





















In Table 3,  $d$  is GPH estimate,  $Sd.as$  is the asymptotic standard deviation and  $Sd.reg$  is the standard error deviation. The value of the differenced parameter  $d$  is  $0.6275 > 0.5$ , is indication of non stationarity long memory in the series of Uyo data, PH and Ibadan data have intermediate memory properties because  $d$  is  $< 0$ ; and Sokoto fractional differenced parameter is  $0.4220$  which is stationary long memory property (that is  $0 < d < 0.5$ ).

Based on that result in Table 2, we applied differencing to Uyo and Ibadan series. The differenced test results are given in Table 4.

**Table 4: Fractional and Unit Root test of Solar Radiation for Differenced Series**

City	Test	ADF				KPSS
		Intercept		Intercept and Trend		
Uyo	Test Statistic	-15.02449		-15.00591		0.108747
	Critical values: 1%	-3.447770	(<0.01*)	-3.982988	(<0.01*)	0.216000
	5%	-2.869113		-3.421983		0.146000
	10%	-2.570871		-3.133816		0.119000
Ibadan	Test Statistic	-6.561011		-7.340125		0.037755
	Critical values: 1%	-3.57446	(<0.01*)	-4.161144	(<0.01*)	0.739000
	5%	-2.923780		-3.506374		0.363000
	10%	-2.5999		-3.183002		0.347000

From table 4, ADF test accept the alternative hypothesis that the series is stationary and the KPSS agree with the null hypothesis. Also, there is a needs to carry out fractional seasonal differencing in all the city.

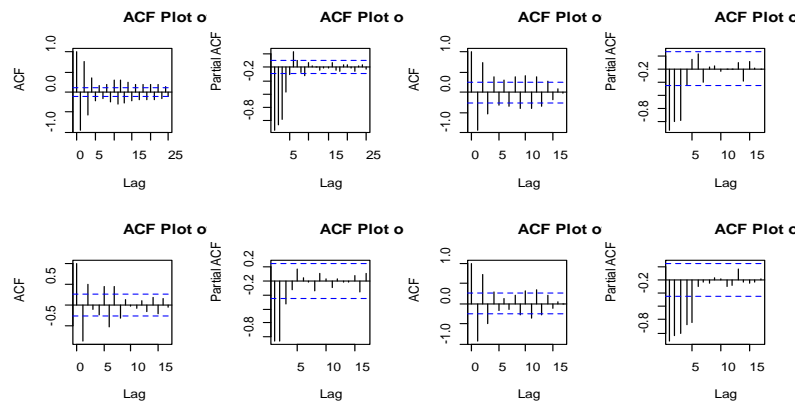


Figure 4: ACF and PACF plots of fractional seasonal differenced

### Construction of the SARIMA model

In this paper, we estimate the parameters of *SARIMA* model using maximum likelihood estimation procedure. We judge the best model base on AIC value.

**Table 5:** The results for the estimated *SARIMA* Models in the four Cities

City	Model	AIC	Log Likelihood
Uyo	<i>SARIMA</i> (1,1,1)(0,1,1) <sub>12</sub>	1210.96	-601.48
	<i>SARIMA</i> (1,1,1)(0,1,2) <sub>12</sub>	1210.29*	-600.15

	<i>SARIMA(0,1,1)(0,1,1)<sub>12</sub></i>	1217.3	-605.65
PH	<i>SARIMA(1,0,1)(0,1,1)<sub>12</sub></i>	412.88	-202.44
	<i>SARIMA(1,0,2)(0,1,1)<sub>12</sub></i>	414.70	-202.35
	<i>SARIMA(1,0,0)(0,1,1)<sub>12</sub></i>	411.57*	-202.78
Ibadan	<i>SARIMA(1,1,1)(1,1,0)<sub>12</sub></i>	375.68*	-183.84
	<i>SARIMA(0,1,1)(1,1,0)<sub>12</sub></i>	377.12	-185.56
	<i>SARIMA(1,1,1)(2,1,0)<sub>12</sub></i>	377.15	-183.58
Sokoto	<i>SARIMA(1,0,1)(0,1,1)<sub>12</sub></i>	440.43*	-216.21
	<i>SARIMA(1,0,2)(0,1,1)<sub>12</sub></i>	442.25	-216.12
	<i>SARIMA(1,0,0)(0,1,1)<sub>12</sub></i>	440.62	-217.31

Table 5 above shows the results of the estimated models of SARIMA, the model with the minimum AIC value is indicated by asterisk and it is considered to be the best model for monthly solar radiation series in the respected Cities. The estimated parameters of those models are given in table 6.

