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Application of Energy with odd Number Functional In Buckling of Simple-Simple- Simple - Simple and Clamped-Clamped-Clamped-Calmped Isotropic Plates

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Abstract

This research work presents the Stability Analysis of rectangular Simple-Simple-Simple-Simple and Clamped-Clamped-Clamped-Clamped Isotropic plate using odd number Functional. For the derivation of the Energy Functional, 3rd order was considered. Upon the derivation of the shape functions, the integral values of the differentiated shape functions of the various boundary conditions were obtained. From these, the stiffness coefficients of the various boundary conditions were derived. The Third order strain energy equation was derived which was further expanded to generate the Third Order Total Potential Energy Functional. The integration of this functional with respect to the amplitude yields the Governing equation. The critical buckling load equations emerged by further minimizing the governing equation. The non-dimensional buckling load parameters were obtained by substituting the different aspect ratios, b/a ranging from 1.0 to 2.0, at the interval of 0.1. The graph of non-buckling load parameters against the aspect ratios was plotted, and it was observed that the increase in one axis brought about the decrease in the other axis.

Notation

S is the simple support

C is the clamped support

a Length of the primary dimension of the plate

b is Width of the secondary dimension of the plate

t is Tertiary dimension (thickness) of the plate

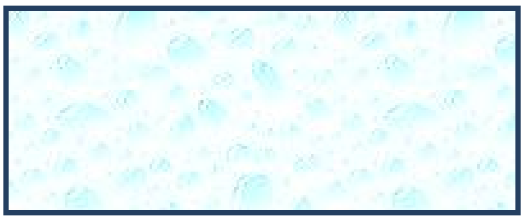
Π is the Total Potential Energy Functional

α is the aspect ratio

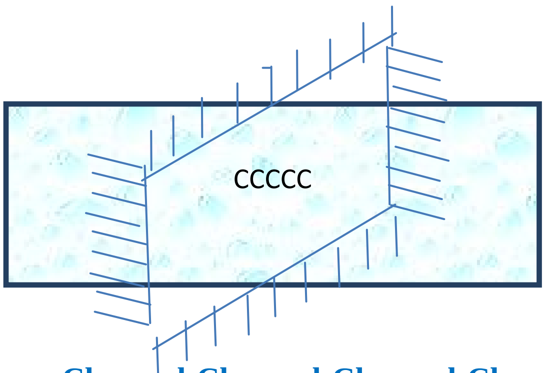
D is the flexural Rigidity

Introduction

Simple-Simple-Simple-Simple and Clamped-Clamped-Clamped-Clamped plates considered are both isotropic plates due to the fact that all their material properties in all directions are the same. In the first plate condition being Simply supported condition, the deflection equation W and the 2nd order derivative of the deflection equation W^{11}, were equated to zero and simultaneous equations were formed by considering R = 0 at the left hand support, for X axis and Q = 0 at the top of the support, for Y axis while R = 1 at the right hand support X-axis and Q = 1 at the bottom support for Y axis. For the Clamped support condition, the deflection equation, W and 1st order derivative of the deflection equation, W^1, were equated to zero and simultaneous equations were formed by considering R = 0 at the left hand support for the X-axis and Q = 0 at the top support for the Y-axis, while at the Right hand support, R = 1 for X-axis while Q = 1 at the bottom support for Y-axis. The stiffness coefficients were generated from the shape functions, from the matrix resolution of the simultaneous equations derived. The final values were substituted into the critical buckling equation which was formed by the minimization of the third order total energy functional and the graph of aspect ratio against corresponding buckling coefficients were plotted for two different aspect ratios a/b and b/a



Simple-Simple-Simple-Simple Plate



Clamped-Clamped-Clamped-Clamped Plate

Stability Analysis Equation

From the first principle we have that Energy is the ability to do work, being a function of force, F gives that

U = Force, F x Displacement ,C

Considering Force , F = Stress,σ x Area, A and

Displacement, C = Strain, ε x Original Length, L

And so for finite displacement, C we have dC = ε x dL

and dU = dF x dC

= dF x ε x dL

where F can be expressed as

dF = σ x dA

combining the equation , the strain energy can be expressed as

dU = σ x dA x ε x dL

dU = σ x ε x dA x dL

considering dA as dx.dy and dL as dz

That implies dU = σ.ε.dx.dy.dz

For the average strain energy, U = (U1 + U2)/2

Therefore, $dU = \frac{1}{2}U$ and finally

$dU = \frac{1}{2}\sigma.\varepsilon.dx.dy.dz$

$dU = \frac{1}{2}(\sigma.\varepsilon.dx.dy.dz)$

The strain energy in 3 dimension is given as

$U = U_x + U_y + U_z$ alternatively as

$\sigma.\varepsilon = \sigma_x.\varepsilon_x + \sigma_y.\varepsilon_y + \tau_{xy}, \gamma_{xy}$

