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# Analytical solution of partial differential equation with variable coefficients by differential transformation technique

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**Abstract**

*We examined the differential transformation (DTM) approach for finding the approximate and actual solutions of some variable-coefficient partial differential equations in this article. A few cases show the effectiveness of the proposed approach. The effects show that the proposed technique represents very good performance and simplicity, and that it could be extended to numerous problems in linear and nonlinear mathematical physics and engineering.*

***Keywords: Differential transform transformation method; partial differential equations with variable coefficients***

## 1.0 Introduction

So far, in many fields such as hydrodynamics, plasma physics, nonlinear optics and others, more and more nonlinear equations representing the motion of distant waves localized on a tiny surface have been presented. Research into the actual answers of certain nonlinear equations is fascinating and extensive. Many authors in recent years have focused on studying nonlinear equation solutions and have used various strategies, including the Backlund transform (Ablowitz and Clarkson, 1991; Coely, 2001), the Darboux transform (Wadati et al., 1975), and the inverse scattering method (Gardner et al., 1967), Hirota's bilinear technique (Hirota, 1971) and the Tanh technique (Malfeit, 1992), the sine-cosine method (Yan, 1996; Yan and Zhang, 2000), the homogeneous fitting method (Wang, 1996; Yan and Zhang, 2001) and the Riccati augmentation technique with continuous coefficients (Yan, 1996; Yan and Zhang, 2001). (January 2001). Fan (2001) proposed an improved tanh characteristic approach and symbolic computation to fix the new related modified KdV equations to obtain four types of soliton solutions. Compared to the tanh feature approach, this technique has some advantages. It does not most effectively use a simplified method to generate an algebraic machine, but it could also discover single soliton responses without additional effort (Fan and Zhang, 1998; Hirota and Satsuma, 1981; Malfliet, 1992; Satsuma and Hirota, 1982; Wu et al., 1999). The Burgers equation has a wide range of programs, consisting of the approximate idea of drift by a surprise wave propagating in a viscous fluid (Cole, 1951) and the Burgers version of turbulence (Burgers, 1948, it could be for any initial conditions be solved analytically (Hopf, 1950) Finite element techniques using a time-based grid, such as Galerkin and Petrov-Galerkin finite element methods, have been performed for hydrodynamic problems

(Caldwell et al., 1981; Herbst et al., 1982. (Rubin and Graves, 1975) Numerical responses were constructed using global capabilities for cubic splines to obtain structures or diagonally dominant equations, which are solved to perceive system evolution. A collocation solution that uses cubic spline interpolation functions to generate three coupled sets of equations for the structured variable and its first derivatives (Caldwell and Hinton, 1987). Aliet al. (1992) used finite element strategies to solve Burger's anger. The finite detail technique is used along with the collocation technique on a constant grid of cubic spline factors. Cubic Spline produces a tridiagonal matrix system solved by Thomas' rule set. Soliman (2000) developed a method to fix the Burgers equation by using similarity reductions for partial differential equations. This approach is based solely on similarity reductions of the Burgers equation on a tiny subdomain. The resulting similarity equation is integrated analytically. The analytical answer is then applied to the Burgers equation to approximate the flux vector. Esipov developed the networked machine (1992). He miles a simple model of gravity-induced sedimentation or evolution of scaled concentrations of particles in liquid suspensions or colloids (Nee and Duan, 1998). In this article we hope to offer a reliable technique for fixing partial differential equations with variable coefficients. The approach is called Differential Rework Technique (DTM) and is based entirely on the extension of the Taylor collection. However, the method of calculation distinguishes it from the same old super order Taylor detection method. This approach is based on similarity reductions of the Burgers equation to a tiny subdomain. A polynomial is used to represent the approach and generate an analytical answer. Pukhov (1986) was the first to introduce the notion of difference transform used to solve linear and nonlinear starting price problems in electrical circuit evaluation. This method for PDEs was further developed by Chen and Ho (1999) who observed closed-loop collection solutions to numerous linear and nonlinear entry fee problems. Halim (Hassan, 2008) recently confirmed that this technique is relevant for a variety of PDEs and that the closed-form solutions can be easily constructed. Halim (Hassan, 2008) also compared very well with the domain decomposition method. The purpose of this article is to extend the DTM approach (Pukhov, 1986; Chen and Ho, 1999; Hassan, 2008; Ali and Raslan, 2009) to solve partial differential equations with variable coefficients (Ali and Raslan, 2009). . The structure of this document is as follows: we started with some simple definitions and how to use the proposed method, and then used the reduced differential transformation method to clarify specific test cases to illustrate its power and efficiency

## 2.0 Research methods

We have considered that u(y, t) is analytic and constantly differentiated in the domain to illustrate the basic idea of the DTM.

