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A COMPARATIVE STUDY OF SOME TIME SERIES MODELS ON THE POPULATION OF NIGERIA

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ABSTRACT

The knowledge of the population of persons living in a given geographical region is of utmost importance for policy making. It further helps to predict the possible population of such region in the nearest future. Population forecasting is an integral part of any nation's planning. This paper studied the suitability of some conventional time series models in forecasting the population of Nigeria. The data used for this study is the estimated population of Nigeria from 1966 to 2016 at 10 year interval. The data was obtained from the website of the National Bureau of Statistics (NBS). The Linear Trend model, Quadratic trend model, Exponential growth model and the Pearl-Reed logistic models were studied. The MSE, MAD and MAPE were used as performance measures to compare the performance of the four models. The study revealed that exponential growth model performed better than the other three models studied. Using this model, forecast made shows that population of Nigeria would raise to about 874.89 million by year 2076. The study therefore recommended that government should utilize its findings and make appropriate policies and planning that would cater for this rapidly growing population.

Keywords: *Forecasting, Population, Time Series, Trend, Models.*

1.0 Introduction

A population census is the counting of all the people living in a country at a particular time. It collects information on the size, distribution, composition and other social/economic characteristics of the population. (National Population Commission, 2005). Population forecasting is a future estimate indicating how the population will increase arbitrary. It is therefore what the future population would be. Population projection is based upon reasonable assumption on the future course of fertility, mortality, and migration.

A good number of researches on population problems has centered on what could be the optimum size and its impact on economic growth and development (see, for example, Caldwell, 1990; National Research Council, 1993; Onokerhoraye, 1995; Bongaarts, 1996; Bloom and

Williamson, 1998; United Nations, 1999; FAO 2000, UNDP, 2001 and Onwuka, 2003). Researchers' interest in this direction can be traced to the question posed by Malthus (1803) as to whether food production could keep pace with the demand of a growing population. He further asserted that food supply grows in arithmetic progression while population grows in geometric progression making the latter outpace the former. However, the Malthusian hypotheses have been criticized to lack universal applicability, especially in industrial countries where agricultural productions have been greatly increased due to technological advances. For those countries, Malthus's positions are somewhat negated, whereas a good number of developing countries like Nigeria are still trapped under conditions capable of validating them (Olofin, 1996; Smil, 2000 in Onwuka, 2012).

Population forecasting are calculations based on models which show possible future development of a population considering past behavior of such population when certain assumption made about the future course of population usually with respect to fertility, mortality and migration. A population forecast is an estimate of future population growth. It is based on a review of historic population growth and assumption about future demographic and economic trends.

Forecast of the size and structure of the population are central to social and economic planning from the provision of services in the short term to policy development in the long term. Developed countries' populations are characterized by large elderly proportions. The major driver of this aging process is the fertility fluctuation of the past, notably the post war baby boom coupled with the low fertility of recent times. One response to population ageing and the attendant shortage of labour to provide for the elderly has been an increase in immigration to replace or make up for past shortfalls in birth.

Population growth is difficult to predict because unforeseen event can alter birth rate, death rate, migration, or the resources limit on population growth. Birth rate may decline faster than predicted due to increased access to contraceptives, later ages of marriage and the growing desire of many women to seek careers that may affect their involvement in child bearing and domestic work. Countries may also choose to undertake mitigation measures to reduce population growth. For example, in China, the government has put policy in place that regulates the number of children allowed to each couple. Other society has already begun to implement social marketing strategies in other to educate the public on over population effect. Certain government policies are making it easier and more socially acceptable to use contraceptives and abortion methods. Population forecasting is of great importance in any nation, they can be used to study human spread and its effects on the nation's economy.

Some of the common methods used for population forecasting include the arithmetic increase method, graphical method, comparative graphical method and geometric increase method (Geometrical Progression Method), etc. This study, however, explores some time series models and compares their performances in forecasting the future population of Nigeria for the next six (6) decades. The models considered include the linear trend model, quadratic model, exponential growth model and the Pear-Real logistic model.