Given that $\sigma_x.\varepsilon_x = \frac{Ez^2}{1-u^2}\left(\left[\frac{\partial^2 w}{\partial x^2}\right]^2 + u\left[\frac{\partial^2 w}{\partial x^2}\right]^2 . \left[\frac{\partial^2 w}{\partial y^2}\right]^2\right)$

$$\sigma_y.\varepsilon_y = \frac{Ez^2}{1-u^2}\left(u\left[\frac{\partial^2 w}{\partial x^2}\right]^2 . \left[\frac{\partial^2 w}{\partial y^2}\right]^2 + \left[\frac{\partial^2 w}{\partial y^2}\right]^2\right)$$

where ε_{x} ,and ε_{y} are $\left[\frac{\partial^2 w}{\partial x^2}\right]^2$ and $\left[\frac{\partial^2 w}{\partial y^2}\right]^2$ respectively

$$\tau_{xy},\ \gamma_{xy} = \frac{Ez^2(1-u)}{(1-u^2)}\left[\frac{\partial^2 w}{\partial x\,\partial y}\right]^2$$

Integrating the Strain Energy ,U with respect to x, y, z ,

$$\text{gives } U = \frac{1}{2}\iint_{xy}\left[\int_{\frac{-t}{2}}^{\frac{t}{2}} \sigma\varepsilon\, dz\right] dxdy$$

$$\text{Therefore } U = \frac{D}{2}\int_0^a\int_0^b \left(\frac{\partial^3 h}{\partial x^3}\cdot\frac{\partial h}{\partial x} + 2\frac{\partial^3 h}{\partial x^2\,\partial y}\cdot\frac{\partial h}{\partial y} + \frac{\partial^3 h}{\partial y^3}\cdot\frac{\partial h}{\partial y}\right) dxdy$$

Generation Of Governing Equation

The total potential energy is given as the sum of the strain energy and the external load . This can be expressed mathematically as

$$\Pi = U + V$$

$$\Pi = \frac{D}{2}\int\int\left(\frac{\partial^3 w}{\partial x^3}\cdot\frac{\partial w}{\partial x} + 2\frac{\partial^3 w}{\partial x^2\,\partial y}\cdot\frac{\partial w}{\partial y} + \frac{\partial^3 w}{\partial y^3}\cdot\frac{\partial w}{\partial y}\right) dx - \frac{1}{2}\int\int(Nx\frac{\partial^2 w}{\partial x^2} + 2Nxy\frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y} dxdy + Ny\frac{\partial^2 w}{\partial y^2})dxdy$$

Substituting Ah for the deflection, w and considering the values of Nxy and Nx as 0, reduces the Overall Total Potential Energy Functional to equation

$$\Pi = \frac{A^2 D}{2}\int_0^a\int_0^b \left(\frac{\partial^3 h}{\partial x^3}\cdot\frac{\partial h}{\partial x} + 2\frac{\partial^3 h}{\partial x\,\partial y^2}\cdot\frac{\partial h}{\partial x} + \frac{\partial^3 h}{\partial y^3}\cdot\frac{\partial h}{\partial y}\right) dxdy - \frac{A^2 Nx}{2}\int\int\frac{\partial^2 h}{\partial x^2}.h dxdy$$

Differentiating the last equation obtained with respect to it with respect to A gives the

Governing Equation as

$$\frac{d\pi}{dA} = 0 = \frac{2AD}{2}\int_0^a\int_0^b \left(\frac{\partial^3 h}{\partial x^3}\cdot\frac{\partial h}{\partial x} + 2\frac{\partial^3 h}{\partial x^2\,\partial y}\cdot\frac{\partial h}{\partial y} + \frac{\partial^3 h}{\partial y^3}\cdot\frac{\partial h}{\partial y}\right) dxdy - \frac{2ANx}{2}\int_0^a\int_0^b\left(\frac{\partial h}{\partial x}\right)^2 dxdy$$

On making Buckling Load Equation (Nx) the subject formula gives

$$Nx = \frac{\frac{2AD}{2}\int_0^1\int_0^1 \left(\left(\left[\frac{\partial^3 h}{\partial x^3}\right]\cdot\frac{\partial h}{\partial y} + 2\left[\frac{\partial^3 h}{\partial x\,\partial y}\right]\cdot\frac{\partial h}{\partial x} + \left[\frac{\partial^3 h}{\partial y^3}\right]\cdot\frac{\partial h}{\partial y}\right) dxdy\right.}{\frac{2A}{2}\int_0^a\int_0^b\left(\frac{\partial h}{\partial x}\right)^2 dxdy}$$

$$Nx = \frac{\frac{D}{a^2}(k_1 + 2\frac{1}{p^2}k_2 + \frac{1}{p^4}k_3)}{k_6}$$

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Formation Of Buckling Coefficient

DETERMINATION OF BUCKLING COEFFICIENT USING ASPECT RATIO, $\boldsymbol{p = \mathrm{a/b}}$
Defining the principal in-plane coordinates (x and y) in terms of non-dimension in-plane coordinates (R and Q) as:

