Global Scientific JOURNALS

GSJ: Volume 13, Issue 6, June 2025, Online: ISSN 2320-9186
www.globalscientificjournal.com

APPLICATION OF EXPONENTIAL-GAMMA-RAYLEIGH DISTRIBUTION TO COVID-19 DEATHS IN NIGERIA

Agbona A.A, Odukoya E. A, Aromolaran A. D, Olawale A. O

Department Statistics, Federal Polytechnic Ede, Osun State Email: agbona.anthony@federalpolyede.edu.ng

Department of Statistics, Faculty of Science, Ekiti State University, Ado-Ekiti, Ekiti State,,Email: ayooluwa88@gmail.com

Department of Statistics yaba college of Technology lagos state Email: adeyemi.aromolaran@yabatech.edu.ng

Department Statistics, Federal Polytechnic Ede, Osun State Email: olawaleabolade@federalpolyede.ehu.ng

ABSTRACT

This research explores the performance of Exponential-Gamma-Rayleigh distribution (EGRD) on Covid-19 deaths in Nigeria from March 2020 to April 2021. The parameters of the distribution were estimated using the method of maximum likelihood estimates. The newly developed Exponential-Gamma-Rayleigh distribution was compared with the existing Exponential-gamma by Ogunwale (2019) ,Exponential and Gamma distributions using the log-likelihood function, Akaike information criterion (AIC), and Bayesian information criterion (BIC) as the criteria for selecting the best fit model. The results show that the newly developed Exponential-Gamma-Rayleigh distribution performed better than Exponential-Gamma developed Ogunwale and the existing Exponential and Gamma distributions in contrast in terms of model fit, this showed that the newly developed Exponential-Gamma-Rayleigh distribution is more flexible and precise in analyzing the Covid-19 data other than Exponential-Gamma and the traditional existing Exponential and Gamma distributions.

KEYWORDS: Exponential-Gamma-Rayleigh Distribution, Maximum likelihood Estimate, AIC, BIC, log-likelihood function, Covid-19

INTRODUCTION

In both theory and practice, statistical distributions serve as the cornerstone of statistical methods. They serve as the foundation for all parametric statistical approaches, including modeling, health data, the process of implementing statistical distributions for analysing real-world data sets is known as modeling statistical distributions. Understanding and interpreting data behaviour more scientifically is an essential stage in every field of life, Statistical methods are used in almost every applied sector, including healthcare sciences; several researchers have generated new adaptable distributions from existing distributions using various modification techniques to increase their flexibility in modeling data. These adaptable distributions are created by adding extra parameters to the baseline distribution with generators or combining two distributions (Ali, *et al*., 2021). Recent emerging data of interest exhibit non-normal characteristics such as high or moderate skewness and high kurtosis, which existing distributions cannot accurately model. Cancer continues to impact the world significantly, and there was recently a Covid-19 virus that spread across the globe. The data sets representing these diseases mortality or infection rate exhibit some of the stated features. The results of fitting these data sets with traditional distributions may be unreliable. Many applications, such as lifetime analysis require extended forms of these distributions. Most statistical distribution modeling approaches are concerned with determining which probability distribution best represents the data. No single probability distribution, however, can fit all types of data. As a result, new classical distributions must be developed or created (Nasiru, 2018). These statistical distributions posed substantial usefulness in modeling and analysing natural life phenomena that encompass uncertainty and riskiness. Research are going on to analyze and predict future trends of various diseases in refining plans for prevention and treatment of such disease occurrence. Researchers have applied statistical distributions on health data to see the performance of the distribution among the researchers are Ogunwale *et a*l (2022), Jin, *et al*., (2021) Albalawi, *et al*., (2022) Xiaofeng, *et al*., (2021) Hisham *et al* (2021), Ayeni *et al* (2019), Ayeni et al(2020), Hui *et al* (2011),. All these researchers explored the applicability of their new models on data of Covid-19. Therefore this study aimed to examine the performance of the Exponential-Gamma-Rayleigh distribution on Covid-19 data in Nigeria

METHODS

One of the concerns of researchers in the modeling and analysis of health data is the interpretation of past health data regarding future probabilities of occurrences. Many probability distributions have been applied to various health data and have proved beneficial for health studies Analysis of health data is mostly determined by its distribution pattern. It has long been a recurrent topic of interest in the field of health in establishing a probability distribution that provides a suitable fit to most health data. In this study, we, therefore, aim to examine and fit the newly developed exponential-gamma-Rayleigh distribution to Covid-19 death in Nigeria from March 2020 to April 2021. The method of maximum likelihood was used to estimate their parameters, while the log-likelihood, Bayesian information criterion (BIC) and Akaike information criterion (AIC) goodness of fit test were employed to determine their goodness of fit, and Python software was used for data analysis.