Nigeria population was estimated 201 million (World Bank, 2019), as of 2020, it rose to 206 million (www.worldmeters.info>Nigeria Population).

2.0 Methodology

This paper compares four conventional time series models to obtain the best performing when used to model the Nigerian population data. The studied models include the linear trend model, quadratic trend model, exponential growth model and the Pearl-Reed logistic (also known as S-curve model).

2.1 The Linear Trend Model

The linear trend model is given as:

$$Y_t = a + bt + e \tag{1}$$

Where:

Y_t is the Population for a given time period t

a = value of the trend when the time t = 0

b = the change in Y_t per unit time.

t = the unit time (t = 1, 2, …)

e = the random error term

The Ordinary Least Squares (OLS) is used to estimate the model coefficients and are given as:

$$b = \frac{n\Sigma t Y_t - (\Sigma Y_t)(\Sigma t)}{n\Sigma t^2 - (\Sigma t)^2} \tag{2}$$

$$a = \overline{Y}_t - b\bar{t} \tag{3}$$

2.2 Quadratic Trend Model

The quadratic trend model is given as
$Y_t = a + bx + cx^2$
where:
Y_t is the Population for a given time period t

2.3 Exponential Growth Model

The *exponential growth trend model* accounts for exponential growth or decay in a time series data. The model is given as:

$$Y_t = a \times b^t \times e \tag{4}$$

Using logarithmic transformation will enable us to fit the data and estimate the model parameters in a much similar way as the linear trend.

2.4 The Pearl-Reed Logistic model

The Pearl-Reed logistic model is given as

$$Y_t = \frac{L}{1+e^{-k(x-x_0)}} \quad (5)$$

where

x_0

L, the curve's maximum value

k, the logistic growth rate or steepness of the curve.

2.5 Performance Measures

The accuracy of the fitted model was assessed using the Mean Squared Absolute Percentage Error (MAPE), Mean Absolute Deviation (MAD), and the Mean Squared Deviation (MAD).

2.5.1 Mean Absolute Percentage Error (MAPE)

Mean absolute percentage error (MAPE) measures the accuracy of fitted time series values. MAPE expresses accuracy as a percentage and is given as

$$MAPE = \frac{\sum_{t=1}^{n}\left|\frac{y_t - \hat{y}_t}{y_t}\right|}{n} \times 100, \quad (y_t \neq 0) \quad (6)$$

where:

y_t = actual value at time t

$\hat{y}_t$ = fitted value

n = number of observations

2.5.2 Mean Absolute Deviation (MAD)

Mean absolute deviation (MAD) measures the accuracy of fitted time series values. MAD expresses accuracy in the same units as the data, which helps conceptualize the amount of error. The Mean Absolute Deviation is given as

$$MAD = \frac{\sum_{t=1}^{n} y_t - \hat{y}_t}{n} \quad (7)$$

where y_t, $\hat{y}_t$, and n are as defined above

2.5.3 Mean Squared Deviation (MSD)

Mean squared deviation (MSD) is always computed using the same denominator, n, regardless of the model. MSD is a more sensitive measure of an unusually large forecast error than MAD. The Mean Squared Deviation is given as

$$MSD = \frac{\sum_{t=1}^{n} \left| y_t - \hat{y}_t \right|^2}{n} \tag{8}$$

where y_t, $\hat{y}_t$, and n are as defined above

The prediction error represented by the Mean Absolute Percentage Error, MAPE (Tratar and Srmcnik, 2016; Chatfield, 2001) is chosen as our performance metric. The MAPE is selected because it provides an accurate and fair comparison of the forecasting methods, and it is not affected by change in the magnitude of time series to be forecast (Gentry *et al*., 1995; Alon *et al*. 2001in Edike *et al*., 2021). Also, it is frequently used in practice (Ravindran and Warsing, 2013). The MAPE is expressed in Equation (5).

