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A THREE LAYERED STEADY MAGNETOHYDRODYNAMIC (MHD) THIRD GRADE BLOOD FLOW IN A STENOSED ARTERY

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Abstract.

This research modeled the steady state MHD third grade blood flow in a three layered stenosed artery. Regular perturbation method was used to obtain the flow characteristics such as the flow velocity, the volume flow rate, the shear stress and the resistance to the flow. We considered a three layered stenosed artery of about 6unit length. We reduced the model into dimensionless parameters and hypothetical values were used. The obtained results showed that, the volume flow rate and the velocity increases with increase in the magnetic field intensity H, the pressure drop P and third grade parameter α. The resistance reduces with increase in the field intensity H, the pressure drop P and third grade parameter α.

Key words: Third grade fluid, Three Layered Flow, stenosis, (MHD), third grade parameter, regular perturbation.

1. Introduction

Studies on blood flow have widely received attention in recent times due to its importance in human anatomy and physiology. It is a complex study because blood flow is a circulatory system where the flow is driven by the pumping action of the heart. (Tortora and Derrickson, *2012*).

In real life, there are many materials that can be seen with characteristics of both elasticity and viscosity. These materials are referred to as non-Newtonian fluids which blood is one of such fluids. These fluids can only be described satisfactorily by the combination of both the theory of elasticity or viscosity. According to Buchanan *et al.* (2000), blood behaves differently when flowing in large vessels, in which Newtonian behavior is expected and in medium and small vessels where non-Newtonian effects appear. Zeb *et al.* (2013) studied steady flow of an incompressible, third-grade fluid in helical screw rheometer (HSR) by "unwrapping or flattening" the channel, lands, and the outside rotating barrel. Hayat *et al*. (2015) considered steady boundary layer axi-symmetric flow of third-grade fluid over a continuously stretching cylinder in the presence of magnetic field. They used homotopy analysis method (HAM) to solve the differential equations.

Several studies have been conducted on three layered fluid flow most of which centered on the flow of blood in a three layered stenosed artery. For example, Chaturani and Biswas (1983) modelledCouette flow of blood as a three layered flow. The model basically consists of a core (red-cell suspension) and plasma (a Newtonian fluid) in the top (near the moving plate) and bottom (near the stationary plate) layers. Flow is assumed to be steady and laminar and fluids are incompressible. Dharmendra (2012) constructed a mathematical model to examine the characteristics of three layered blood flow through the oscillatory cylindrical tube (stenosed arteries). His analysis was restricted to propagation of small-amplitude harmonic waves, generated due to blood flow whose wave length is larger compared to the radius of the arterial segment. The impacts of viscosity of fluid in peripheral layer and intermediate layer on the interfaces, average flow rate, mechanical efficiency, trapping and reflux were discussed with the help of numerical and computational results. Pandey *et al.*

(2011) studied the theoretical study of two-dimensional peristaltic flow of power-law fluids in three layers with different viscosities. The analysis is carried out under low Reynolds number and long wavelength approximations.

In recent time, perturbation method has been applauded for solving non-linear models in fluid dynamics. Sankar and Hemlatha (2006) Sankar and Lee (2009), Ikpakyegh *et al*, (2018), used the perturbation method to obtain the flow variables in their studies of the pulsatile flow of non –Newtonian fluid in stenosed artery. The method seemed consistent.

In all the above studies with a few on three layered fluid model, we are motivated from the above interesting models and results on blood flow through the stenosed arteries, to develop a mathematical model to study blood flow in three layers in presence of magnetic field. Perturbation method will be used to obtain approximate analytic solution to the model.

2. Formulation of the model

Consider a fully developed flow of blood axially symmetric, and laminar in the axial direction through a circular tube with an axially symmetric mild stenosis which is influenced by magnetic field. It is assumed that the body fluid (blood) is flowing in three layers with the inner layer as a Casson fluid, the central layer of suspension of all erythrocytes as a third grade fluid and the external layer of plasma as a Newtonian fluid. Consider blood as a magnetic fluid since red blood cells are a major bio-magnetic substance, blood flow will be influenced by the magnetic field. Hence in the present study, we considered the flow of blood to be unidirectional and in the axial direction as can be seen by the flow diagram below. (Fernando, 2008), presented the Cauchy stress tensor for both Newtonian and non-Newtonian fluids by

$$\tau = -PI + \sum_{j=1}^{n} S_j \qquad 1$$

S_j, $j = 1,2,3$, are called the stress tensors, P is the pressure force due to fluid flow. For the third grade fluid we have $n = 3$ and the first three tensors S_j are given by

$$S_1 = \overline{\mu} A_1 \qquad 2$$

$$S_2 = \alpha_1 A_2 + \alpha_2 A_1^2 \qquad 3$$

$$S_3 = \beta_1 A_3 + \beta_2 \left(A_2 A_1 + A_1 A_2 \right) + \beta_3 \left(tr A_1^2 \right) A_1 \qquad 4$$