Let $U_n = \frac{1}{n!}\left[\frac{\partial^n u(y,t)}{\partial t^n}\right]$ (1)

The transformed curve, regularly referred to as the T-curve, is plotted using the Great Britain(y) spectrum. $U_k(y)$ has a differential inverse remodeling defined as follows:

$$u(y,t) = \sum_{n=0}^{\infty} U_n(y)(t-t_0)^n \quad (2)$$

Combining (1) and (2), it can be obtained that

$$u(y,t)=\sum_{n=0}^{\infty}\frac{1}{n!}\left[\frac{\partial^n u(y,t)}{\partial t^n}\right](t-t_0)^n \quad (3)$$

When $(t_0)$ are taken as $(t_0=0)$ then equation (3) is expressed as

$$u(y,t)=\sum_{n=0}^{\infty}\frac{1}{n!}\left[\frac{\partial^n u(y,t)}{\partial t^n}\right]t^n, \quad (4)$$

and equation (2) can be written as

$$u(y,t)=\sum_{n=0}^{\infty}U_n(y)t^n \quad (5)$$

In real application, the function $u(y,t)$ by a finite series of equation (5) can be written as

$$u(y,t)=\sum_{n=0}^{k}U_n(y)t^n \quad (6)$$

usually, the values of $k$ is decided by convergence of the series coefficients. The table below was deduced from equation (3) and (4)

**Table 1:** *The differential transforms identities used in this study*

| Differentiable Function | DTM expression |
|---|---|
| $u(y,t)=w(y,t)\pm v(y,t)$ | $U_n(y)=w_n(y)\pm V_n(y)$ |
| $u(y,t)=\lambda v(y,t)$ | $U_n(y,)=\lambda V_n(y)$ |
| $u(y,t)=\frac{\partial^m w(y,t)}{\partial t^m}$ | $U_n(y)=\frac{(n+m)!}{n!}w_n(y)$ |
| $u(y,t)=\frac{\partial\ w(y,t)}{\partial t}$ | $U_n(y)=\frac{\partial}{\partial y}w_n(y)$ |

To illustrate the aforementioned identities, some Examples of partial differential equations with Variable coefficients are discussed in details and the obtained results are in excellent agreement with the one found by the variation iteration method.

***Application***

Here, the extended differential transformation method (DTM) is used to find the solutions of the PDEs in one, two and three dimensional variable coefficients, and compared with other method

***Example 1**:* Consider the heat equation of the form $u_{ty}=-ku_{yy}$ and the initial condition $U_0=\ell^{-y^2}$ where $u=u(y,t)$ is a function of the variables y and t. we can find the transformed form of equation as;

$$(n+1)\ \frac{d}{dy}U_{n+1}=-k\frac{d^2}{dy^2}U_n$$

$$if \quad n=0,$$

$$(0+1)\ \frac{d}{dy}U_{0+1}=-k\frac{d^2}{dy^2}U_0$$

$$U_1=-k\ell^{-y}$$

$$n=1$$

$$(1+1)\ \frac{d}{dy}U_{1+1}=-k\frac{d^2}{dy^2}U_1$$

$$2U_2=-k..k\frac{d^2}{dy^2}(\ell^{-y})$$

$$U_2=\frac{k^2}{2!}\ell^{-y}$$

$$n=2$$

$$(2+1)\ \frac{d}{dy}U_{2+1}=-k\frac{d^2}{dy^2}U_2$$

$$3U_3 = -\frac{k^3}{2!}\ell^{-y}, \qquad U_3 = -\frac{k^3}{3!}\ell^{-y}$$

Then, the general solution is given as:

$$U_n = \frac{(-1)^n k^n}{n!}\ell^{-y}$$

***Example 2**:* Consider the heat equation of the form $u_{ty} = m^2 u_y + u_{yy}$ and the initial condition $U_0 = \ell^y$

where $u = u(y,t)$ is a function of the variables y and t. we can find the transformed form of equation as;

$$(k+1)\frac{d}{dy}U_{K+1} = m^2\frac{d}{dy}U_k + \frac{d^2}{dy^2}U_k,$$

$$if \quad k = 0,$$

$$(0+1)\frac{d}{dy}U_{0+1} = m^2\frac{d}{dy}U_0 + \frac{d^2}{dy^2}U_0,$$

$$U_1 = (m^2+1)\ell^y$$

$$for \quad k = 1$$

$$(1+1)\frac{d}{dy}U_{1+1} = m^2\frac{d}{dy}U_1 + \frac{d^2}{dy^2}U_1,$$

$$2U_2 = m^2\frac{d}{dy}(m^2+1)\ell^y + \frac{d^2}{dy^2}(m^2+1)\ell^y,$$

$$2U_2 = m^2(m^2+1)\ell^y + (m^2+1)\ell^y$$

$$U_2 = \frac{(m^2+1)^2\ell^y}{2!},$$

$$for \quad k = 2,$$

$$(2+1)\frac{d}{dy}U_{2+1} = m^2\frac{d}{dy}U_2 + \frac{d^2}{dy^2}U_2,$$

$$3U_3 = \frac{1}{2!}\left[m^2\frac{d}{dy}(m^2+1)\ell^y + \frac{d^2}{dy^2}(m^2+1)\ell^y\right]$$

$$U_3 = \frac{(m^2+1)^3\ell^y}{2!}$$

Then, the general solution is given as

$$U_k = \frac{(m^2+1)^k}{k!}\ell^y$$

***Example 3**:* Consider the heat equation of the form $u_{ty} = m^2 u_y + n^2 u_{yy}$ and the initial condition and the initial condition $U_0 = \ell^{-y}$

where $u = u(y,t)$ is a function of the variables y and t. we can find the transformed form of equation as;

$$(k+1)\frac{d}{dy}U_{K+1} = m^2\frac{d}{dy}U_k + n^2\frac{d^2}{dy^2}U_k,$$

$$if \quad k = 0,$$

$$(0+1)\frac{d}{dy}U_{0+1} = m^2\frac{d}{dy}U_0 + n^2\frac{d^2}{dy^2}U_0,$$

$$U_1 = (n^2 - m^2)\ell^{-y}$$

$$for \quad k = 1$$

$$(1+1)\frac{d}{dy}U_{1+1} = m^2\frac{d}{dy}U_1 + n^2\frac{d^2}{dy^2}U_1,$$

$$2U_2 = m^2\frac{d}{dy}(n^2-m^2)\ell^{-y} + \frac{d^2}{dy^2}(n^2-m^2)\ell^{-y},$$

$$2U_2 = n^2(n^2-m^2)\ell^{-y} - m^2(n^2-m^2)\ell^{-y}$$

$$U_2 = \frac{(n^2-m^2)^2\ell^{-y}}{2!},$$

$$for \quad k = 2,$$

$$(2+1)\frac{d}{dy}U_{2+1} = m^2\frac{d}{dy}U_2 + n^2\frac{d^2}{dy^2}U_2,$$

$$3U_3 = \frac{1}{2!}\left[m^2\frac{d}{dy}(n^2-m^2)^2\ell^{-y} + n^2\frac{d^2}{dy^2}(n^2-m^2)^2\ell^{-y}\right]$$

$$U_3 = \frac{(n^2-m^2)^3\ell^{-y}}{3!}$$

Then, the general solution is given as

$$U_k = \frac{(n^2-m^2)^k\ell^{-y\ y}}{k!}$$

***Example 4****:* Consider the heat equation of the form $u_{ty} = n^2 u_{yy}$ and the initial condition and the initial condition $U_0 = \ell^{-y}$

where $u = u(y,t)$ is a function of the variables y and t. we can find the transformed form of equation as;