$R = \frac{x}{a}$. That is $x = aR$

$Q = \frac{y}{b}$. That is $y = bQ$

Where "a" and "b" are plate dimensions in x and y directions. The aspect ratio $\boldsymbol{\alpha}$ (ratio of length in y direction to length in x direction) of $\frac{a}{b}$, where the α ranges from 1.0 to 2.0 was considered.. The value of a is less or equal to b. (i. e $a \leq b$).
While D is flexural rigidity and w is the deflection, Π is the total potential energy functional. R and Q are non-dimensional axis (quantity) parallel to x and y axis. Substituting aR and bQ for x and y respectively into equation

$$Nx = \frac{D\int_0^1\int_0^1 \left(\left[\frac{\partial^3 h}{\partial a^3 R^3}\right].\frac{\partial h}{\partial aR} + \left[\frac{\partial^3 h}{\partial aR\,\partial b^2 Q^2}\right].\frac{\partial h}{\partial aR} + \left[\frac{\partial^3 h}{\partial b^3 Q^3}\right].\frac{\partial h}{\partial bQ}\right)dRdQ}{\frac{1}{a^2}\int_0^1\int_0^1 \left(\frac{\partial h}{\partial R}\right)^2 dRdQ}$$

$$Nx = \frac{D\int_0^1\int_0^1 \left(\frac{1}{a^4}\left[\frac{\partial^3 h}{\partial R^3}\right].\frac{\partial h}{\partial R} + 2\frac{1}{a^2 b^2}\left[\frac{\partial^3 h}{\partial R\partial Q^2}\right].\frac{\partial h}{\partial R} + \frac{1}{b^4}\left[\frac{\partial^3 h}{\partial Q^3}\right].\frac{\partial h}{\partial Q}\right)dRdQ}{\frac{1}{a^2}\int_0^1\int_0^1 \left(\frac{\partial h}{\partial R}\right)^2 dRdQ}$$

Substituting pa in place of b in the last equation

$$Nx = \frac{D\int_0^1\int_0^1 \left(\frac{1}{a^4}\left[\frac{\partial^3 h}{\partial R^3}\right].\frac{\partial h}{\partial R} + 2\frac{1}{a^4 p^2}\left[\frac{\partial^3 h}{\partial R\,\partial Q^2}\right].\frac{\partial h}{\partial R} + \frac{1}{a^4 p^4}\left[\frac{\partial^3 h}{\partial Q^3}\right].\frac{\partial h}{\partial Q}\right)dRdQ}{\frac{1}{a^2}\int_0^1\int_0^1 \left(\frac{\partial h}{\partial R}\right)^2 dRdQ}$$

$$Nx = \frac{\frac{D}{a^4}\int_0^1\int_0^1 \left(\left[\frac{\partial^3 h}{\partial R^3}\right].\frac{\partial h}{\partial R} + 2\frac{1}{p^2}\left[\frac{\partial^3 h}{\partial R\partial Q^2}\right].\frac{\partial h}{\partial R} + \frac{1}{p^4}\left[\frac{\partial^3 h}{\partial Q^3}\right].\frac{\partial h}{\partial Q}\right)dRdQ}{\frac{1}{a^2}\int_0^1\int_0^1 \left(\frac{\partial h}{\partial R}\right)^2 dRdQ}$$

Dividing the numerator and the denominator by $\frac{1}{a^2}$

$$Nx = \frac{\frac{D}{a^2}\int_0^1\int_0^1 \left(\left[\frac{\partial^3 h}{\partial R^3}\right].\frac{\partial h}{\partial R} + 2\frac{1}{p^2}\left[\frac{\partial^3 h}{\partial R\partial Q^2}\right].\frac{\partial h}{\partial R} + \frac{1}{p^4}\left[\frac{\partial^3 h}{\partial Q^3}\right].\frac{\partial h}{\partial Q}\right)dRdQ}{\int_0^1\int_0^1 \left(\frac{\partial h}{\partial R}\right)^2 dRdQ}$$

This can be expressed in terms of stiffness coefficients as

$$Nx = \frac{\frac{D}{a^2}\left(k_1 + 2\frac{1}{p^2}k_2 + \frac{1}{p^4}k_3\right)}{k_6}$$

where $k_1 = \left[\frac{\partial^3 h}{\partial R^3}\right].\frac{\partial h}{\partial R}$, $k_2 = \left[\frac{\partial^3 h}{\partial R\partial Q^2}\right].\frac{\partial h}{\partial R}$, $k_3 = \left[\frac{\partial^3 h}{\partial Q^3}\right].\frac{\partial h}{\partial Q}$

But substituting $\frac{b^2}{p^2}$ for a^2 in Equation 3.52 we have

$$Nx = \frac{\frac{D}{\frac{b^2}{p^2}}\int_0^1\int_0^1 \left(\left[\frac{\partial^3 h}{\partial R^3}\right].\frac{\partial h}{\partial R} + 2\frac{1}{p^2}\left[\frac{\partial^3 h}{\partial R\partial Q^2}\right].\frac{\partial h}{\partial R} + \frac{1}{p^4}\left[\frac{\partial^3 h}{\partial Q^3}\right].\frac{\partial h}{\partial Q}\right)dRdQ}{\int_0^1\int_0^1 \left(\frac{\partial h}{\partial R}\right)^2 dRdQ}$$