Where $\overline{\mu}$ is the coefficient of sheer viscosity and $\alpha_i, (i = 1,2), \beta_i, (i = 1,2,3)$ are material constants. A_n Are called Rivlin Ericksen tensors and are defined by the recursion relation

$$A_n = \frac{D}{Dt} A_{n-1} + A_{n-1}\left(\nabla \overline{u}\right) + \left(\nabla \overline{u}\right)^T A_{n-1},\ n > 1 \qquad 5$$

$$A_1 = \left(\nabla \overline{u}\right) + \left(\nabla \overline{u}\right)^T \qquad 6$$

When $\beta_j = 0, (j = 1,2,3)$, then, the above model reduces to second grade fluid model and if $\alpha_i = 0, (i = 1,2)$ and $\beta_j = 0, (j = 1,2,3)$, the model reduces to classical Navier stokes viscous fluid model (Fernando, 2008).

Let the velocity field for the fluid flow be given as a vector field, we assume the flow to be in the positive z-direction.

This implies that the velocity field and the shear stresses are functions of r alone. Hence, the constitutive equation of motion for a third grade fluid flow is

$$\tau = \tau_{zr} = \overline{\mu} \frac{\partial \overline{u}}{\partial \overline{r}} + 2\beta_3 \left(\frac{\partial \overline{u}}{\partial \overline{r}} \right)^3 \qquad 7$$

the tunica media contains a large amount of smooth muscles that allows a more efficient exchange of gases and nutrients in blood within the capillary beds. Thus, due to the presence of hemoglobin (iron compound) in the red blood cells in that layer, we regard blood in the layer as a suspension of magnetic particles. Hence, the continuity equation and the momentum equations are respectively given by

$$div \overline{u} = 0 \qquad 8$$

$$\frac{\partial}{\partial \overline{z}} \overline{P} = -\frac{1}{\overline{r}} \frac{\partial}{\partial \overline{r}} \left(\overline{r}\, \overline{\tau}_c \right) \qquad 9$$

$$\frac{\partial}{\partial \overline{z}} \overline{P} = -\frac{1}{\overline{r}} \frac{\partial}{\partial \overline{r}} \left(\overline{r}\, \overline{\tau}_T \right) - \sigma \beta_0^{\ 2} \overline{u}_T \qquad 10$$

$$\frac{\partial}{\partial \bar{z}}\bar{P} = -\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\,\bar{\tau}_N\right) \qquad 11$$

The relations between the shear stress and the strain rate of the fluid in motion in the three layers, that is, the constitutive equation of motion of the fluid flow are given by

$$\sqrt{\bar{\tau}_c} = \sqrt{\bar{\tau}_y} + \sqrt{-\bar{\mu}\frac{\partial}{\partial \bar{r}}\bar{u}_c}\text{, if }\quad \sqrt{\bar{\tau}_c} \ge \sqrt{\bar{\tau}_y}$$

and $\bar{R}_p(\bar{z}) \le \bar{r} \le \bar{R}_2(\bar{z})$ 12

$$\bar{\tau}_T = -\left(\bar{\mu}_T \frac{\partial}{\partial \bar{r}}\bar{u}_T + 2\beta_3\left(\frac{\partial}{\partial \bar{r}}\bar{u}_T\right)^3\right),$$

If $\bar{R}_2(\bar{z}) \le \bar{r} \le \bar{R}_1(\bar{z})$ 13

$$\bar{\tau}_N = -\bar{\mu}\frac{\partial}{\partial \bar{r}}\bar{u}_N\text{, if } \bar{R}_1(\bar{z}) \le \bar{r} \le \bar{R}(\bar{z}) \qquad 14$$

$$\frac{\partial}{\partial \bar{r}}\bar{u} = 0, \qquad \text{if} \qquad 0 \le \bar{r} \le \bar{R}(\bar{z}) \qquad 15$$

The boundary conditions are given as follows.

