying this research, I would like to offer compact information about prime numbers and, on the other hand, to provide further study and understanding.

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References

[1]A. David, "Number Theory", Hopking University, York, p 40, 2005.

[2] M.B David, "Elementary Number Theory", McGraw-Hill New York, p 56" 2011.

[3] J. James, "Elementary Number Theory", Cambridge University, Pp. 115, 2005.

[4] C. Harvey, "Advance Number Theory", Dover Publications Inc. p 160, 1992.

[5] H. Kenneth,"Elementary Number Theory and its Applications", Addision-wesley. Pp. 258-259, 2011.

[6]م. شهریاری “ , نظریه اعداد , دانشگاه تبریز" ,ص20, 1387.

[7] د. صادق زاده“,حل مسایل نظریه اعداد, انتشارات گلباد تبریز”, ص 189, 1382.

[8] ج. گولدشتین, مترجم: ن.آ," آشنایی با نطریه اعداد, مرکز دانشگاهی تهران”, ص 174, 1381.