The methods applied to this research work as described above were implemented using MINITAB 17 Statistical software.

2.6 Data collection

The data used for this study consists of the population of Nigeria from 1966 to 2016 at 10 years interval. It is a secondary data collected from The National Bureau of Statistics (N.B.S) website. (www.nigerianstat.org, 2018).

Table 1: Population of Nigeria (1966 – 2016)

Year	Population (in millions)
1966	51.22
1976	65.23
1986	85.82
1996	110.70
2006	142.60
2016	186.00

Source: National Bureau of Statistics, 2018

The trend plot of the data collected is given in the figure below.

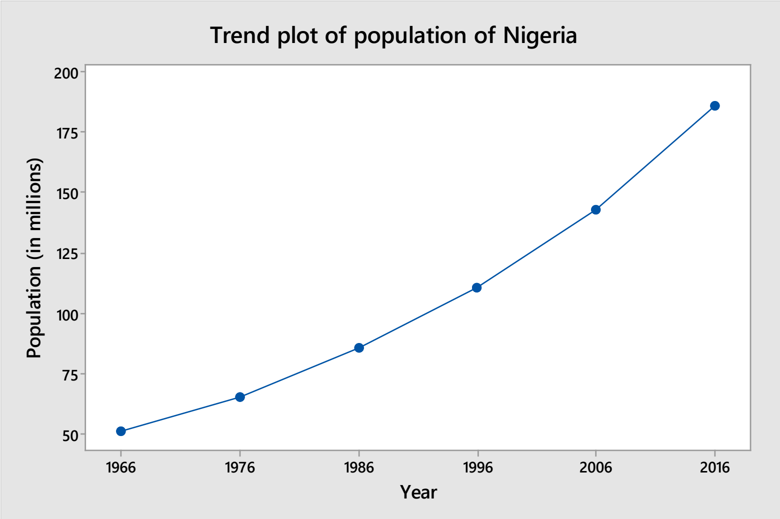

Figure 1: Trend plot of population of Nigeria (1966 – 2016)

From Figure 1 above, the trend plot of the Nigerian population (1966 – 2016) reveals an approximately linear trend. However this study will fit different models to the data to find which model most closely fits the series. The obtained model will then the used to make forecast of the series for the next six (6) decades.

3.0 Results and Discussion

This section presents the results of the analyzed data. Four different time series models were fitted to the data collected in order to determine the best performing among the four. Following are the results obtained.

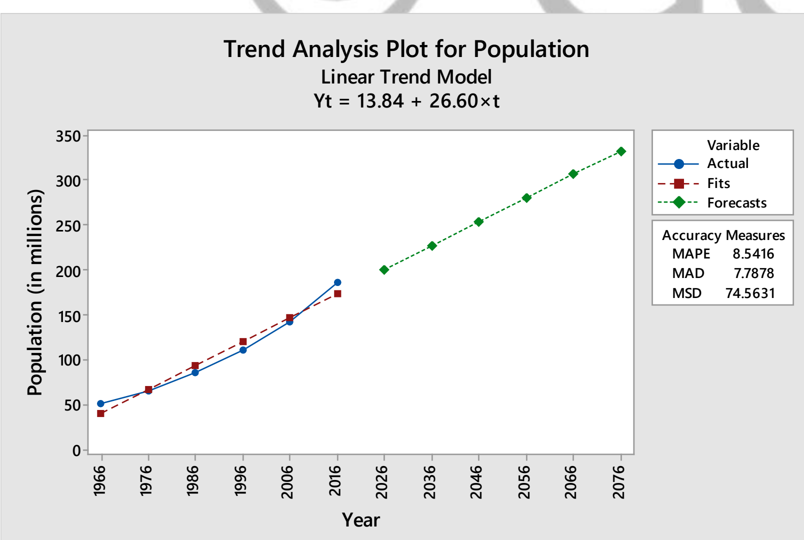

Figure 2: Linear Trend Plot for Population

Figure 2 above shows the trend plot using the linear trend model. The model was fitted to the data using equations (1), (2) and (3). The fitted trend is found to be $Y_t = 13.84 + 26.60t$. The fitted model gave a Mean Absolute Percentage Error (MAPE) of 8.5416, Mean Absolute Deviation (MAD) of 7.7878 and a Mean Square Deviation MSD) of 74.5631