$\bar{\tau}_C$ is finite at $\bar{r} = 0$ 16

$\bar{u}_N = 0$ at $\bar{r} = \bar{R}(\bar{z})$ 17

$\bar{\tau}_C = \bar{\tau}_T$ and $\bar{u}_C = \bar{u}_T$ at $\bar{r} = \bar{R}_2(\bar{z})$ 18

$\bar{\tau}_N = \bar{\tau}_T$ and $\bar{u}_N = \bar{u}_T$ at $\bar{r} = \bar{R}_1(\bar{z})$ 19

$\bar{u}_C = 0$ at $\bar{r} = 0$ 20

The geometry of the stenosis is given in figure1

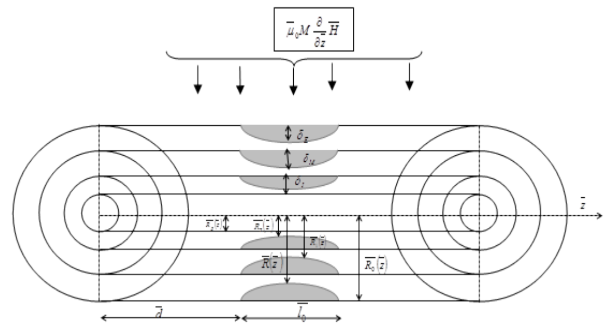

Figure 1: Geometry of the three layered stenosed artery

and is defined by

$$\bar{R}(\bar{z}) = \begin{cases} \bar{R}_0 - (\bar{\delta}_E)e^{\left(\frac{m\varepsilon\bar{z}}{\bar{R}_0}\right)^2} & in\ \bar{d} \le \bar{z} \le \bar{d} + \bar{l}_0 \\ \bar{R}_0 & \text{in the normal artery region} \end{cases} \qquad 21$$

$$\bar{R}_1(\bar{z}) = \begin{cases} \beta\bar{R}_0 - (\bar{\delta}_M)e^{\left(\frac{m\varepsilon\bar{z}}{\bar{R}_0}\right)^2} & in\ \bar{d} \le \bar{z} \le \bar{d} + \bar{l}_0 \\ \beta\bar{R}_0 & \text{in the normal artery region} \end{cases} \qquad 22$$

$$\bar{R}_2(\bar{z}) = \begin{cases} \beta_1\bar{R}_0 - (\bar{\delta}_I)e^{\left(\frac{m\varepsilon\bar{z}}{\bar{R}_0}\right)^2} & in\ \bar{d} \le \bar{z} \le \bar{d} + \bar{l}_0 \\ \beta_1\bar{R}_0 & \text{in the normal artery region} \end{cases} \qquad 23$$

Where β is the ratio of the tunica media radius to the normal artery radius. β_1 is the ratio of the tunica intima radius to the normal artery radius $\bar{\delta}_E, \bar{\delta}_M$ and $\bar{\delta}_I$ are respectively the height of the stenosis in the tunics.

$$\left.\begin{array}{l} z = \dfrac{\bar{z}}{\bar{R}_0}, r = \dfrac{\bar{r}}{\bar{R}_0}, R(z) = \dfrac{\bar{R}(\bar{z})}{\bar{R}_0}, R_1(z) = \dfrac{\bar{R}_1(\bar{z})}{\bar{R}_0}, \\ \delta_E = \dfrac{\bar{\delta}_E}{\bar{R}_0}, \delta_M = \dfrac{\bar{\delta}_M}{\bar{R}_0}, \delta_I = \dfrac{\bar{\delta}_I}{\bar{R}_0}, u_C = \dfrac{\bar{u}_C}{\bar{R}_0^2/4\bar{\mu}_C}, \\ \alpha_C^2 = \alpha_N^2 = \alpha_T^2 = \dfrac{\bar{R}_0^2\bar{\omega}\bar{\rho}_T}{\bar{\mu}_T}, \dfrac{\partial}{\partial \bar{z}}\bar{P} = P, H = \dfrac{\bar{H}}{q_0}, \\ u_T = \dfrac{\bar{u}_T}{\bar{R}_0^2/4\bar{\mu}_T}, u_N = \dfrac{\bar{u}_N}{\bar{R}_0^2/4\bar{\mu}_N}, \tau_C = \dfrac{\bar{\tau}_c}{q_0\bar{R}_0/2}, \\ \tau_T = \dfrac{\bar{\tau}_T}{q_0\bar{R}_0/2}, \tau_N = \dfrac{\bar{\tau}_N}{q_0\bar{R}_0/2}, \theta = \dfrac{\bar{\tau}_y}{q_0\bar{R}_0/2}, \end{array}\right\} \qquad 24$$