The critical buckling load, Nx for the aspect ratio p = b/a is

$$Nx = \frac{\frac{D}{b^2}\int_0^1\int_0^1 \left(p^2\left[\frac{\partial^3 h}{\partial R^3}\right]\cdot\frac{\partial h}{\partial R} + 2\left[\frac{\partial^3 h}{\partial R\,\partial Q}\right]\cdot\frac{\partial h}{\partial R} + \frac{1}{p^2}\left[\frac{\partial^3 h}{\partial Q^3}\right]\cdot\frac{\partial h}{\partial Q}\right)dRdQ}{\int_0^1\int_0^1 \left(\frac{\partial h}{\partial R}\right)^2 dRdQ}$$

$$Nx = \frac{\frac{D}{b^2}\int_0^1\int_0^1 \left(p^2\overline{K1} + 2\overline{K2} + \frac{1}{p^2}\overline{K3}\right)dRdQ}{\int_0^1\int_0^1 \overline{K6}\, dRdQ}$$

The equation can be written as

$$Nx = \frac{\frac{D}{b^2}\left(p^2 k_1 + 2k_2 + \frac{1}{p^2}k_3\right)}{k_6}$$

SHAPE FUNCTIONS FOR SIMPLE- SIMPLE- SIMPLE- SIMPLE PLATE

SHAPE FUNCTION FOR SIMPLE- SIMPLE- SIMPLE- SIMPLE PANEL

Fig 1.1a SSSS RECTANDULAR SHAPE

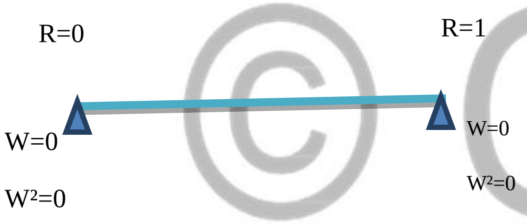

Fig 1.1b SIMPLE–SIMPLE SUPPORT ON X-X AXIS

Considering the X- X axis

But $w_x = a_o + a_1R + a_2 R^2 + a_3R^3 + a_4R^4$

$W^1 = a_1 + 2a_2 R^2 + 3a_3R^3 + 4a_4R^4$

$W^2 = 2a_2 + 6a_3R + 12a_4R^4$

At the left support, **R =0**

When w=0

$w = 0 = a_o + 0 + 0 + 0 + 0$

$a_o = 0$

Also when $W^2 = 0$

$W^2 = 0 = 2a_2 + 6a_3R + 12a_4R^4$

$2a_2 = 0$

$a_2 = 0$

At the right support, **R =1**

When w=0

$w = a_o + a_1R + a_2 R^2 + a_3R^3 + a_4R^4$

$w = a_o + a_1 + a_2 + a_3 + a_4$ (where $a_o = a_2 = 0$)

$0 = a_1 + a_3 + a_4$

$a_1 + a_3 = - a_4$

Also when $W^2 = 0$,

$W^2 = 0 = 0 + 6a_3 + 12a_4$ (where $a_2 = 0$)

$6a_3 = -12a_4$

$a_3 = -2a_4$

Substituting $-2a_4$ for a_3 into equation (3.56)

$a_1 + (-2a_4) = -a_4$

$a_1 = 2a_2 - a_4$

$a_1 = a_4$

Substituting back in the general equation

$w_x = a_o + a_1R + a_2 R^2 + a_3R^3 + a_4R^4$

$0 = 0 + a_4R + 0 + (-2a_4) R^3 + a_4R^4$.

$w_x = R(R-2R^3+R^4)$

Considering the Y- Y axis

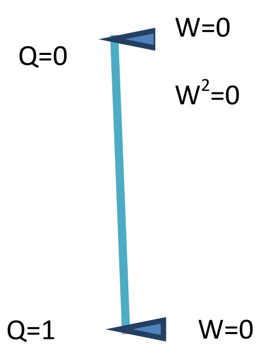

Fig 1.2c SIMPLE – SIMPLE SUPPORT ON Y-Y AXIS

But $w_y = b_o + b_1Q + b_2 Q^2 + b_3Q^3 + b_4Q^4$

$W^1 = b_1 + 2b_2 Q^2 + 3b_3Q^3 + 4b_4Q^4$

$W^2 = 2b_2 + 6b_3Q + 12b_4Q^4$

At the left support, **$Q = 0$**

When $w=0$

$w = 0 = b_o + 0 + 0 + 0 + 0$

$b_o = 0$

Also when $W^2 = 0$

$W^2 = 0 = 2b_2 + 6b_3Q + 12b_4Q^4$

$2b_2 = 0$

$b_2 = 0$

At the right support, **$Q = 1$**

When $w=0$

$w = b_o + b_1Q + b_2 Q^2 + b_3Q^3 + b_4Q^4$

$w = b_o + b_1 + b_2 + b_3 + b_4$ (where $b_o = b_2 = 0$)

$0 = b_1 + b_3 + b_4$

$b_1 + b_3 = - b_4$

Also when $W^2 = 0$,

$W^2 = 0 = 0 + 6b_3 + 12b_4$ (where $b_2=0$)

$6b_3 = -12b_4$

$b_3 = -2b_4$

Substituting $-2b_4$ for b_3 into equation

$b_1 + (-2b_4) = -b_4$

$b_1 = 2b_2 - b_4$

$b_1 = b_4$

Substituting back in the general equation $w_y = b_o + b_1Q + b_2 Q^2 + b_3Q^3 + b_4Q^4$

$0 = 0 + b_4Q + 0 + (-2b_4) Q^3 + b_4Q^4$.