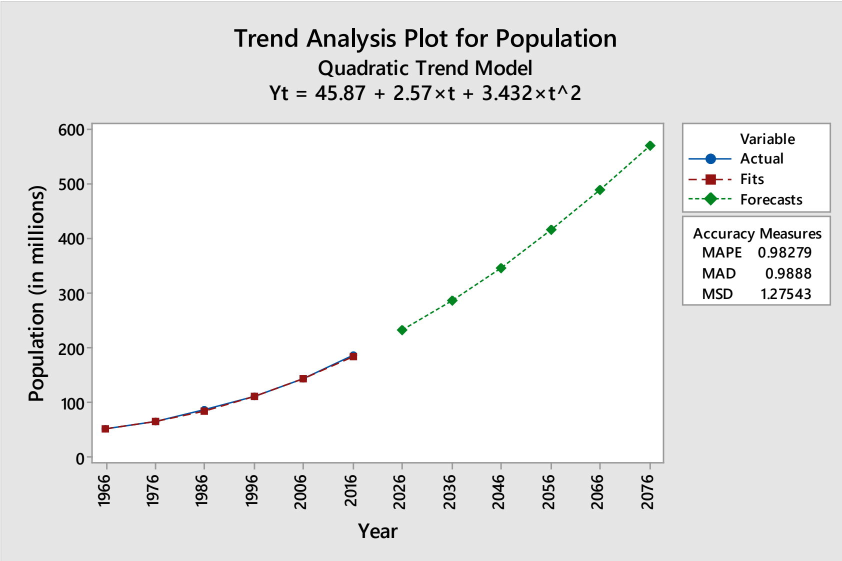

Figure 3: Quadratic trend plot for Population

From the visual presentation in figure 3 above, it can be seen that the quadratic model fits better to the data than the linear trend model. This can be observed from the closeness of the actual data and fits as they, infact, superimposed each other. The quadratic trend model gave the fitted trend to be $Y_t = 45.87 + 2.57t + 3.432t^2$. The fitted model gave a Mean Absolute Percentage Error (MAPE) of 0.9828, Mean Absolute Deviation (MAD) of 0.9888 and a Mean Square Deviation MSD) of 12.7543.