We introduce the above non-dimensional variables to reduce our model into a model with dimensionless variables for easy computation.

Thus, Substituting (24) into (8) to (23) gives

$$2P = -\frac{1}{r}\frac{\partial}{\partial r}(r\tau_c) \quad 25$$

$$4P = -\frac{2}{r}\frac{\partial}{\partial r}(r\tau_T) - Mu_T \quad 26$$

$$2P = -\frac{1}{r}\frac{\partial}{\partial r}(r\tau_N) \quad 27$$

and the constitutive equations becomes

$$-\frac{\partial}{\partial r}u_c = 2\left(\tau_c - 2\sqrt{\theta\tau_c} + \theta\right), \text{ if } \sqrt{\tau_c} \geq \sqrt{\theta} \text{ and } R_p(z) \leq r \leq R_2(z) \quad 28$$

$$\tau_T = -\frac{1}{2}\frac{\partial u_T}{\partial r} - \alpha_T\left(\frac{\partial u_T}{\partial r}\right)^3, \text{ if } R_2(z) \leq r \leq R_1(z) \quad 29$$

$$\tau_N = -\frac{1}{2}\frac{\partial}{\partial r}u_N, \text{ if } R_1(z) \leq r \leq R(z) \quad 30$$

With the boundary conditions becoming

τ_C is finite at $r = 0$ 31

$u_N = 0$ at $r = R(z)$ 32

$\tau_C = \tau_T$ and $u_T = u_C$ at $r = R_2(z)$ 33

$\tau_T = \tau_N$ and $u_T = u_N$ at $r = R_1(z)$ 34

$u_C = 0$ at $r = 0$ 35

3. Solution of the Three Layered Model Equations using Regular Perturbation Method

We present the following perturbation series as thus,

$$\begin{aligned}
\tau_N &= \tau_{N0} + \alpha_N^2\tau_{N1} + \ldots \\
\tau_T &= \tau_{T0} + \alpha_T^2\tau_{T1} + \ldots \\
\tau_C &= \tau_{C0} + \alpha_C^2\tau_{C1} + \ldots \\
u_N &= u_{N0} + \alpha_N^2 u_{N1} + \ldots \\
u_T &= u_{T0} + \alpha_T^2 u_{T1} + \ldots \\
u_C &= u_{C0} + \alpha_C^2 u_{C1} + \ldots
\end{aligned} \quad 36$$

Expanding (25) to (30) in the perturbation series of α^2 and equate coefficients of α we have

$$\alpha^0: \begin{cases}
2P = -\frac{2}{r}\frac{\partial}{\partial r}(r\tau_{C0}) \\
4P = -\frac{2}{r}\frac{\partial}{\partial r}(r\tau_{T0}) - Mu_{T0} \\
2P = -\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{N0}) \\
-\frac{\partial}{\partial r}(u_{C0}) = 2\left(\tau_{C0} - \sqrt{(\theta\tau_{C0})} + \theta\right) \\
\tau_{T0} = -\frac{1}{2}\frac{\partial}{\partial r}(u_{T0}) \\
\tau_{N0} = -\frac{1}{2}\frac{\partial}{\partial r}(u_{N0})
\end{cases} \quad 37$$

$$\alpha^2: \begin{cases}
-\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{C1}) = 0 \\
-\frac{2}{r}\frac{\partial}{\partial r}(r\tau_{T1}) - Mu_{T1} = 0 \\
-\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{N1}) = 0 \\
-\frac{\partial}{\partial r}(u_{C1}) = 2\tau_{C1}\left(1 - \sqrt{\left(\frac{\theta}{\tau_{C0}}\right)}\right) \\
\tau_{T1} = -\frac{1}{2}\frac{\partial}{\partial r}(u_{T1}) - \left(\frac{\partial}{\partial r}(u_{T0})\right)^3 \\
\tau_{N1} = -\frac{1}{2}\frac{\partial}{\partial r}(u_{N1})
\end{cases} \quad 38$$

