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A Mathematical Model for Transmission Dynamics of Ebola Virus with Control Measures

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Abstract: Ebola virus disease (EVD) is a very deadly and highly infectious disease that affect human immune system, EVD is also known as Ebola Hemorrhagic fever (EHF) or called Ebola and also a human and other primates viral hemorrhagic fever caused by ebolaviruses, the incubation period of Ebola is between 2 – 21 days. In this paper we developed a deterministic mathematical model for transmission dynamics of Ebola virus disease control, that incorporating the constant the recruitment and numerical simulation of four control strategies: public enlighten campaign on personal hygiene, quarantine, isolation and proper burial. From the model we compute the effective reproduction number R_C and numerical simulation of effective reproduction number was computed using different control strategies which verifies the existence of LAS of DFE if $R_C < 1$. it was observed that absence of control in a community the disease will persist within short period of time and also we observed that with moderate and high control strategies the disease will be curbed in a long run, we also verify that effective public enlighten campaign on personal hygiene is the best strategy in curbing Ebola followed by isolation strategy then quarantine and proper burial strategies.

Keywords: Modeling, Effective Reproductive Number, Ebola, Virus, Disease

1. Introduction

Ebola virus disease (EVD) is a very deadly and highly infectious disease that affect human immune system, EVD is also known as Ebola Hemorrhagic fever (EHF) or called Ebola and also a human and other primates viral hemorrhagic fever caused by ebolaviruses, the incubation period of Ebola is between 2 – 21 days [6] and has effective period of 4 – 10 days. Ebola is categorized by fever, weakness, reduced appetite, muscular pain, joint pain, headache, and sore throat [6] and [14], usually the fever is 38.3 ^{0}C or 101 ^{0}F higher [15], then followed by nausea, vomiting, diarrhea, abdominal pain, and sometimes hiccups, diarrhea, bleeding which lead to death. Identifying Ebola is a very difficult challenge because of misdiagnosing as malaria and typhoid. Infected individuals of EVD has a death risk between 25 $\%$ and 90 $\%$ [6] and [10] and average death risk of infected individuals is 50 $\%$ [6]. There is vaccine now for the virus, an investigational vaccine called rVSVZEBOV, it was genetically engineered from Vascular Stomatitis Virus (VSV) and Zaire Ebola Virus (ZEBOV) and has shown to be highly protective against Ebola virus infection and the treatments are still supportive care.

According to [17], the most populous country in Africa is Nigeria and the seventh most populous country in the world with more than two hundred and three million people. Between March 2014 and July 2015, more than 10,500 Ebola virus disease (EVD) cases, including over 4,800 deaths, occurred in Liberia; the majority of these cases was identified in Montserrado County, where the capital city of Liberia, Monrovia, is located [9]. It was fortunate only 19 (EVD) cases were conferred in Nigeria [20].

Transmission of Ebola virus eventuate from having contact with body fluids containing blood that is infected [13] and [12], and the possible form of transmission include saliva, mucus, vomit, feces, sweat, tears, breast milk, urine and semen [7]. WHO states that only people who are very sick are able to spread Ebola virus via saliva, and whole virus has not been reported to be transmitted through sweat. Most people spread the virus through blood, feces and vomit [11]. Entry points for the virus include the nose, mouth, eyes, open wounds, cuts and abrasions. Ebola may be spread through large droplets; however, this is believed to occur only when a person is very sick, the contamination can only

happen if a person is being splashed with droplets and contact with surfaces or objects contaminated by the virus, particularly needles and syringes, may also transmit the infection according and the virus is able to survive on objects for few hours in a dried state, and can survive for a few days within body fluids outside of a person [2]. The Ebola virus may be able to persist for more than 3 months in the semen after recovery, which could lead to infections via sexual intercourse [8] and [21]. Virus persistence in semen for over a year has been recorded in a national screening program [16]. Ebola may also occur in the breast milk of women after recovery, and it is not known when it is safe to breastfeed again [4]. The virus was also found in the eye of one patient in 2014, two months after it was cleared from his blood [19]. Otherwise, people who have recovered are not infectious [5].

Ebola virus disease have been a very big burden in some parts of the world and it has been affecting humanity by reducing population growth, it has affected tourism, agricultural production has decreases in the affected areas, it has affected mining by fewer investors, high unemployment and less tread and transportation e.t.c

Many people have carried out analysis and many others are on the way, to find a cure to Ebola virus disease. The study analyzed a mathematical model to study the transmission dynamics of Ebola virus disease; some of the existing models were discussed.

[1] developed a deterministic mathematical model on transmission dynamics of Ebola virus disease (EVD) reoccurrence in Nigeria was developed and analyzed from a population that is sub-divided into eight sub-population (compartment), namely susceptible individuals (*S*), quarantine susceptible individuals (Q_S) latently infected individuals (*L*), quarantine latently infected individuals (Q_L), infectious individuals (*I*), isolated individuals (*J*), recovered individuals due to permanent recovery from the infection (*R*), and Ebola induced dead bodies before burial (D). Mathematical equations where generated from the corresponding sub-population (compartment).

He concluded that the effective reproductive number R_C was obtained and numerical simulation was obtained using demographic and epidemiological relevant data and results showed that quarantine, isolation of infected individuals as well as proper burial of the dead bodies of those that were infected was suggested to be the practicable way to control the spread of the highly infectious disease and recommended that;

i. Strong surveillance should be put to quarantined, isolated and treat infected individuals.
ii. Proper burial of the infected dead bodies.