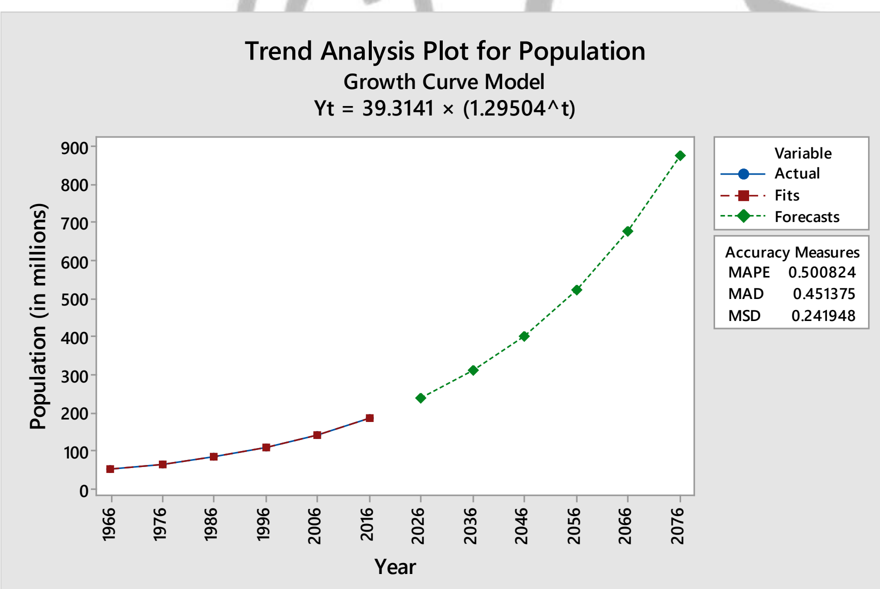

Figure 4: Exponential growth curve for population

Figure 4 above shows the fitted exponential growth curve model to the Nigerian population data The exponential growth gave the fitted trend to be $Y_t = 39.3141(1.2950)^t$. The fitted model gave a Mean Absolute Percentage Error (MAPE) of 0.5008, Mean Absolute Deviation (MAD) of 0.4514 and a Mean Square Deviation MSD) of 0.2419.

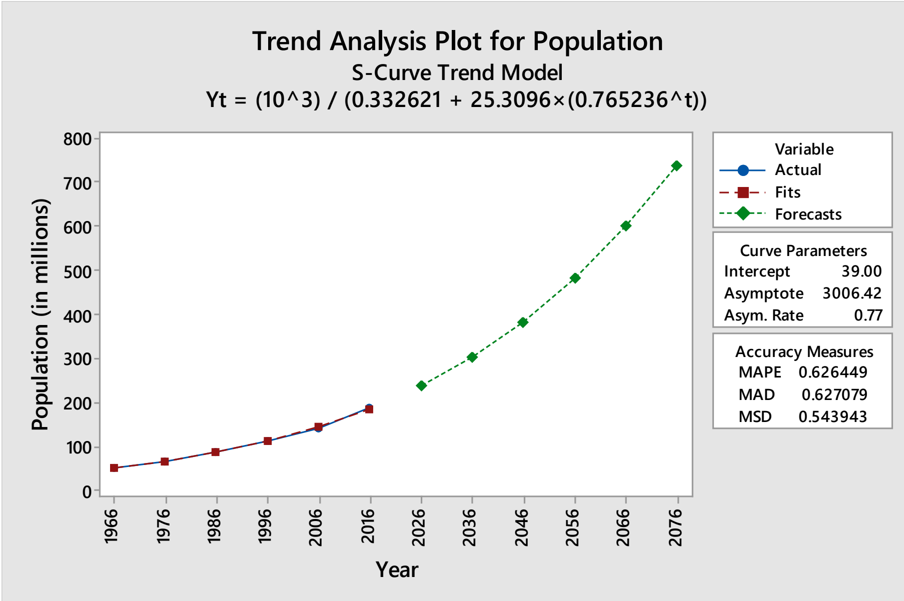

Figure 5: Pearl-Reed logistic model for population

Figure 5 above shows the fitted Pearl-Reed model to the Nigerian population data The model gave the fitted trend to be $Y_t = 10^3/(0.3326 + 25.3096 \times 0.7652^t)$. The fitted model gave a Mean Absolute Percentage Error (MAPE) of 0.6264, Mean Absolute Deviation (MAD) of 0.6271 and a Mean Square Deviation MSD) of 0.5439.

Table 2: Summary of Performance Metrics of the studied models

Measures	Linear trend model	Quadratic trend model	Exponential growth model	Pearl-Reed logistic (S-curve) model
Fitted Model	$Y_t = 13.84 + 26.60t$	$Y_t = 45.87 + 2.57t + 3.432t^2$	$Y_t = 39.3141(1.2950)^t$	$Y_t = \frac{10^3}{(0.3326 + 25.3096 \times 0.7652^t)}$
MAPE	8.5416	0.9828	0.5008	0.6264
MAD	7.7878	0.9888	0.4514	0.6271
MSD	74.5631	12.7543	0.2419	0.5439