Solving (37) and (38) as system together with the boundary conditions we have the flow characteristics as follows

$$\tau_C = -\frac{r}{2}P \quad 39$$

$$\tau_T = \frac{1}{4}\left(F\frac{\partial}{\partial z}H - 4P\right)r + \frac{1}{4}\left(2P - F\frac{\partial}{\partial z}H\right)\frac{R_2^2}{r} \quad 40$$

$$\tau_N = \frac{1}{4}\left(F\frac{\partial}{\partial z}H\right)\frac{R_1^2}{r} + \frac{1}{4}\left(2P - F\frac{\partial}{\partial z}H\right)\frac{R_2^2}{r} - (r)P \quad 41$$

$$u_N = P\left(r^2 - R^2\right) + \frac{1}{2}\left(2P - F\frac{\partial}{\partial z}H\right)R_2^2\ln\left(\frac{R}{r}\right) + \frac{1}{2}\left(F\frac{\partial}{\partial z}H\right)R_1^2\ln\left(\frac{R}{r}\right) \quad 42$$

$$\begin{aligned} u_T &= P\left(2R_1^2 - R^2 - r^2\right) + \frac{1}{4}\left(F\frac{\partial}{\partial z}H\right)\left(r^2 - R_1^2\right) \\ &+ \frac{1}{2}\left(2P - F\frac{\partial}{\partial z}H\right)R_2^2\ln\left(\frac{r}{R_1}\right) \\ &+ \frac{1}{2}\left(2P - F\frac{\partial}{\partial z}H\right)R_2^2\ln\left(\frac{R}{R_1}\right) + \frac{1}{2}\left(F\frac{\partial}{\partial z}H\right)R_1^2\ln\left(\frac{R}{R_1}\right) \\ &+ \frac{3}{8}\alpha_T\left(4P + F\frac{\partial}{\partial z}H\right)^2\left(2P - F\frac{\partial}{\partial z}H\right)R_2^2\left(r^2 - R_1^2\right) \\ &+ \frac{1}{16}\alpha_T\left(4P + F\frac{\partial}{\partial z}H\right)^3\left(R_1^4 - r^4\right) \\ &+ \frac{3}{4}\alpha_T\left(4P + F\frac{\partial}{\partial z}H\right)\left(2P - F\frac{\partial}{\partial z}H\right)^3 R_2^4\ln\left(\frac{R_1}{r}\right) \\ &+ \frac{1}{8}\alpha_T\left(2P - F\frac{\partial}{\partial z}H\right)^3\left(\frac{R_2^6}{R_1^2} - \frac{R_2^6}{r^2}\right) \end{aligned} \quad 43$$

$$\begin{aligned} u_C &= 2\theta\left(R_2 - r\right) + \frac{2}{3}\sqrt{2(\theta P)}(r)^{\frac{3}{2}} - \frac{2}{3}\sqrt{2(\theta P)}\left(R_2\right)^{\frac{3}{2}} \\ &+ P\left(2R_1^2 - R^2 - R_2^2\right) + \frac{1}{4}\left(F\frac{\partial}{\partial z}H\right)\left(R_2^2 - R_1^2\right) \\ &+ \frac{1}{2}\left(2P - F\frac{\partial}{\partial z}H\right)R_2^2\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{2}\left(2P - F\frac{\partial}{\partial z}H\right)R_2^2\ln\left(\frac{R}{R_1}\right) \\ &+ \frac{1}{2}\left(F\frac{\partial}{\partial z}H\right)R_1^2\ln\left(\frac{R}{R_1}\right) + \frac{1}{16}\alpha_C\left(4P + F\frac{\partial}{\partial z}H\right)^3\left(R_1^4 - R_2^4\right) \\ &+ \frac{3}{8}\alpha_C\left(4P + F\frac{\partial}{\partial z}H\right)^2\left(2P - F\frac{\partial}{\partial z}H\right)R_2^2\left(R_2^2 - R_1^2\right) \\ &+ \frac{3}{4}\alpha_C\left(4P + F\frac{\partial}{\partial z}H\right)\left(2P - F\frac{\partial}{\partial z}H\right)^3 R_2^4\ln\left(\frac{R_1}{R_2}\right) \\ &+ \frac{1}{8}\alpha_C\left(2P - F\frac{\partial}{\partial z}H\right)^3\left(\frac{R_2^6}{R_1^2} - R_2^4\right) \end{aligned} \quad 44$$