$$(k+1)\frac{d}{dy}U_{k+1} = \frac{d^2}{dy^2}U_{k,}$$

*if* $k = 0,$

$$(0+1)\frac{d}{dy}U_{0+1} = n^2 \frac{d^2}{dy^2}U_0$$

$$U_1 = n^2 \ell^{-y}$$

*if* $k = 1,$

$$(1+1)\frac{d}{dy}U_{1+1} = n^2 \frac{d^2}{dy^2}U_1,$$

$$2U_2 = n^4 \ell^{-y},$$

$$U_2 = \frac{n^2}{2!}\ell^{-y},$$

*if* $k = 2$

$$(2+1)\frac{d}{dy}U_{1+1} = n^2 \frac{d^2}{dy^2}U_2,$$

$$3U_3 = n^2 \frac{d^2}{dy^2}(\frac{n^2}{2!}\ell^{-y}) \ ,$$

$$U_3 = \frac{n^6}{3!}\ell^{-y},$$

Then, the general solution is given as

$$U_k = \frac{n^{2k}\ell^{-y\ y}}{k!}$$

***Example 5*** Consider the heat equation of the form $u_{ty} = (n^2 - 1)u_{yy}$ and the initial condition and the initial condition $U_0 = \ell^{y}$

$$(k+1)\frac{d}{dy}U_{k+1} = \frac{d^2}{dy^2}U_k$$

*if* $k = 0,$

$$(k+1)\frac{d}{dy}U_{0+1} = (n^2 - 1)\frac{d^2}{dy^2}U_0$$

$$U_1 = (n^2 - 1)\ell^{y}$$

*if* $k = 1$

$$(1+1)\frac{d}{dy}U_{1+1} = (n^2 - 1)\frac{d^2}{dy^2}(n^2 - 1)\ell^{y}$$

$$2U_2 = (n^2 - 1)^2 \ell^{y}$$

$$U_2 = \frac{(n^2 - 1)^2 \ell^{y}}{2!}$$

*if* $k = 2$

$$(2+1)\frac{d}{dy}U_{2+1} = (n^2 - 1)\frac{d^2}{dy^2}\frac{(n^2 - 1)^2 \ell^{y}}{2!}$$

$$3U_3 = \frac{(n^2 - 1)^3 \ell^{y}}{2!}$$

$$U_3 = \frac{(n^2 - 1)^3 \ell^{y}}{3!}$$

Then, the general solution is given as

$$U_k = \frac{(n^2 - 1)^k \ell^{y}}{k!}$$

## 3.0 Conclusion

The differential post-processing method was used efficiently for partial differential equations with variable coefficients. For correct initial conditions, the answer provided by the differential rework technique is an innumerable collection of energies that could describe the exact closed-form solutions. The results show that the differential revision technique can be used to fix partial differential equations with variable coefficients Differential remodeling approaches reliability, and reducing the length of the computational domain expands its applicability. As a result, we find that the method can be used with caution to solve a variety of PDEs with variable coefficients visible in physical and engineering application

## References

[1] Ablowitz MJ, Clarkson PA (1991). Nonlinear Evolution Equations and Inverse Scattering, Soliton. Cambridge University Press.
[2] Ali AHA, Gardner GA, Gardner LRT (1992). Comput. Methods Appl. Mech. Eng., 100: 325-337.
[3] Ali AHA, Raslan KR (2009). Variational iteration method for solving partial differential equations. Chaos, Soliton Fractals, 40: 1520-1529.
[4] Burgers J (1948). Advances in Applied Mechanics. Academic Press, New York, 171: 199.
[5] Caldwell J, Hinton E, Owen J, Onate E (1987). Numerical Methods for Nonlinear Problems. Pineridge, Swansea, 3: 253-261.
[6] Caldwell J, Wanless P, Cook AE (1981) A finite element approach to Burgers' equation. Appl. Math. Modell., 5: 189-193.
[7] Chen CK, Ho SH (1999). Solving partial differential equations by two-differential transform method. Appl Math Comput,, 106: 171-179.
[8] Coely A (2001). Backlund and Darboux Transformations. American Mathematical Society, Providence, Rhode Island, pp. 458-468.
[9] Cole JD (1951). On a quasi-linear parabolic equation occurring in aerodynamics. Quart. Appl. Math., 9: 225-236.
[10] Esipov SE (1992). Coupled Burgers' equations: A model of polydispersive sedimentation. Phys. Rev. E, 52: 3711-3718.