$W_y = b_4(Q-2Q^3+Q^4)$

But $W = W_X W_y$

$W = a_4(R-2R^3+R^4)\, b_4(Q-2Q^3+Q^4)$

$= a_4b_4(R-2R^3+R^4)(Q-2Q^3+Q^4)$

$W = Ah$

Therefore the shape function h for SSSS panel is $(R-2R^3+R^4)(Q-2Q^3+Q^4)$

SHAPE FUNCTION FOR CLAMPED- CLAMPED- CLAMPED- CLAMPED **PANEL**

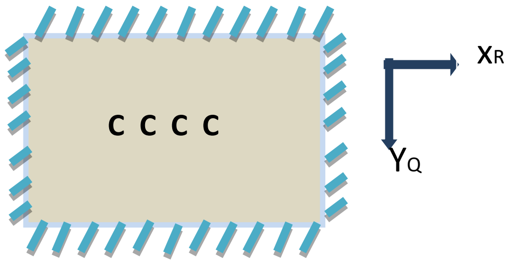

Fig 1.2a CCCC RECTANGULAR SHAPE

Considering the X- X axis

$w = a_o + a_1R + a_2 R^2 + a_3R^3 + a_4R^4$

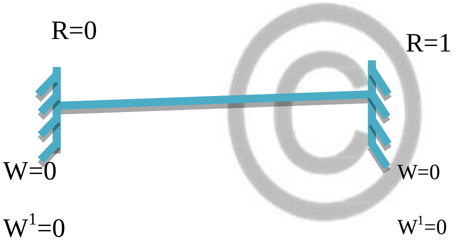

Fig 1.2b CLAMPED – CLAMPED SUPPORT ON X=X AXIS

But $w_x = a_o + a_1R + a_2 R^2 + a_3R^3 + a_4R^4$

$W^1 = a_1 + 2a_2 R^2 + 3a_3R^3 + 4a_4R^4$

At the left support, **R =0**

When w=0

$w = 0 = a_o + 0 + 0 + 0 + 0$

$a_o = 0$

Also when $W^1 = 0$

$W^1 = 0 = a_1 + 0 + 0 + 0$

$a_1 = 0$

At the right support, **R =1**

When w=0

$w = 0 + 0 + a_2 + a_3 + a_4$

$a_2 + a_3 + a_4$ (where $a_o = a_2 = 0$)

$a_2 + a_3 = - a_4$

Also when $W^1 = 0$

$W^1 = 0 = 0 + 2a_2 + 3a_3 + 4a_4$ (where $a_2 = 0$)

$2a_2 + 3a_3 = -4a_4$

Solving two equations simultaneously using matrix method

$$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} * \begin{bmatrix} a2 \\ a3 \end{bmatrix} = \begin{bmatrix} -a4 \\ -4a4 \end{bmatrix}$$

$a_3 = -2a_4$

Substituting $-2a_4$ for a_3 into equation

$a_2 + (-2a_4) = -a_4$

$a_2 = 2a_2 - a_4$

$a_2 = a_4$

Substituting back in the general equation $w_x = a_o + a_1R + a_2 R^2 + a_3R^3 + a_4R^4$

$0 = 0 + 0 + a_4R^2 + (-2a_4) R^3 + a_4R^4.$

$w_x = a_4 (R^2 - 2R^3 + R^4)$

Considering the Y- Y axis

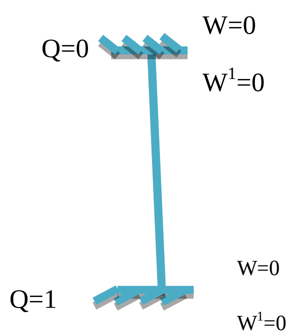

Fig 1.2c CLAMPED–CLAMPED SUPPORT ON Y-Y AXIS

But $w_y = b_o + b_1Q + b_2 Q^2 + b_3Q^3 + b_4Q^4$

$W^1 = b_1 + 2b_2 Q^2 + 3b_3Q^3 + 4b_4Q^4$

$W^2 = 2b_2 + 6b_3Q + 12b_4Q^4$

At the left support, **Q =0**

When w=0

$w = 0 = b_o + 0 + 0 + 0 + 0$

$b_o = 0$

Also when $W^1 = 0$

$W^1 = 0 = 2b_2 + 6b_3Q + 12b_4Q^4$

$2b_1 = 0$

$b_1 = 0$

At the right support, **Q =1**

When w=0

$w = b_o + b_1Q + b_2 Q^2 + b_3Q^3 + b_4Q^4$

$w = b_o + b_1 + b_2 + b_3 + b_4$ (where $b_o = b_1 = 0$)

$0 = b_2 + b_3 + b_4$

$b_2 + b_3 = - b_4$

Also when $W^1 = 0$,

$W^1 = 0 = 0 + 6b_3 + 12b_4$ (where $b_2 = 0$)

$6b_3 = -12b_4$

$b_3 = -2b_4$

Substituting $-2b_4$ for b_3 into equation

$b_2 + (-2b_4) = -b_4$

$b_2 = 2b_4 - b_4$

$b_2 = b_4$

Substituting back in the general equation $w_y = b_o + b_1Q + b_2 Q^2 + b_3Q^3 + b_4Q^4$

$0 = 0 + 0 + b_4Q^2 + (-2b_4) Q^3 + b_4Q^4$.