The total velocity of the flow is given by

$$u = u_C + u_T + u_N \quad 45$$

And the volume flow rate is

$$\begin{aligned} Q &= 2\pi\int_0^{R_2(z)} u_C(r,t)\,rdr \\ &+ 2\pi\int_{R_2(z)}^{R_1(z)} u_T(r,t)\,rdr \\ &+ 2\pi\int_{R_1(z)}^{R(z)} u_N(r,t)\,rdr \end{aligned} \quad 46$$

The resistance to the flow is

$$\xi = \frac{-P}{Q} \quad 47$$

The wall shear stress of the three layered fluid flow is given by

$$\tau_w = \tau_N\big|_{r=R} \quad 48$$

$$\tau_w = \frac{1}{4}\left(F\frac{\partial}{\partial z}H\right)\frac{R_1^2}{R} + \frac{1}{4}\left(2P - F\frac{\partial}{\partial z}H\right)\frac{R_2^2}{R} - (R)P \quad 49$$

4. Results and Discussion

Numerical simulations are very useful tools in research. With numerical experiments, it is possible to extract information difficult or impossible to obtain in the laboratory, in most cases, giving a better understanding of the problem under study. We have used the following to simulate the model; The value of β is taken as 0.8, β_1 as 0.6 respectively. The value 0.18 is used for δ_N, 0.17 for δ_T and 0.16 for δ_C.

In what follows, blood is a very complex fluid. It is a mixture of cells, proteins, lipoproteins, and ions by which nutrients and wastes are transported as a result, does not exhibit a constant viscosity at all flow rates and can be regarded as non-Newtonian and it is best studied under the field of bio-Rheology. The rheological properties are modeled into the determining third grade parameter $\alpha = \alpha_C = \alpha_T = \alpha_N$ and the model parameters with the variables were reduced to dimensionless parameters and variable, hypothetical values are thought of as a good guide. We used maple computer software to numerically simulate the model.

In the solution approach, Figure 2 shows that the volume flow rate increases as the magnetic field increase.