The new Exponential-Gamma-Rayleigh distribution was developed by Adisa *et al* (2025) and its probability distribution function, pdf is defined as

$$g(x)=\left[\frac{\lambda}{2\sigma^2}\right]^{\alpha}\frac{2\lambda}{\Gamma(\alpha)}x^{2\alpha-1}\exp\left[-\frac{2\lambda x}{\sigma^2}\right],\ x,\lambda,\alpha,\sigma>0 \qquad (1)$$

The mean and the Variance are defined as

$$\mu_1'=\frac{2\alpha\lambda\Gamma(2\alpha)}{2^{2\alpha}\theta^{\alpha+1}\Gamma(\alpha)} \qquad (2)$$

$$V(x)=\frac{\alpha\lambda(2\alpha+1)2^{2\alpha}\theta^{\alpha}-4\alpha^2\lambda^2}{2^{4\alpha}\theta^{2\alpha+2}} \qquad (3)$$

Cumulative Distribution Function (CDF

$$F(x)=\frac{2\lambda\gamma(2\alpha,x)}{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha)} \qquad x,\theta,\alpha,\lambda>0 \qquad (4)$$

The reliability function also known as survival function $S(x)=1-F(x)$

$$S(x)=\frac{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha)-2\lambda\gamma(2\alpha,x)}{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha)} \qquad (5)$$

The hazard function also called the *force of mortality, instantaneous failure rate*

$$h(x)=\frac{f(x)}{S(x)} \qquad (6)$$

$$h(x)=\frac{2^{2\alpha+1}\theta^{2\alpha}\lambda x^{2\alpha-1}e^{-2\theta.x}}{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha)-2\lambda\gamma(2\alpha,x)} \qquad (7)$$

Cumulative hazard function; The cumulative hazard function is the integral of the hazard function

$$H(x) = W(F(x)) = -\log(1 - F(x)) \equiv \int_0^x h(x)dx \quad \text{it was obtain as} \tag{8}$$

$$H(x) = \frac{2\lambda\gamma(2\alpha, x)}{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha) - 2\lambda\gamma(2\alpha, x)}. \tag{9}$$

Let $x_1, x_2, ...x_n$ be a random sample of size n from Exponential-Gamma-Rayleigh distribution (EGRD) with probability distribution function, pdf

$f(x) = \frac{2\lambda\theta^{\alpha}}{\Gamma(\alpha)} x^{2\alpha-1} \exp\left(-2\theta x\right), \ \forall \ x, \theta, \alpha, \lambda > 0$ then, the likelihood function of EGRD is given by:

$$\mathrm{L}(\alpha, \lambda, \beta; x) = \left(\frac{2\lambda\theta^{\alpha}}{\Gamma(\alpha)}\right)^n \prod_{i=1}^{n} x_i^{2\alpha-1} \exp\left(-2\theta \sum x_i\right) \tag{10}$$

By taking the natural logarithm of (3.44), the log-likelihood function is obtained as;

$$\log_e(L) = \alpha n \log_e \theta + n \log_e 2\lambda - n \log_e \Gamma(\alpha) + \left(2\alpha - 1\right) \sum \log_e x_i - 2\theta \sum x_i \tag{11}$$

Differentiating equation (11) with respect to α , λ and θ give the maximum likelihood.

Therefore, the MLE which maximizes (11) must satisfy the following normal equations;

$$\frac{\partial \log_e L}{\partial \alpha} = n \log_e \theta - \frac{n\Gamma^{'}(\alpha)}{\Gamma(\alpha)} + 2\sum_{i=1}^{n} \log_e x_i = 0 \tag{12}$$

$$\frac{\partial \log_e L}{\partial \lambda} = \frac{n}{\lambda} = 0 \tag{13}$$

$$\frac{\partial \log_e L}{\partial \theta} = \frac{\alpha n}{\theta} - 2\sum x_i = 0 \tag{14}$$

Differentiating equation (11) with respect to α , λ and θ give the maximum likelihood estimates of the model parameter that generates the solution of the nonlinear system of equations. The parameters can be estimated numerically by solving (12), (13), and (14), while solving it analytically is very cumbersome and tasking. The numerical solution can also be obtained directly using some data sets in Python.