$W_y = b_4(Q^2 - 2Q^3 + Q^4)$

But $W = W_X W_y$

$W = a_4(R^2 - 2R^3 + R^4) b_4(Q^2 - 2Q^3 + Q^4)$

$= a_4b_4 (R^2 - 2R^3 + R^4) (Q^2 - 2Q^3 + Q^4)$

Where $a_4b_4 = A$

$W = AH$

Therefore the shape function H for CCCC panel is $(R^2-2R^3+R^4)(Q^2-2Q^3+Q^4)$

DERIVATION OF THE DIFFERENTIAL VALUES

The shape functions derived were differentiated to different degrees and that gave rise to the differential values $(\overline{K}-values)$. The was done with respect to the non-dimensional parameters R and Q in the equations.

Differential values for SSSS shape

From the derivations, the shape function, h for SSSS was given as

$$(R - 2R^3 + R^4)(Q - 2Q^3 + Q^4)$$

that means $\frac{\partial h}{\partial R} = (1 - 6R^2 + 4R^3)(Q - 2Q^3 + Q^4)$

$$\overline{K6} = \frac{\partial^2 h}{\partial R^2} = (-12R + 12R^2)(Q - 2Q^3 + Q^4)$$

$$\frac{\partial h}{\partial Q} = (R - 2R^3 + R^4)(1 - 6Q^2 + 4Q^3)$$

$$\frac{\partial^2 h}{\partial Q^2} = (R - 2R^2 + R^4)(-12Q + 12Q^2)$$

$$\frac{\partial^2 h}{\partial R\,\partial Q} = (1 - 6R^2 + 4R^3)(1 - 6Q^2 + 4Q^3)$$

$$\frac{\partial^3 h}{\partial R^3} = (-12 + 24R)(Q - 2Q^3 + Q^4)$$

$$\overline{K1} = \frac{\partial^3 h}{\partial R^3} * \frac{\partial h}{\partial R} = (-12 + 24R)(Q - 2Q^3 + Q^4) * (1 - 6R^2 + 4R^3)(Q - 2Q^3 + Q^4)$$

$$\frac{\partial^3 h}{\partial Q^3} = (R - 2R^2 + R^4)(-12 + 24Q)$$

$$\frac{\partial^3 h}{\partial R\,\partial Q^2} = (1 - 6R^2 + 4R^3)(-12Q + 12Q^2)$$

$$\overline{K3} = \frac{\partial^3 h}{\partial Q^3} * \frac{\partial h}{\partial Q} = (R - 2R^2 + R^4)(-12 + 24Q) * (R - 2R^3 + R^4)(1 - 6Q^2 + 4Q^3)$$

$$\overline{K2} = \frac{\partial^3 h}{\partial R\,\partial Q^2} * \frac{\partial h}{\partial R} = (1 - 6R^2 + 4R^3)(-12Q + 12Q^2) * (1 - 6R^2 + 4R^3)(Q - 2Q^3 + Q^4)$$

Stiffness components for SSSS plates:

The values of k_1, k_2, k_3 and k_6 were derived below by integrating the values of $\overline{K1}$, $\overline{K2}$, $\overline{K3}$ *and* $\overline{K6}$ respectively.

$$k_1 = \int_0^1 \int_0^1 (1 - 12R^2 + 8R^3 + 36R^4 - 48R^5 + 16R^6)(Q^2 - 4Q^4 + 2Q^5 + 4Q^6 - 4Q^7 + Q^8)dRdQ = 12\left(\frac{2}{5}\right)\left(\frac{31}{630}\right) = \frac{124}{525}$$

$$k_2 = \int_0^1 \int_0^1 12(R^3 - 2R^5 + R^6 - R^2 + 2R^4 - R^5)(1 - 12Q^2 + 8Q^3 + 36Q^4 - 48Q^5 + 16Q^6)dRdQ = 12\left(\frac{-17}{420}\right)\left(\frac{-17}{35}\right) = \frac{289}{1225}$$

$$k_3 = \int_0^1 \int_0^1 12(R^2 - 4R^4 + 2R^5 + 4R^6 - 4R^7 + R^8)(2Q + 6Q^2 - 16Q^3 + 8Q^4 - 1)dRdQ = 12\left(\frac{31}{630}\right)\left(\frac{2}{5}\right) = \frac{124}{525}$$

$$k_6 = \int_0^1 \int_0^1 \left(\frac{\partial h}{\partial R}\right)^2 dRdQ$$

$$= \int_0^1 \int_0^1 (1 - 12R^3 + 8R^3 + 36R^4 - 48R^5 + 16R^6)(Q^2 - 4Q^4 + 2Q^5 + 4Q^6 - 4Q^7 + Q^8)\,dRdQ = \left(\frac{17}{35}\right)\left(\frac{31}{630}\right) = \frac{527}{22050}$$