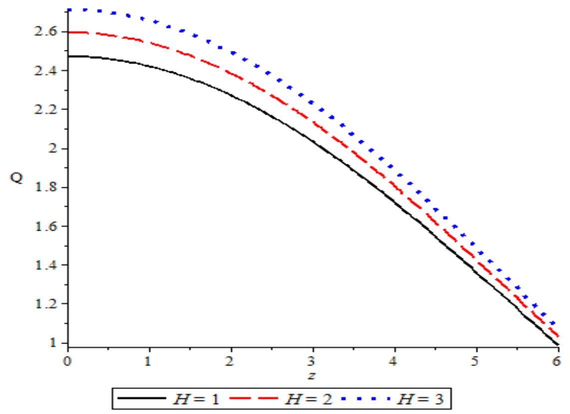

Figure 2: Effect of the Magnetic Field on the Volume Flow Rate

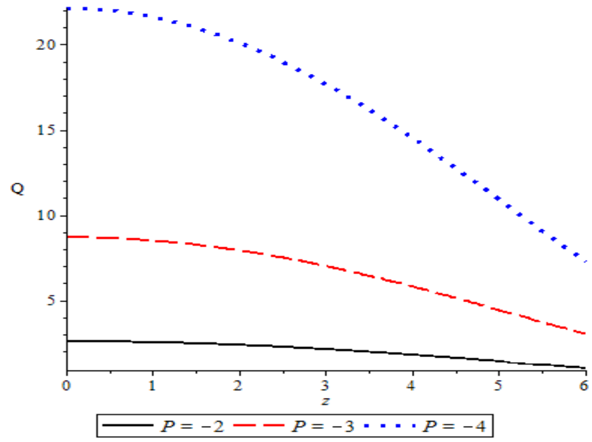

Figure 3: Effect of Pressure Drop on the Volume Flow Rate

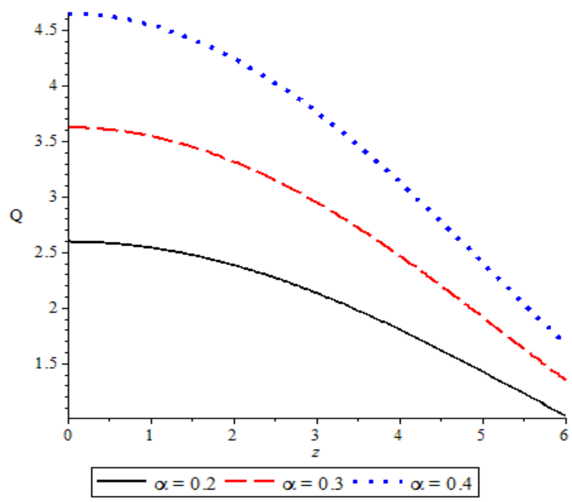

Figure 4: Effect of the Third grade Parameter on the Volume Flow Rate

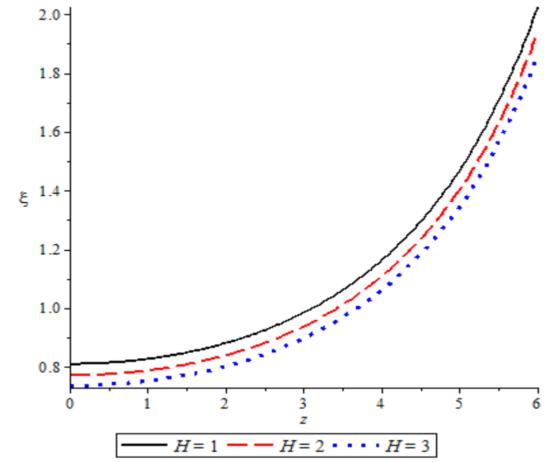

Figure 5: Effect of the Magnetic Field on the Resistance to the Flow

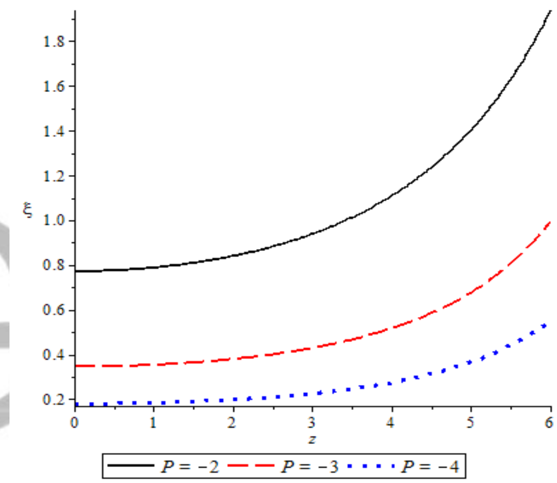

Figure 6: Effect of the Pressure Drop on the Resistance to the Flow

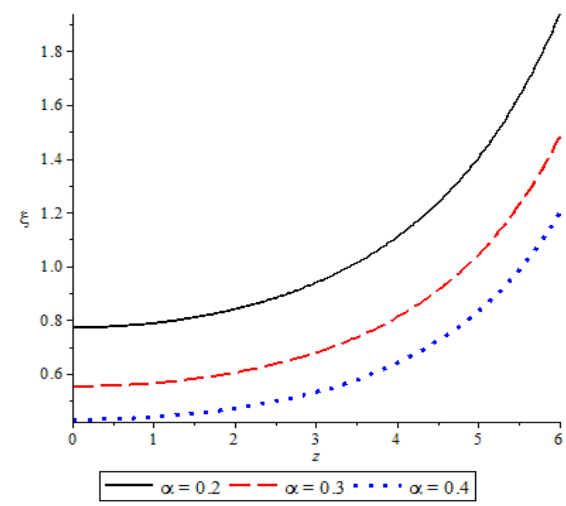

Figure 7: Effect of the Third Grade Parameter of the Resistance to the Flow

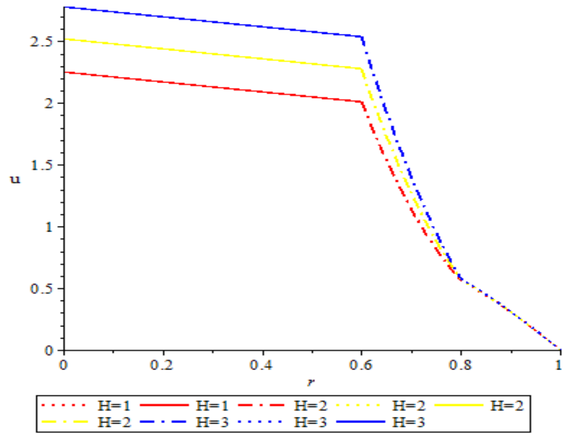

Figure 8: The Effect of the Magnetic Field on the Velocity Profile

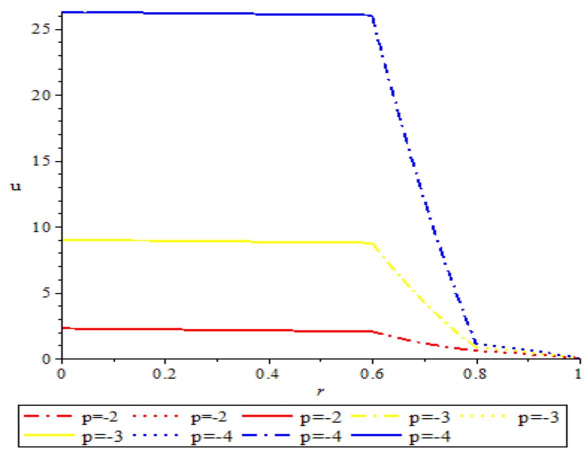

Figure 9: The Effect of the Pressure Drop on the Velocity Profile

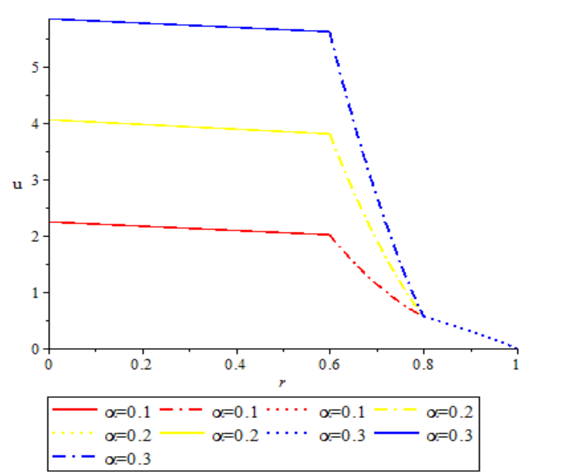

Figure 10: The Effect of the Third Grade Parameter on the Velocity Profile

The blood pressure in the circulation is principally due to the pumping action of the heart (Caro, 1978). The mean blood flow rate depends on both blood pressure and the resistance to flow offered by the flow channels (vessels). Blood pressure drops over the entire circulation as can be seen by Figure 3, although most of the fall occurs along the small arteries (Klabunde, 2005). Due to viscous losses of energy (third grade parameter), mean blood pressure decreases as the circulating blood moves away from the heart through arteries and increases steadily to capillaries (Mahler, *et al*,1979) as shown by Figure 4.