Table 6: Estimate of Parameters of of SARIMA Models for the series.

City	Models	Parameter	ar1	ma1	sma1	sma2
Uyo	<i>SARIMA(1,1,1)(0,1,2)<sub>12</sub></i>	Estimate	0.1680	-0.9423	-0.8652	-0.0883
		s.e.	0.0566	0.0227	0.0616	0.0549
PH	<i>SARIMA(1,0,0)(0,1,1)<sub>12</sub></i>	Estimate	0.5226		-0.9996	
		s.e.	0.1368		0.3470	
Ibadan	<i>SARIMA(1,1,1)(1,1,0)<sub>12</sub></i>	Estimate	0.3238	-0.8963	-0.4693	
		s.e.	0.1705	0.0865	0.1496	
Sokoto	<i>SARIMA(1,0,1)(0,1,1)<sub>12</sub></i>	Estimate	0.8601	-0.4561	-0.6060	
		s.e.	0.1622	0.2638	0.2919	

## Construction of the SARFIMA model

In SARFIMA model, the order  $(p, d, q)$  and the seasonal components  $(P, D, Q)$  are specified same as the SARIMA above. Unfortunately, the method of estimation we used to estimate the order of fractional differencing cannot estimate all parameters in the model simultaneously, and we cannot identify the parameter  $d$  and  $D$  when using GPH estimation method. Therefore, we used exact maximum likelihood estimator to estimate SARFIMA model.

This method estimates the memory parameter and the parameters of the appropriate SARFIMA Model orders simultaneously. In this work, we choose the best model based on Akaike Information Criterion (AIC). The results are shown in Table 7 and the parameters estimate for the adequate models in table 8

Table 7: Estimate of Parameters of of SARFIMA Models for the series.

City	Model	AIC	Log Likelihood
Uyo	<i>SARFIMA(1,0.1117,0)(0,0.4560,2)<sub>12</sub></i>	166.824*	-76.4122
	<i>SARFIMA(0,0.0758,1)(1,0.3519,1)<sub>12</sub></i>	179.519	-83.7594
	<i>SARFIMA(0,0.1364,1)(0,0.4580,2)<sub>12</sub></i>	167.354	-76.6772
PH	<i>SARFIMA(1, -0.3975,1)(0,0.3988,1)<sub>12</sub></i>	355.763	-169.881
	<i>SARFIMA(1, -0.4685,2)(0,0.3910,1)<sub>12</sub></i>	354.458	-169.229
	<i>SARFIMA(1, -0.2316,2)(1,0.4033,1)<sub>12</sub></i>	352.429*	-17.214
Ibadan	<i>SARFIMA(1,0.1137,1)(1,0.4657,0)<sub>12</sub></i>	316.598	-151.299
	<i>SARFIMA(1, -0.0588,1)(1,0.4491,0)<sub>12</sub></i>	315.305	-151.653
	<i>SARFIMA(1, -0.8637,1)(2,0.4720,0)<sub>12</sub></i>	314.263*	-149.132

	$SARFIMA(1, -0.4374, 1)(0, 0.3868, 1)_{12}$	382.695	-184.347
Sokoto	$SARFIMA(1, -0.5304, 2)(0, 0.3843, 1)_{12}$	384.507	-184.253
	$SARFIMA(1, -0.3688, 2)(1, 0.3519, 1)_{12}$	380.743*	-184.372