Differential values for Clamped-Clamped-Clamped-Clamped shape

From the derivations, the shape function, h for Clamped-Clamped-Clamped-Clamped was given as

$$(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)$$

That means

$$\frac{\partial h}{\partial R} = (2R - 6R^2 + 4R^3)(Q^2 - 2Q^3 + Q^4)$$

$$\overline{K6} = \frac{\partial^2 h}{\partial R^2} = (2 - 12R + 12R^2)(Q^2 - 2Q^3 + Q^4)$$

$$\frac{\partial h}{\partial Q} = (R^2 - 2R^3 + R^4)(2Q - 6Q^2 + 4Q^3)$$

$$\frac{\partial^2 h}{\partial Q^2} = (R^2 - 2R^3 + R^4)(2 - 12Q + 12Q^2)$$

$$\frac{\partial^2 h}{\partial R\,\partial Q} = (2R - 6R^2 + 4R^3)(2Q - 6Q^2 + 4Q^3)$$

$$\frac{\partial^2 h}{\partial R\,\partial Q^2} = (2R - 6R^2 + 4R^3)(2 - 12Q + 12Q^2)$$

$$\frac{\partial^3 h}{\partial R^3} = (-12 + 24R)(Q^2 - 2Q^3 + Q^4)$$

$$\overline{K1} = \frac{\partial^3 h}{\partial R^3} * \frac{\partial h}{\partial R} = (-12 + 24R)(Q^2 - 2Q^3 + Q^4) * (2R - 6R^2 + 4R^3)(Q^2 - 2Q^3 + Q^4)$$

$$\frac{\partial^3 h}{\partial Q^3} = (R^2 - 2R^3 + R^4)(-12 + 24Q)$$

$$\overline{K2} = \frac{\partial^3 h}{\partial R\,\partial Q^2} * \frac{\partial h}{\partial R} = (2R - 6R^2 + 4R^3)(2 - 12Q + 12Q^2) * (2R - 6R^2 + 4R^3)(Q^2 - 2Q^3 + Q^4)$$

$$\overline{K3} = \frac{\partial^3 h}{\partial Q^3} * \frac{\partial h}{\partial Q} = (R^2 - 2R^3 + R^4)(-12 + 24Q) * (R^2 - 2R^3 + R^4)(2Q - 6Q^2 + 4Q^3)$$

Stiffness components for Clamped-Clamped-Clamped-Clamped plates:

The values of k_1, k_2, k_3 and k_6 were derived below by integrating the values of $\overline{K1}$, $\overline{K2}$, $\overline{K3}$ *and* $\overline{K6}$ respectively.

$$k_1 = \int_0^1 \int_0^1 12(4R^2 - 12R^3 + 8R^4 - 2R + 6R^2 - 4R^4)(Q^2 - 4Q^5 + 6Q^6 + 4Q^7 - Q^8 + Q^8)dRdQ = 12\left(\frac{-1}{15}\right)\left(\frac{-1}{630}\right) = \frac{2}{1575}$$

$$K_2 = \int_0^1 \int_0^1 (2R^2 - 16R^3 + 38R^4 - 36R^5 + 12R^6)(4Q^2 - 24Q^3 + 52Q^4 - 48Q^5 + 16Q^6)dRdQ = \left(\frac{-2}{105}\right)\left(\frac{-2}{105}\right) = \left(\frac{4}{11025}\right)$$

$$k_3 = \int_0^1 \int_0^1 12(R^4 - 4R^5 + 6R^6 + 4R^7 - 4R^7 + R^8)(4Q^2 - 12Q^3 + 8Q^4 - 2Q + 6Q^2 - 4Q^4)dRdQ = -\ 12\left(\frac{-1}{630}\right)\left(\frac{-1}{15}\right) = \left(\frac{2}{1575}\right)$$

$$k_6 = \int_0^1 \int_0^1 \left(\frac{\partial h}{\partial R}\right)^2 dRdQ$$

$$= \int_0^1 \int_0^1 (4R^2 - 24R^3 + 52R^4 - 48R^5 + 16R^6)(Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8)\, dRdQ = \left(\frac{2}{105}\right)\left(\frac{1}{630}\right) = \frac{2}{66150}$$

DETERMINATION OF CRITICAL BUCKLING LOAD OF THE VARIOUS PLATES

Critical Buckling Load of Simple-Simple-Simple-Simple Plates

Considering b^2 as a^2/p^2, when the stiffness coefficients k_1, k_2, k_3 and k_6 were introduced into Critical Buckling Equation, we obtain various Critical Buckling Load Coefficients corresponding to the aspect ratios of p = b/a

$$Nx = \frac{D(0.23621 + \frac{2}{p^2}(0.23591) + \frac{1}{p^4}(0.23621))}{0.0239a^2}$$

Also for the aspect ratios of p = a/b ranging from 0.5 to 1.0 the corresponding Critical Buckling Coefficient were obtained by substituting the stiffness coefficients into equation.

$$Nx = \frac{D(0.23621 + 2p^2(0.23591) + p^4(0.23621)}{0.0239a^2}$$

Critical Buckling of Clamped-Clamped-Clamped-Clamped Plates

Just same way when the stiffness coefficients k_1, k_2, k_3 and k_6 for the Clamped-Clamped-Clamped-Clamped shape were introduced into Critical Buckling Equation, we obtain various Critical Buckling Load Coefficients corresponding to the aspect ratio of b/a

$$Nx = \frac{D(0.00127 + \frac{2}{p^2}(0.00036) + \frac{1}{p^4}(0.00127))}{0.00003a^2}$$

When the aspect ratios of p = a/b ranging from 0.5 to 1.0 were introduced into the Critical Buckling Equation at the usual interval, we obtain various Critical Buckling Coefficient corresponding to the aspect ratio and the results are as

$$Nx = \frac{D(0.00127 + 2p^2(0.00036) + p^4(0.00127))}{0.00003\,a^2}$$

Results And Discussion

For each shape function, the results were presented in two tables. The first table on the left hand side, represents the values of the critical buckling coefficients for the aspect ratio of b/a while the second tables on the right hand side, contains the critical buckling coefficients for the aspect ratio a/b. It was observed that in both aspect ratios, the results are the same.