Figures 5, 6 and 7 shows the resistance to flow as the magnetic field intensity increases, the pressure drops, and the third grade parameter increases. The vessels, offers low resistance to the flow as the magnetic field intensity and the third grade parameter increases. As the pressure drops, the resistance also reduces.

Looking at Figures 8, 9 and 10, you can observe three very different velocity profiles depending on the fluid layer. For all these fluid layers, the shear rate at the walls (i.e. the slope of the velocity profile near the walls) will determine the viscosity. The ideal characterization of viscosity is key in determining if a fluid is Newtonian or non-Newtonian, and what range of shear rates needs to be considered for a specific application. Any constriction of the arteries (stenosis) increases the velocity with increase in the magnetic field intensity and the third grade parameter and hence the pressure drops across all the layers. In such instances, the heart may have to work harder to compensate. And at times of stress, where an increased flow rate is required, there can be a breakdown.

5 Conclusions

In the research, we considered a three layered stenosed artery. We reduced the model into dimensionless parameters and hypothetical values were used. The obtained results showed that, there is a decrease in pressure across each section of the artery. the volume flow rate and the velocity increases with increase in the magnetic field intensity H, the pressure drop P and third grade parameter α. The resistance reduces with increase in the field intensity H, the pressure drop P and third grade parameter α.

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