Therefore, our objective is to improve the model of [1], by incorporating the constant the recruitment and numerical simulation of four control strategies: public enlighten campaign on personal hygiene, quarantine, isolation and proper burial. In general, these control measures are functions of time. compute the effective reproduction number R_C of the model. Use mathematical tools to analyze disease-free equilibrium state of the model for stability or otherwise. obtain the numerical simulations of the model using reliable data.

This paper is structured as follows; in section 2. We present the model. Positivity of solution and effective reproductive number and mathematical tools were used to analyze disease-free equilibrium state of the model for stability or otherwise are given in section 3. We perform numerical simulations of the model using reliable data in section 4. We discuss numerical simulation of the model to verify the analytical analysis of the model in section 5. The paper ends with conclusion and some recommendations in section 6.

2.0 Model Formulation

In this study, the total population (*N*) is divided into seven (*7*) sub-populations (compartments). Namely susceptible individuals (*S*), latently infected individuals (*L*), quarantined latently infected individuals (*Q*), infectious individuals (*I*), isolated individuals (*J*), recovered individuals (*R*), and infectious dead bodies before burial (*D*).

The *S* compartment represents individuals at risk of disease contamination and is generated from constant recruitment through birth of uninfected individuals at the rate Λ, then individuals move to L compartment due to infection via effective contact with infectious individuals from *I* and *D* compartments, and individuals in the compartment decreases due to effective contact at the rate $\left[\frac{pc(1-\varepsilon)I+\eta pc(1-\varepsilon)D}{N}\right]S$ where *p* is probability of Ebola infection, *c* is

daily average contact of *I* and *D*, η is modification parameter that reduced association with infectious dead bodies and $(1-\varepsilon)$ is public enlightenment campaign on personal hygiene impact on Ebola transmission, $0 < \varepsilon < 1$.

The *L* compartment represents individuals that are infected but not infectious and have not shown any Ebola symptoms and is generated by effective contact between *S* with *I* and *D* compartments as discussed earlier. They decrease due to disease progression at the rate σ and quarantine at the rate q.

The *Q* compartment represents latently infected individuals who are quarantine at the rate *q* because they have contact with infectious individuals or infectious dead body. The sub population decreases by disease progression at the rate σ.

The *I* compartment represent the symptomatic individuals that are infected and also infectious. The sub-population is generated by disease progression at the rate σ due to clinical symptom of Ebola development by *L* compartment member and decreases by isolation at the rate τ and disease induced death at the rate δ_1.

The *J* compartment represents isolated individuals who have developed Ebola clinical symptoms to be given treatment. The sub-population is generated from quarantine *Q* and isolation *I* compartments at the rates σ and τ respectively as discussed earlier. The sub-population decreases by recovery at the rate γ and disease induced death at the rate δ_2 .The *R* compartment represents individuals that recovered from Ebola virus disease and assumed to have permanent immunity against Ebola.

The *D* compartment represents the infectious dead bodies that died due to disease infection. The sub-population is generated from both *I* and *J* compartments. The sub-population decreases by proper burial at the rate ρ and natural death at the rate μ in all compartments excluding *D* compartment.

2.1 Model Assumptions

The formulation of our model is guided by the following assumptions:

i. There is homogeneous mixing of the population, where all people equally likely to be infected if they have contact with the infectious individuals.
ii. Effective public enlightenment campaign on personal hygiene.
iii. Quarantined individuals are latently infected.
iv. Recovered individuals are assumed to have permanent immunity against Ebola.

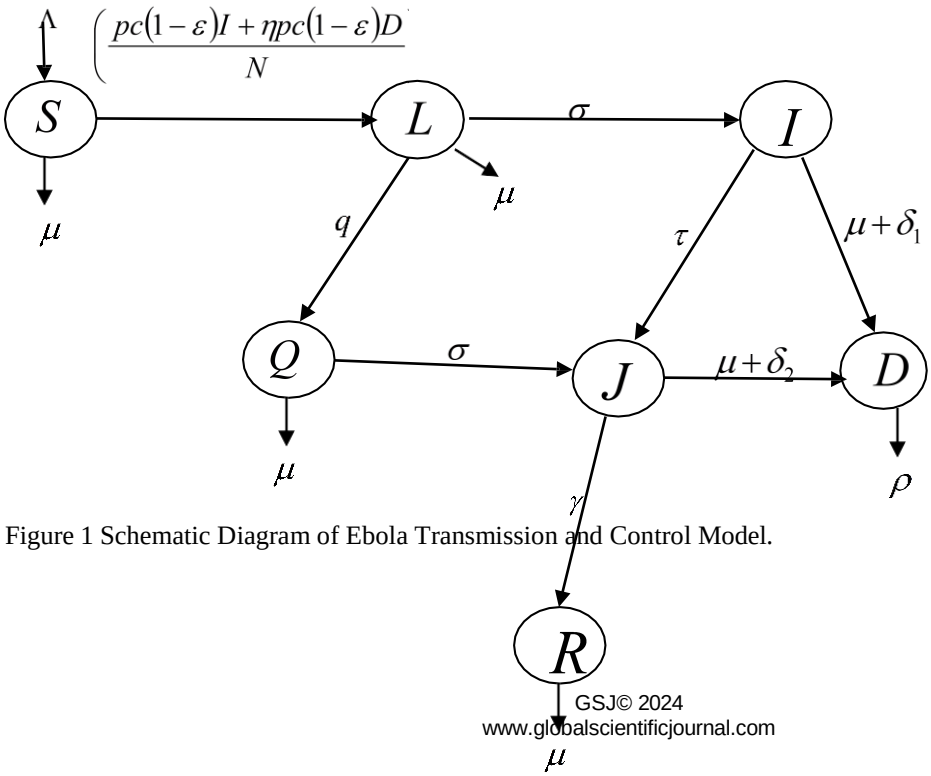

Figure 1 Schematic Diagram of Ebola Transmission and Control Model.