4.0 Conclusion

This paper compares the performance of some basic time series models on the Nigerian population data taken at 10 year interval from 1966 to 2016. Four different models were fitted into the data and comparison was made using the Mean Absolute Percentage Error (MAPE), Mean Absolute Deviation (MAD) and the Mean Squared Deviation (MSD). The results in Table 2 above show that the exponential growth model gave a MAPE, MAD, and MSD of 0.5008, 0.4514 and 0.2419 respectively. These are the least error measures among the four studied models, we therefore conclude that the exponential growth model is the best performing time series model among the four studied models for the population figure of Nigeria. Tables A1 to A8 in the Appendix shows the fitted values and forecasts made from the four models. It can therefore be said the most reliable forecast among all is that obtained using the exponential growth model, and this shows that the population of Nigeria would rise to approximately 874.89 million by year 2076. It is therefore recommended that government should utilize this information and make appropriate policies and planning that would cater for this rapidly growing population.

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APPENDIX

Table A1: Fitted values and residuals for the linear trend model

LINEAR TREND $Y_t = 13.84 + 26.60t$				
Index (t)	Year	Population (in million)	Fitted Values (in million)	Residuals
1	1966	51.22	40.43619	10.78381
2	1976	65.23	67.03305	-1.80305
3	1986	85.82	93.6299	-7.8099
4	1996	110.7	120.2268	-9.52676
5	2006	142.6	146.8236	-4.22362
6	2016	186	173.4205	12.57952

Table A2: 6-decade forecast using the linear trend model

Index (t)	Year	Forecasts (in million)
7	2026	200.0173
8	2036	226.6142
9	2046	253.211
10	2056	279.8079
11	2066	306.4048
12	2076	333.0016

Table A3: Fitted values and residuals for the quadratic trend model

QUADRATIC TREND $Y_t = 45.87 + 2.57t + 3.432t^2$				
Index (t)	Year	Population (in million)	Fitted Values (in million)	Residuals
1	1966	51.22	51.87607	-0.65607
2	1976	65.23	64.74507	0.484929
3	1986	85.82	84.478	1.342
4	1996	110.7	111.0749	-0.37486
5	2006	142.6	144.5356	-1.93564
6	2016	186	184.8604	1.139643

Table A4: 6-decade forecast using the quadratic trend model

Index (t)	Year	Forecasts (in million)
7	2026	232.049
8	2036	286.1016
9	2046	347.0181
10	2056	414.7985
11	2066	489.4429
12	2076	570.9511

Table A5: Fitted values and residuals for the exponential growth model

EXPONENTIAL TREND $Y_t = 39.3141(1.2950)^t$				
Index (t)	Year	Population (in million)	Fitted Values (in million)	Residuals
1	1966	51.22	50.91344	0.306562
2	1976	65.23	65.93507	-0.70507
3	1986	85.82	85.38873	0.431269
4	1996	110.7	110.582	0.117953
5	2006	142.6	143.2085	-0.60847
6	2016	186	185.4611	0.538929

Table A6: 6-decade forecast using the exponential growth model

Index (t)	Year	Forecasts (in million)
7	2026	240.18
8	2036	311.0433
9	2046	402.8144
10	2056	521.6618
11	2066	675.5743
12	2076	874.8975

Table A7: Fitted values and residuals for the S-Curve model

S-CURVE MODEL $Y_t = 10^3/(0.3326 + 25.3096 \times 0.7652^t)$.				
Index (t)	Year	Population (in million)	Fitted Values (in million)	Residuals
1	1966	51.22	50.76033	0.459673
2	1976	65.23	65.99106	-0.76106
3	1986	85.82	85.65935	0.16065
4	1996	110.7	110.9685	-0.26845
5	2006	142.6	143.3883	-0.78832
6	2016	186	184.6757	1.324321

Table A8: 6-decade forecast using the S-Curve model

Index (t)	Year	Forecasts (in million)
7	2026	236.8678
8	2036	302.2303
9	2046	383.1342
10	2056	481.8363
11	2066	600.1486
12	2076	739.0079