Results and Discussion

This research work used secondary data collected based on the total monthly amount of deaths of covid-19 patients in Nigeria from the period of March 2020 to April 2021. The data were collected from the website of the National Centre for Disease Control (NCDC).

The newly developed Exponential-Gamma-Rayleigh distribution was applied to the data to determine its flexibility in modeling such data. The data were analyzed using the Python software package. The flexibility and the measures of goodness fit were determined by using selection criteria such as the Akaike information criterion (AIC), Bayesian information criterion (BIC), and the log-likelihood function (l). The probability distribution with the least Akaike information criterion (AIC), Bayesian information criterion (BIC), or the highest log-likelihood function (l) value will be considered the most flexible and best fit model, and the results are obtained as follows;

TABLE 1: Summary of Data (Covid-19)

Parameters	**Value**
N	428
Min	0
Max	24
Mean	4.8178
Skewness	-2.2087
Kurtosis	7.48541

Source: Covid-19 Deaths, NCDC (2020-2021)

The results from Table 1 indicated that the data distribution is skewed to the left with a skewness of -2.2087. This shows that the Exponential-Gamma-Rayleigh distribution can fit left-skewed data. Also, the kurtosis value is 7.48541, which is greater than 3. This implies that the distribution has longer and fatter tails with a heavy peakedness compared to the Normal distribution.

Table 2: Estimates and performance of the distributions (Covid-19)

Distribution	**Parameter Estimate**	**AIC**	**BIC**	**Log-likelihood(l)**
EGRD	$\hat{\alpha} = 1.6781$ $\hat{\lambda} = 8.1477$ $\hat{\theta} = 0.1471$	**52.1475**	**58.0323**	**478.4112**

EGD	$\hat{\alpha} = 1.1472$ $\hat{\lambda} = 3.5874$	85.8683	87.1441	293.9463
GD	$\hat{\alpha} = 1.0179$ $\hat{\lambda} = 2.1471$	218.8993	218.9811	47.1963
ED	$\hat{\lambda} = 2.1474$	514.7681	517.3015	96.3115
RD	$\hat{\alpha}$ =0.7478	108.6715	109.8255	42.8138

The estimates of the parameters, Akaike information criterion (AIC), Bayesian information criterion (BIC), and log-likelihood, for the data sets on the covid-19 death in Nigeria from March 2020 to April 2021 are presented in Table 2. The results showed that the Exponential-Gamma –Rayleigh Distribution provides a better fit than the Exponential, Gamma and Rayleigh distributions. Likewise, the Exponential-Gamma-Rayleigh yields the overall performance since it has the lowest value of the Akaike information criterion (AIC) and Bayesian information criterion (BIC) and the highest value of log-likelihood (l). Hence, the Exponential-Gamma-Rayleigh distribution performed better than other distributions compared.

CONCLUSION

When comparing the newly developed Exponential-Gamma distribution by Ogunwale, *et al*., (2019), to the existing Exponential, Gamma and Rayleigh distributions, the results from the AIC, BIC, and log-likelihood values for the data revealed that the newly developed Exponential-Gamma-Rayleigh distribution has the smallest AIC, BIC values, and highest log-likelihood function values for the data used. Regarding model fit, these results reveal that the newly developed Exponential-Gamma-Rayleigh distribution outperformed the Exponential, Gamma and Rayleigh distributions. This shows that the Exponential-Gamma-Rayleigh distribution is more adaptable and tractable of evaluating real-world data than the classical Exponential, Gamma and Rayleigh distributions.