2.2 The Model Equations

The model equations are given below

$$\frac{dS}{dt} = \Lambda - \left(\frac{pc(1-\varepsilon)I + \eta pc(1-\varepsilon)D}{N} \right) S - \mu S \quad (1)$$

$$\frac{dL}{dt} = \left(\frac{pc(1-\varepsilon)I + \eta pc(1-\varepsilon)D}{N} \right) S - (q + \sigma + \mu) L \quad (2)$$

$$\frac{dQ}{dt} = qL - (\sigma + \mu) Q \quad (3)$$

$$\frac{dI}{dt} = \sigma L - (\tau + \mu + \delta_1) I \quad (4)$$

$$\frac{dJ}{dt} = \tau I + \sigma Q - (\gamma + \mu + \delta_2) J \quad (5)$$

$$\frac{dR}{dt} = \gamma J - \mu R \quad (6)$$

$$\frac{dD}{dt} = (\mu + \delta_1) I + (\mu + \delta_2) J - \rho D \quad (7)$$

2.3 Basic Properties of the Model
2.3.1 Invariant Region

Consider the region,

$$\Omega = \left\{ (S, L, Q, I, J, R, D) \in \Re_+^7 : N \le \frac{\Lambda}{\mu} \right\} \quad (8)$$

In order to study the dynamics of the system (1 – 7) in Ω, the positive-invariance and attractiveness of Ω with respect to the system (1 – 7) is established as follows. Now, the rate of change of the total population, obtained by adding all the equations in the system (1 – 7), is given by:

$$\frac{dN}{dt} \le \Lambda - \mu N \quad (9)$$

As standard comparison theorem, so we have

$$\frac{dN}{dt} + \mu N \le \Lambda \quad (10)$$

Using integrating factor method, we have

$$\frac{dN}{dt} e^{\mu t} + \mu e^{\mu t} \le \Lambda e^{\mu t} \quad (11)$$

$$\frac{d}{dt}\left(N e^{\mu t} \right) \le \Lambda e^{\mu t} \quad (12)$$

$$\int d\left(Ne^{\mu t}\right) \le \int \Lambda e^{\mu t} dt \quad 13$$

$$Ne^{\mu t} \le \frac{\Lambda}{\mu} e^{\mu t} + c \quad 14$$

At $t = 0$, $C = N_0 - \frac{\Lambda}{\mu}$

$$Ne^{\mu t} \le \frac{\Lambda}{\mu} e^{\mu t} + N_0 - \frac{\Lambda}{\mu} \quad 15$$

$$N \le \frac{\Lambda}{\mu} + \left(N_0 - \frac{\Lambda}{\mu}\right) e^{-\mu t} \quad 16$$

$$N = N_0 e^{-\mu t} + \frac{\Lambda}{\mu}\left(1 - e^{-\mu t}\right) \quad 17$$

In particular, if $N_0 \le \frac{\Lambda}{\mu}$ then $N \le \frac{\Lambda}{\mu}$ hence, Ω is positively invariant set and an attractor so that no solution path leaves through any boundary of Ω.

2.3.2 Positivity of the Solutions

Lemma 3.1 Let the initial data for equations (3.1) – (3.7) be S(0)>0, L(0)>0, Q(0)>0, I(0)>0, J(0)>0, R(0)>0 and D(0)>0 then the solution S(t), L(t), Q(t), I(t), J(t), R(t) and D(t) with positive initial data will remain positive for all time t>0

Proof:

Let,

$$t^1 = \sup\left\{ t > 0 : \mathrm{S}(0) > 0, \mathrm{L}(0) > 0, \mathrm{Q}(0) >, \mathrm{I}(0) > 0, \mathrm{J}(0) > 0, \mathrm{R}(0) > 0, \mathrm{D}(0) > 0 \right\} > 0 \quad 18$$

$$\frac{dS}{dt} = \Lambda - (\lambda + \mu)S \quad 19$$

We use integrating factor method to solve equation (19)

$$\mathrm{I.F} = \exp\left[\mu t + \int_0^t (\lambda(\tau)) d(\tau)\right] \quad 20$$

$$\frac{d}{dt}\left[S(t)\exp\left\{\mu t + \int_0^t (\lambda(\tau)) d(\tau)\right\}\right] = \Lambda\left[\exp\left\{\mu t + \int_0^t (\lambda(\tau)) d(\tau)\right\}\right] \quad 21$$

$$S(t_1)\exp\left\{\left|\mu t+\int_0^{t_1}\left(\lambda(\tau)\right)d(\tau)\right\}\right|=S(0)+\int_0^{t_1}\Lambda\left|\exp\left\{\left|\mu y+\int_0^{t_1}\left(\lambda(\tau)\right)d(\tau)\right\}\right|dy \quad 22$$

$$S(t_1)=S(0)\exp\left\{\left|-\mu t-\int_0^{t_1}\left(\lambda(\tau)\right)d(\tau)\right\}\right|$$

$$+\left|\exp\left\{\left|-\mu t-\int_0^{t_1}\left(\lambda(\tau)+q(\tau)\right)d(\tau)\right\}\right|\int_0^{t_1}\Lambda\left|\exp\left\{\left|\mu y+\int_0^{t_1}\left(\lambda(\tau)+q(\tau)\right)d(\tau)\right\}\right|dy>0 \quad 23$$