Critical Buckling Load Coefficients

The critical buckling load coefficients for Simple-Simple-Simple-Simple plate for the aspect ratios of b/a and a/b are presented in the Table 1.1a while the critical buckling load coefficients for Clamped-Clamped-Clamped-Clamped plates for the aspect ratio of b/a and a/b are contained in the Table 1.1b.

Table 1.1a **Critical Buckling load coefficients for Simple-Simple-Simple-Simple plates for aspect ratios of b/a and a/b**

Critical Buckling load coefficients for SSSS plate for aspect ratio of b/a		**Critical Buckling load coefficients for SSSS plate for aspect ratio of a/b**	
b/a	$Nx = N.D/a^2$	a/b	$Nx = N.D/a^2$
2	15.43632	0.5	15.43632
1.9	16.11018	0.5263	16.11018
1.8	16.91777	0.5556	16.91777
1.7	17.89753	0.5882	17.89753
1.6	19.10282	0.625	19.10282
1.5	20.60948	0.6667	20.60948
1.4	22.52811	0.7143	22.52811
1.3	25.02498	0.7692	25.02498
1.2	28.35882	0.8333	28.35882
1.1	32.94889	0.9091	32.94889
1	39.50795	1	39.50795

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Table 1.1b **Critical Buckling load coefficients for Clamped-Clamped-Clamped-Clamped plates for aspect ratios of b/a and a/b**

Critical Buckling load coefficients for CCCC plate for aspect ratio of b/a

b/a	$N_x = ND/a^2$ N - Values
2.0	50.97917
1.9	52.22992
1.8	53.77341
1.7	55.70642
1.6	58.16789
1.5	61.36214
1.4	65.59795
1.3	71.35659
1.2	79.41538
1.1	91.08228
1	108.6667

Critical Buckling load coefficients for CCCC plate for aspect ratio of a/b

a/b	$N_x = ND/a^2$ N - Values
0.5	50.97917
0.5263	52.22913
0.5556	53.77588
0.5882	55.7042
0.625	58.16789
0.6667	61.36488
0.7143	65.59932
0.7692	71.35309
0.8333	79.41078
0.9091	91.08383
1	108.6667

The graph of the Critical Buckling Load against Aspect Ratio of **b/a was plotted,** The aspect Ratio is of the ranges 1.0 to 2.0 with arithmetic increment of 0.1, From the graph plotted, it was observed that as the critical buckling load parameters decreases the aspect ratio increases from 1.0 to 2.0,. The graph is of the 2^{nd} degree polynomial.

COMPARISM WITH THE PREVIOUS WORK

N-values from present study compared with previous work for Simple-Simple-Simple-Simple rectangular plate buckling.

Aspect Ratios (p = b/a)	N-Values from Present Study (i)	N-Values from Ibearugbulem et al. (2014) (ii)	Percentage Difference Between (i) and (ii)
1	39.508	39.508	0
1.1	32.9489	32.9492	-0.00091
1.2	28.3588	28.3593	-0.00176
1.3	25.025	25.0256	-0.0024
1.4	22.5281	22.5288	-0.00311
1.5	20.6095	20.6102	-0.0034
1.6	19.1028	19.1036	-0.00419
1.7	17.8975	17.8983	-0.00447
1.8	16.9178	16.9186	-0.00473
1.9	16.1102	16.111	-0.00497
2	15.4363	15.4371	-0.00518

N-values from present study compared with previous work for Clamped-Clamped-Clamped-Clamped rectangular plate buckling.

Aspect Ratios (p = b/a)	N-Values from Present Study (i)	N-Values from Ibearugbulem et al. (2014) (ii)	Percentage Difference Between (i) and (ii)
1	108.6667	108.667	-0.00028
1.1	91.0823	91.082	0.000329
1.2	79.41538	79.415	0.000478
1.3	71.3566	71.3565	0.00014
1.4	65.598	65.5979	0.000152
1.5	61.3621	61.3621	0
1.6	58.1679	58.167	0.001547
1.7	55.7064	55.706	0.000718
1.8	53.7734	53.773	0.000744
1.9	52.2299	52.229	0.001723

2	50.9792	50.979	0.000392

Conclusions; The research work conducted shows that the non-dimensional buckling load parameters generated in the course of the work were very close to those from previous works, with the highest percentage difference very insignificant. The odd energy functional for the plates were generated, which made it possible for the calculations of the critical buckling load equation. The research work also provided stiffness coefficients for Simple-Simple-Simple-Simple and Clamped-Clamped-Clamped-Clamped plates using the odd Energy Functional

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