Similarly, the AIC, BIC, and log-likelihood values for Covid-19 data revealed that the newly developed Exponential-Gamma-Rayleigh distribution has the smallest AIC, BIC, and highest log-likelihood function values of all the distributions compared. As regards model fit, the newly developed Exponential-Gamma-Rayleigh distribution outperformed the new Exponential-Gamma distribution developed by Ogunwale, *et al*. (2019) and the previously existing Exponential, Gamma and Rayleigh distributions. This demonstrated that, in addition to the novel

Exponential-Gamma distribution and the current Exponential and Rayleigh distributions, the Exponential-Gamma-Rayleigh distribution efficiently analyzes real-world data; this substantiates that modification and parameter addition to existing probability distributions is beneficial. Therefore, for flexibility and greater precision in analyzing Covid-19 death data, the use of Exponential-Gamma-Rayleigh distribution is greatly suggested and also persistently useful in various fields where analysis of health data is essential

REFERENCES

Albalawi, O., Kabdwal, N. C., Azhad, Q. J., Hora, R., and Alsaedi, B. S. O. (2022).Estimation of the Generalized Logarithmic Transformation Exponential Distribution under Progressively Type-II Censored Data with Application to the COVID-19 Mortality Rates. *Mathematics 10, (1015), https://doi.org/10.3390/ math10071015.*

Agbona, A.A., Odukoya, E. A., Amalare. A., and Ayeni T. M. (2025). Exponential-Gamma-Rayleigh Distribution: Theory and Properties. *Asian Journal of Probability and Statistics. Volume 27, Issue 3,Pp 134-144.ISSN 2582-0230*

Ali T. M., Mayssa J. M., Muna, D. S., and Rabha W. I. (2021). The inverse exponentialRayleigh distribution and related concepts. *Italian Journal of Pure And Applied Mathematics, 47(2), 852–861.*

Ayeni, T.M., Ogunwale, O.D., Adewusi, O.A. (2020) Application of Exponential-Gamma Distribution in Modeling Queuing Data. *International Journal of Trend in Scientific Research and Development 4(2); 839-842*

Ayeni, T.M., Ogunwale, O.D., Adewusi, O.A., Odukoya, E.A. (2020). Estimating the Performance Measure of Exponential-Gamma Distribution with Application to Failure Rate Data. *International Journal of Scientific and Research Publications;10(3); 768-771*

Ayeni, T.M., Ogunwale, O.D., Adewusi, O.A. (2019). Exponential-Gamma Distribution and its Applications. *International Journal of Research and Innovation in Applied Science; IV (XII):116-120*

Hisham M. Almongy, Ehab M. Almetwally, Hassan M. Aljohani, Abdulaziz S. Alghamdi, E.H. Hafez (2021) A new extended Rayleigh distribution with applications of COVID-19 data Results in Physics 23 (2021) 104012

Hui P Zhu, Xin Xia, Chuan H Yu, Ahmed Adnan, Shun F Liu, Yu K Du (2011) Application of Weibull model for survival of patients with gastric cancer *BMC Gastroenterology 2011 11:1.*

Ihekweazu C. Steps Nigeria is taking to Prepare for Cases of Coronavirus. 2020. *http://theconversation.com/steps-nigeria-is-taking-to-prepare-for-cases-ofcoronavirus-130704*

Jin, Z., Zubair, A., and Zahra, A. (2021). Modeling the COVID-19 Pandemic Dynamics inIran and China, *Computers, Materials & Continua DOI:10.32604/cmc.2021.014259, 67(2), 2111-2122.*

Najm Obaid Salim Alghazali and Dhelal Adnan Hussein Alawadi (2014) Fitting Statistical Distributions of Monthly Rainfall for Some Iraqi Stations. *Civil and Environmental Research www.iiste.org ISSN 2224-5790 (Paper) ISSN 2225-0514 (Online) Vol.6, No.6, 2014*

Nasiru, S., (2018). Extended Odd Frechet-G family of Distributions, *Journal of Probability and Statistics*. 3(1): 23-30 doi.org/10.1155/2018/2932326.

Ogunwale, O.D., Adewusi, O.A., Ayeni, T.M. (2019). Exponential-Gamma Distribution. *International Journal of Emerging Technology and Advanced Engineering; 9(10):245-249*

Ogunwale, O. D., Ayeni, T. M., and Odukoya, E. A. (2022). Modeling Covid19 Deaths in Nigeria using Exponential-Gamma Distribution. *International Journal of Multidisciplinary Research and Analysis,* 5(6), 1456-1460 DOI: 10.47191/ijmra/v5 i6-32.

Xiaofeng, L., Zubair A., Saima, K. K., Yusuf, M., Osama, A. A., Walid, E. (2021). A New Flexible Statistical Model: Simulating and Modeling the Survival Times of COVID- 19 Patients in China *Hindawi Wiley Complexity, Article ID 6915742, https://doi.org/10.1155/2021/6915742*.