For $\frac{dL}{dt}=\lambda S-(q+\sigma+\mu)L$ we have $\frac{dL}{dt}\geq-(q+\sigma+\mu)L$

For $\frac{dQ}{dt}=qL-(\sigma+\mu)Q$ we have that $\frac{dQ}{dt}\geq-(\sigma+\mu)Q$

For $\frac{dI}{dt}=\sigma L-(\tau+\mu+\delta_1)I$ we have $\frac{dI}{dt}\geq-(\tau+\mu+\delta_1)I$

For $\frac{dJ}{dt}=\tau I+\sigma Q-(\gamma+\mu+\delta_2)J$ we have $\frac{dJ}{dt}\geq-(\gamma+\mu+\delta_2)J$

For $\frac{dR}{dt}=\gamma J-\mu R$ we have $\frac{dR}{dt}\geq-\mu R$

For $\frac{dD}{dt}=(\mu+\delta_1)I+(\mu+\delta_2)J-\rho D$ we have $\frac{dD}{dt}\geq-\rho D$

Similarly, we can show that *L(0)>0,Q(0)>0, I(0)>0, J(0)>0, R(0)>0 and D(0)>0*

3. Model Analysis

3.1 Disease-Free Equilibrium State (E^0)

At disease-free equilibrium state the entire population is susceptible because there is no disease infection in the population, in other words all the compartments are zero except susceptible compartment because no disease infection.

Lemma 3.2 A disease-free equilibrium of the model (1-7) exist at

$$E^0=\left(S^0,L^0,Q^0,I^0,J^0,R^0,D^0\right)=\left|\frac{\Lambda}{\mu},0,0,0,0,0,0\right| \quad 24$$

Proof: Let $E^0=\left(S^0,L^0,Q^0,I^0,J^0,R^0,D^0\right)$

$$S^0=\frac{\Lambda}{\mu} \quad 25$$

$$L^0=Q^0=I^0=J^0=R^0=D^0=0 \quad 26$$

Note: that $R^0=0$, because of the fact that only individuals that are infected becomes permanently recovered from the disease.

Therefore, the lemma is being proved.

3.2 Basic Reproduction Number (R_0)

The basic reproduction number (R_0) is a threshold that is used to measure the increase or decrease of a disease in a society, community or environment. Whenever $R_0 > 1$ it means the disease is increasing and one person is infecting more than one person. In this case the infection may persist on a long run. Conversely $R_0 < 1$ means the disease is decreasing and more than one person are infecting one person. In this case the infection may stop on a long run. The effective reproduction number (R_C) is the average number of secondary infections due one single infection in a community without immunity or without control [1].

Effective reproduction number R_C is obtained by taking the maximum eigenvalue as spectral radius of the next generation matrix of [3] of the model equations (1 – 7). It is given as $R_C = \rho K$, where ρ is the spectral radius of the next generation matrix and $K = FV^{-1}$ while F is the coefficients of the matrix of the new infection terms and is a non-negative matrix while V is the coefficients of the matrix of the transition terms and a non-singular matrix.

$$F = \begin{pmatrix} 0 & 0 & \alpha & 0 & \alpha\eta \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad 27$$

And

$$V = \begin{pmatrix} k_1 & 0 & 0 & 0 & 0 \\ -q & k_2 & 0 & 0 & 0 \\ -\sigma & 0 & k_3 & 0 & 0 \\ 0 & -\sigma & -\tau & k_4 & 0 \\ 0 & 0 & -k_5 & -k_6 & \rho \end{pmatrix} \quad 28$$

Thus, matrix V^{-1} was determined as

$$V^{-1} = \begin{pmatrix} \frac{1}{k_1} & 0 & 0 & 0 & 0 \\ \frac{q}{k_1 k_3} & \frac{1}{k_2} & 0 & 0 & 0 \\ \frac{\sigma}{k_1 k_4} & 0 & \frac{1}{k_3} & 0 & 0 \\ \frac{\sigma(qk_3 + \tau k_2)}{k_1 k_2 k_3 k_4} & \frac{\sigma}{k_2 k_4} & \frac{\tau}{k_3 k_4} & \frac{1}{k_4} & 0 \\ \frac{\sigma(qk_3 k_6 + \tau k_2 k_6 + k_2 k_4 k_5)}{\rho k_1 k_2 k_3 k_4} & \frac{\sigma k_6}{\rho k_2 k_4} & \frac{\tau k_6 + k_4 k_5}{\rho k_3 k_4} & \frac{k_6}{\rho k_4} & 1 \end{pmatrix} \quad 29$$