Table 8: Estimated Parameters of SARFIMA models for the series

City	Model	Parameter	Estimate	Std. Error	z-value	Pr(> z )
Uyo	$SARFIMA(1, 0.1117, 0)(0, 0.4560, 2)_{12}$	phi(1)	0.1346702	0.0891628	1.51039	0.130945
		theta.12(1)	0.2984788	0.0686513	4.34775	1.3754e-05 ***
		theta.12(2)	0.1247637	0.0571489	2.18313	0.029026 *
		d.f	0.1116659	0.0674833	1.65472	0.097982 .
		d.f.12	0.4559956	0.0222194	20.52245	< 2.22e-16 ***
		zbar	10.6114583			
		AIC	166.824			
		Log	-76.4122			
		Likelihood				
	$\sigma^2$	1.41906				
PH	$SARFIMA(1, -0.2316, 0)(0, 0.4033, 1)_{12}$	phi(1)	0.757209	0.136492	5.54765	2.8953e-08 ***
		theta.12(1)	0.000000	NA	NA	NA
		d.f	-0.231608	0.192406	-1.20375	0.22869
		d.f.12	0.403324	0.039950	10.09572	< 2.22e-16 ***
		zbar	153.291083			
		AIC	352.429			
		Log	-170.214			
		Likelihood				
			$\sigma^2$	254.446		
Ibadan	$SARFIMA(1, -0.8637, 1)(2, 0.4720, 0)_{12}$	phi(1)	0.9277675	0.0890505	10.41844	< 2.22e-16 ***
		theta(1)	-0.6459958	0.1750143	-3.69110	0.00022328 ***
		phi.12(1)	-0.2861464	0.1897894	-1.50771	0.13163002
		phi.12(2)	-0.0284297	0.2186778	-0.13001	0.89656080
		d.f	-0.8636865	0.2164948	-3.98941	6.6238e-05 ***
		d.f.12	0.4719875	0.0214476	22.00653	< 2.22e-16 ***
		zbar	142.2441710			
		AIC	314.263			
		Log	-149.132			
	Likelihood					
	$\sigma^2$	108.351				
Sokoto	$SARFIMA(1, -0.3688, 2)(1, 0.3880, 1)_{12}$	phi(1)	0.8795555	0.1219016	7.21529	5.3819e-13 ***
		theta.12(1)	0.0000000	NA	NA	NA
		d.f	-0.3687582	0.2147790	-1.71692	0.085994 .
		d.f.12	0.3879681	0.0458476	8.46213	< 2.22e-16 ***
		zbar	235.0096517			
		AIC	380.743			
		Log	-184.372			
		Likelihood				
			$\sigma^2$	418.582		

### Comparison between SARIMA and SARFIMA models performance

According to table 9 below, SARFIMA model failed to produce better forecast estimates compared to SARIMA indicated by the high values of MSE, RMSE, MAE and MAPE for the cities of Uyo, PH and Sokoto. In Ibadan SARFIMA model has a better forecast estimate than SARIMA model. The smaller the estimated error value, the better the forecasting performance of the model.

Table 9: Forecast performance measures for SARIMA model and SARFIMA model.

City	Model	MSE	RMSE	MAE	MAPE
------	-------	-----	------	-----	------

Uyo	<i>SARIMA</i> (1,1,1)(0,1,2) <sub>12</sub>	1.3287	1.1527	0.8510	8.15634
	<i>SARFIMA</i> (1,0.1117,0)(0,0.4560,2) <sub>12</sub>	1.4006	1.1853	0.8940	8.6939
PH	<i>SARIMA</i> (1,0,1)(0,1,1) <sub>12</sub>	145.4653	12.0609	8.9849	5.9344
	<i>SARFIMA</i> (1, -0.2316,0)(0,0.4033,1) <sub>12</sub>	237.0938	15.3979	12.6610	8.3903
Ibadan	<i>SARIMA</i> (1,1,1)(1,1,0) <sub>12</sub>	104.5883	10.2266	7.7164	5.7821
	<i>SARFIMA</i> (1, -0.8637,1)(2,0.4720,0) <sub>12</sub>	97.51615	9.8750	7.6418	5.6207
Sokoto	<i>SARIMA</i> (1,0,1)(0,1,1) <sub>12</sub>	339.4733	18.4248	13.5063	5.6704
	<i>SARFIMA</i> (1, -0.3688,0)(0,0.3880,1) <sub>12</sub>	390.6763	19.7655	16.1129	10.8752

## Conclusion

This work studies and analyze the nature of monthly solar radiation in four metropolises in Nigeria using SARIMA and SARFIMA processes.

Examining the two models from their AIC values, which are 1210.29, 411.57, 375.68 and 440.43 in SARIMA models while SARFIMA models have 165.492, 352.429, 314.263 and 380.743 for Uyo, PH, Ibadan and Sokoto, respectively. We observed that SARFIMA models have the least values of AIC for solar radiation compare to SARIMA models in all the cities. In terms of forecast, performance measures: MSE, RMSE, MAE and MAPE results show that SARIMA models have the minimum error values in Uyo, PH and Sokoto while SARFIMA model is better for the analysis solar radiation in Ibadan.

Judging from the AIC values of the two appropriate models, we can conclude that SARFIMA was best model for analyzing solar radiation in the four metropolises except in Ibadan. Even though the forecasts SARFIMA models was adjudged to be better than SARIMA in three of the four series, the forecast performance is poor except in Ibadan where long memory was exhibited.

It should also be noticed that good and best fit models may not yield good forecast values for the future.

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