Therefore,

$$FV^{-1}=\begin{pmatrix}0&0&\alpha&0&\eta\alpha\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix}\times\begin{pmatrix}\frac{1}{k_1}&0&0&0&0\\\frac{q}{k_1k_3}&\frac{1}{k_2}&0&0&0\\\frac{\sigma}{k_1k_4}&0&\frac{1}{k_3}&0&0\\\frac{\sigma\left(qk_3^{\,3}+\tau k_2^{\,2}\right)}{k_1k_2k_3k_4}&\frac{\sigma}{k_2k_4}&\frac{\tau}{k_3k_4}&\frac{1}{k_4}&0\\\frac{\sigma\left(qk_3k_6+\tau k_2k_6+k_2k_4k_5\right)}{\rho k_1k_2k_3k_4}&\frac{\sigma k_6}{\rho k_2k_4}&\frac{\tau k_6+k_4k_5}{\rho k_3k_4}&\frac{k_6}{\rho k_4}&\frac{1}{\rho}\end{pmatrix} \quad 30$$

$$FV^{-1}=\begin{pmatrix}\frac{\sigma\alpha\left(\rho k_2k_4+\eta qk_3k_6+\eta\tau k_2k_6+\eta k_2k_4k_5\right)}{\rho k_1k_2k_3k_4}&\frac{\eta\alpha\sigma k_6}{\rho k_2k_4}&\frac{\alpha\left(\rho k_4+\eta\tau k_6+\eta k_4k_5\right)}{\rho k_3k_4}&\frac{\eta\alpha k_6}{\rho k^4}&\frac{\eta\alpha}{\rho}\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix} \quad 31$$

Therefore, the effective reproductive number is:

$$\rho FV^{-1}=R_C=\frac{\alpha\sigma\left(\rho k_2k_4+\eta\left(qk_3k_6+\tau k_2k_6+k_2k_4k_5\right)\right)}{\rho k_1k_2k_3k_4} \quad 32$$

3.3 Local Stability of DFE (E^0)

The Jacobian stability technique was used to verify, consider the Jacobian matrix from (1 – 7) at disease free equilibrium, E^0 is given by

$$J(E^0)=\begin{pmatrix}-\mu&0&0&-\alpha&0&0&-\eta\alpha\\0&-k_1&0&\alpha&0&0&\eta\alpha\\0&q&-k_2&0&0&0&0\\0&\sigma&0&-k_3&0&0&0\\0&0&\sigma&\tau&-k_4&0&0\\0&0&0&0&\gamma&-\mu&0\\0&0&0&k_5&k_6&0&-\rho\end{pmatrix} \quad 33$$

Elementary row-transformation was used.

$$J(E^0)=\begin{pmatrix}-\mu&0&0&-\alpha&0&0&-\eta\alpha\\0&-k_1&0&\alpha&0&0&\eta\alpha\\0&0&-k_2&\frac{q\alpha}{k_1}&0&0&\frac{q\eta\alpha}{k_1}\\0&0&0&-\frac{\alpha\sigma-k_1k_3}{k_1}&0&0&\frac{\eta\alpha\sigma}{k_1}\\0&0&0&0&-k_4&0&-\frac{\eta\alpha\sigma\left(qk_3^{\,3}+\tau k_2^{\,2}\right)}{k_2\left(\alpha\sigma-k_1k_3^{\,2}\right)}\\0&0&0&0&0&-\mu&-\frac{\eta\alpha\sigma\gamma\left(qk_3+\tau k_2^{\,2}\right)}{k_2k_4\left(\alpha\sigma-k_1k_3^{\,2}\right)}\\0&0&0&0&0&0&\frac{\alpha\sigma\left(\rho k_2k_4+\eta\left(qk_3^{\,3}k_6+\tau k_2^{\,2}k_6+k_2k_4k_5\right)\right)-\rho k_1k_2k_3k_4}{k_2k_4\left(\alpha\sigma-k_1k_3^{\,2}\right)}\end{pmatrix} \quad 34$$

Here are the eigenvalues

$$\lambda_1 = -k_4 < 0 \quad (35)$$

$$\lambda_2 = -k_2 < 0 \quad (36)$$

$$\lambda_3 = -k_1 < 0 \quad (37)$$

$$\lambda_4 = -\frac{\alpha\sigma - k_1 k_3}{k_1} < 0 \quad (38)$$

$$\lambda_5 = \frac{\alpha\sigma\left(\rho k_2 k_4 + \eta\left(q k_3 k_6 + \tau k_2 k_6 + k_2 k_4 k_5\right)\right) - \rho k_1 k_2 k_3 k_4}{k_2 k_4\left(\alpha\sigma - k_1 k_3\right)} \quad (39)$$

$$\lambda_6 = -\mu < 0 \quad (40)$$

$$\lambda_7 = -\mu < 0 \quad (41)$$

Thus, all the eigenvalues are negative excluding λ_5 but λ_5 which is less than zero.

$$\frac{\alpha\sigma\left(\rho k_2 k_4 + \eta\left(q k_3 k_6 + \tau k_2 k_6 + k_2 k_4 k_5\right)\right) - \rho k_1 k_2 k_3 k_4}{k_2 k_4\left(\alpha\sigma - k_1 k_3\right)} < 0 \quad (42)$$

$$\alpha\sigma\left(\rho k_2 k_4 + \eta\left(q k_3 k_6 + \tau k_2 k_6 + k_2 k_4 k_5\right)\right) - \rho k_1 k_2 k_3 k_4 < 0 \quad (43)$$

$$\alpha\sigma\left(\rho k_2 k_4 + \eta\left(q k_3 k_6 + \tau k_2 k_6 + k_2 k_4 k_5\right)\right) < \rho k_1 k_2 k_3 k_4 \quad (44)$$

$$\frac{\alpha\sigma\left(\rho k_2 k_4 + \eta\left(q k_3 k_6 + \tau k_2 k_6 + k_2 k_4 k_5\right)\right)}{\rho k_1 k_2 k_3 k_4} < 1 \quad (45)$$

i.e

$$R_C < 1 \quad (46)$$

Therefore the conditions are said to be satisfied $\lambda_5 < 0$ if $R_C < 1$ wherefore we claim the following result.

Lemma: 3.3

The disease-free equilibrium (E^0) of the model (1 – 7) is locally asymptotically stable (LAS) if $R_C < 1$.

4. Simulation and Discussion

We estimated the model variables based on EVD epidemiological and demographic profile of Nigeria's population in case of re-occurrence. The Nigerians total population is given as 202,802,887 in the year 2019 (NBS, 2019). For the initial variables states, the study reported that in the year 2014 (WHO, 2014) reported that 20 individuals were latently infected, 8 individuals were infectious, 6 individuals were isolated, 8 individuals were dead and the study assumed no individual was quarantine and 12 individuals were recovered. Thus N = 202,802,887; S = 202,802,867; L = 20; Q = 0; I = 8; J = 6; R = 12; D = 8.

Table 1 Variables value of the model

Variables	Values	Source
S	202,802,867	Calculated
L	20	WHO (2014)
Q	0	Assumed
I	8	WHO (2014)
J	6	WHO (2014)
R	12	Assumed
D	8	WHO (2014)
N	202,802,887	NBS (2019)

Parameters values have to be estimated based on EVD epidemiology and published data. The life expectancy of a Nigerian is 58.85 years (NBS, 2019), thus the death rate is equal to $\frac{1}{58.85} \times \frac{1}{366}$, that is 0.000046 per day. Thus, the recruitment rate is $\Lambda = N \times \mu$ which is $\Lambda = 9340$ per day. Ebola is highly infectious. It is far more infectious than HIV. The study observed that [18] assumed the probability of transmission (*p*) to be 0.9. People frequently interact in different places and different events. Through, average of contact between individuals (*c*) depend on several factors, the study observed that [1] assumed it is 10 per day. Which means, an individual on average have contact with 10 individuals daily, as the contact rate with dead bodies will not be as much as that of individuals, The study observed that [1] assumed the modification parameter associated with reduced contact with dead bodies (η) to be 0.56, so that contact with dead is 5.6 on average daily.

Individuals exposed to the virus that become infectious do so between 1 – 21 days. Thus, you have $\sigma = \frac{1}{21`}$,that is 0.048 per day.

The rate of recovery from Ebola depends on so many factors, assuming a recovery (γ **)** of 90% in 60 days for isolated individuals who are treatment, you have $\gamma = 0.015$ and death rate of isolated infectious individuals (δ_2) to be 0.0017, furthermore, Ebola typically cause death within 10 days after appearance of symptoms if ther is no any medical attention. Thus the rate of disease-induced date of infectious individuals (δ_2) is 0.1.

The average rate of proper burying of dead bodies and personal hygiene against been infected by Ebola should at least be greater than zero. Thus, $0 < \rho, \varepsilon \leq 1$. And the rate quarantining infected but asymptomatic individuals q as well as the rate of isolating symptomatically infected individuals (τ) can be greater or equal to zero.

Table 2 Parameters Value of the Model

Parameters	Values/day	Source
Λ	9340	Calculated
P	0.9	Ndanusa *et al.* (2015)
C	10	Abdulrahman (2016)
η	0.005	Abdulrahman (2016)

σ	0.048	Abdulrahman (2016)
γ	0.015	Assumed
μ	0.000046	NBS (2019)
δ_1	0.1	Abdulrahman (2016)
δ_2	0.0017	Abdulrahman (2016)
ε	(0,1)	Control Parameter
ρ	(0,1)	Control Parameter
q	(0.1)	Control Parameter
τ	(0,1)	Control Parameter

We used maple software to obtain the numerical simulation of the model equations (1–7), using variable and parameter values from tables 1 and 2 respectively, we also computed different values of the related effective reproduction number R_C in other to check the analytical results on local stability of DFE (E^*).

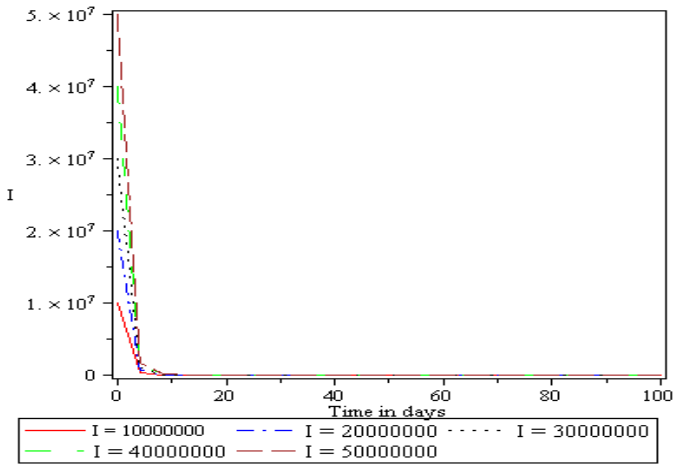

Figure 2: The total number of infectious individuals with different initial variable conditions I = 10000000, I = 20000000, I = 30000000, I = 40000000, I = 50000000 variables and parameters used are from tables 1 and 2 respectively $q = \varepsilon = \tau = \rho = 0.75$ ***which gives*** $R_C = 0.2974928862.$

Clearly Figure 2 confirms our analytical result shows that the disease-free equilibrium is locally asymptotically stable, where $R_C < 1$, which means R_C tends towards zero that means more than one individual is infecting one individual. Clearly we observed that with control the disease will be controlled.

Public enlightenment campaign on personal hygiene and compliance to public enlightenment campaign (effective campaign), quarantine and compliance to quarantine (effective quarantining), isolation and compliance to isolation (effective isolation) and proper burial and compliance to proper burial (effective proper burial) are the basic strategies for curtailing the menace of EVD in any population. Public enlightenment campaign serves as prevention from contacting the disease by susceptible individuals, quarantine serves as prevention from getting many infectious individuals, treatment can be given to those who are infectious and isolated and proper burial is also prevention as well. We compared the effect of these control strategies on the transmission of EVD.

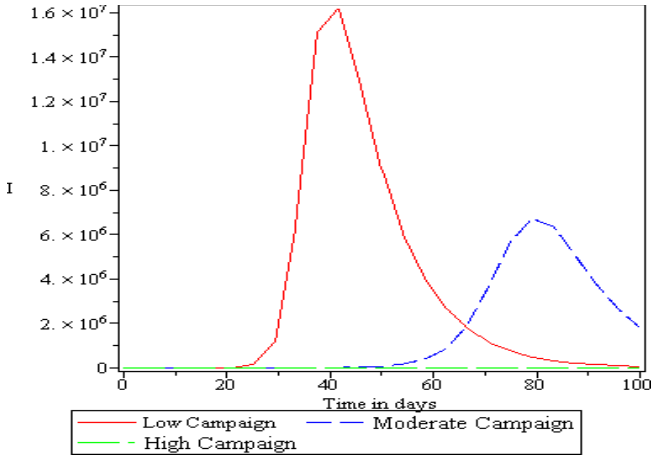

Figure 3: Comparison between level of public campaign strategies, variables and parameters used are as in table 1 and 2 respectively with $\varepsilon = 0.25$ ***for low public campaign,*** $\varepsilon = 0.50$ ***for moderate public campaign and*** $\varepsilon = 0.75$***for high public campaign.***

Clearly Figure 3 observed that EVD will be brought under control with high effective public campaign strategy on personal hygiene $\varepsilon = 0.75$ in other more than cure.

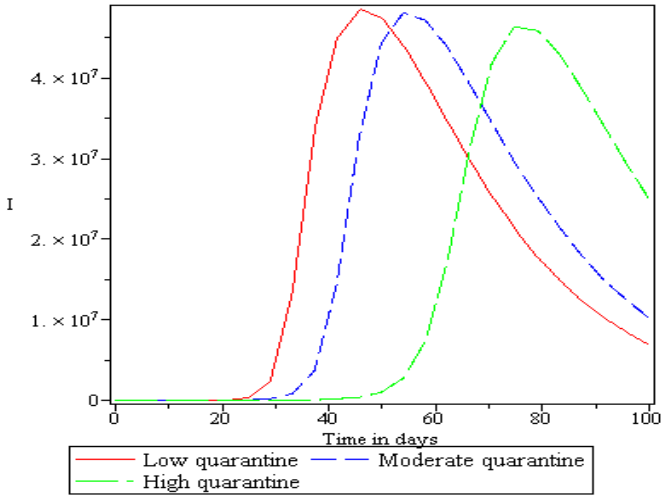

Figure 4: Comparison between level of quarantine strategies, variables and parameters used are as in table 1 and 2 respectively with $q = 0.25$ ***for low quarantine,*** $q = 0.50$ ***for moderate quarantine and*** $q = 0.75$***for high quarantine.***

Clearly Figure 4 observed that EVD will not be brought under control even with high with effective quarantine strategy. $q = 0.75$due to the fact that many individuals will not comply to submit themselves for quadrating centers or quarantine themselves willingly, with these assumptions quarantine strategy is not the best to control EVD.

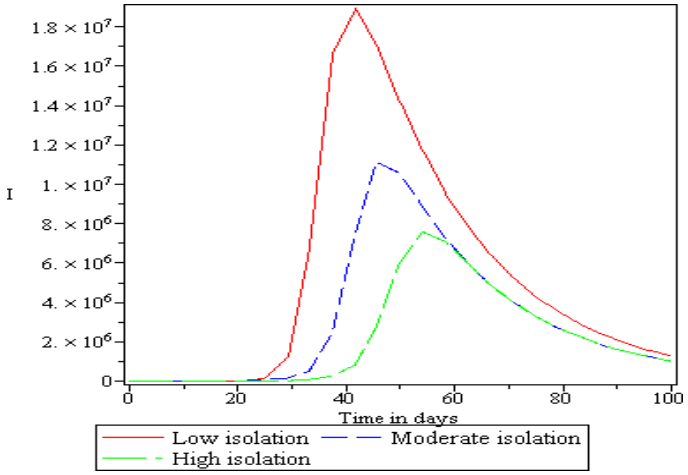

Figure 5: Comparison between level of isolation strategies, variables and parameters used are as in table 1 and 2 respectively with $\tau = 0.25$ ***for low isolation,*** $\tau = 0.50$ ***for moderate isolation and*** $\tau = 0.75$***for high isolation.***

Clearly Figure 5 confirms that EVD will be brought under control with high effective isolation strategy $\tau = 0.75$, due to the fact that infectious individual will show clinical symptoms will be reported to isolation centers willingly or unwillingly, with these assumptions isolation strategy is also a good strategy to control EVD.

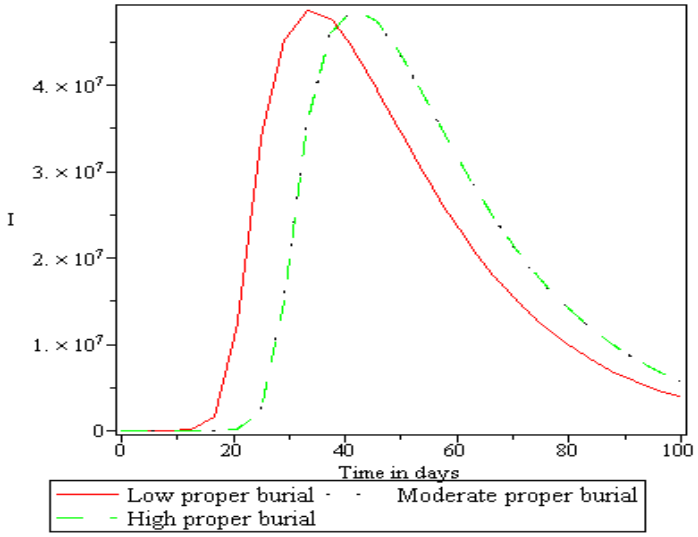

Figure 6: Comparison between level of proper burial strategies, variables and parameters used are as in table 1 and 2 respectively with $\rho = 0.25$ ***for low proper burial,*** $\rho = 0.50$ ***for moderate proper burial and*** $\rho = 0.75$ ***for high proper burial.***

Clearly Figure 6 confirms that EVD will not be brought under control even with high effective proper burial strategy $\rho = 0.75$, due to the fact that EVD is hard to be diagnose and many infectious individuals will die to infection unknowing to the individuals that will observed their burial and they will be infected too, with these assumptions proper burial strategy is not the best strategy to control EVD.

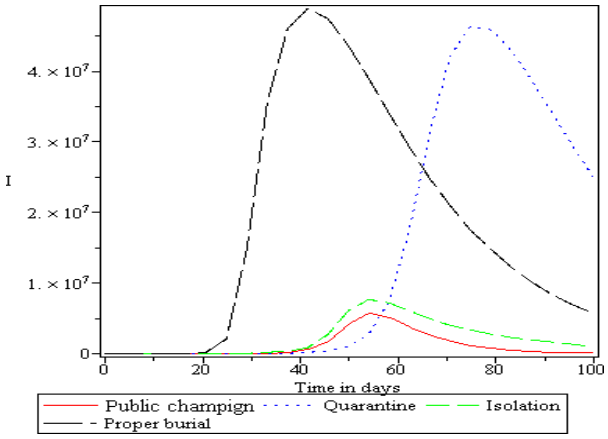

Figure 7: Comparison between combined effective control, public campaign, quarantine, isolation and proper burial control strategies, variables and parameters used is as in table 1 and 2 respectively with $\varepsilon = 0.75, q = 0, \tau = 0, \rho = 0$, $\varepsilon = 0, q = 0.75, \tau = 0, \rho = 0$, $\varepsilon = 0, q = 0, \tau = 0.75, \rho = 0$ ***and*** $\varepsilon = 0, q = 0, \tau = 0, \rho = 0.75$ ***for high control.***

Clearly Figure 7 observed that comparison between the above strategies at $\varepsilon = 0.75, q = 0, \tau = 0, \rho = 0$, $\varepsilon = 0, q = 0.75, \tau = 0, \rho = 0, \varepsilon = 0, q = 0, \tau = 0.75, \rho = 0$ and $\varepsilon = 0, q = 0, \tau = 0, \rho = 0.75$ public campaigning is highly effective followed by isolation then quarantine but proper burial is less effective in curbing EVD.

5. Conclusion

In this paper, we developed a deterministic mathematical model for transmission dynamics of Ebola virus disease control, that incorporating the constant the recruitment and numerical simulation of four control strategies: public enlighten campaign on personal hygiene, quarantine, isolation and proper burial. From the model we compute the effective reproduction number R_C and numerical simulation of effective reproduction number (R_C) was computed using different control strategies which verifies the existence of LAS of DFE if $R_C < 1$. it was observed that absence of control in a community the disease will persist within short period of time and also we observed that with moderate and high control strategies the disease will be curbed in a long run, we also verify that effective public enlighten campaign on personal hygiene is the best strategy in curbing Ebola followed by isolation strategy then quarantine and proper burial strategies.

6. Recommendations

i. Government and non-governmental organizations should strengthen routine on public enlightenment campaign on personal hygiene/sanitation, quarantining, isolation and proper burial of infectious dead bodies to the general public.
ii. Government and non-governmental organizations should encourage Ebola outbreak volunteers with proper Ebola virus disease educational training including appropriate clothing and equipment.
iii. Government and non-governmental organizations should encourage epidemiologist and researchers with funding to carry out projects and researches.
iv. Strong surveillance should be put to quarantined and isolated individuals and proper burial of infectious dead bodies.

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