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CLASSES OF FACTOR-TYPE ESTIMATORS FOR ESTIMATING POPULATION MEAN USING AUXILIARY ATTRIBUTES UNDER SIMPLE RANDOM SAMPLING

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ABSTRACT

This study proposes an enhanced generalized class of factor-type estimators that incorporate auxiliary attributes to improve the accuracy of population mean estimates in survey sampling. Building upon existing work by Singh and Shukla (1987) and Saini and Kumar (2015), a modified estimator was developed and evaluated for its ability to yield more precise estimates compared to conventional estimators. The statistical software R (version 3.4.1) was employed to calculate mean square errors (MSE) and assess the percent relative efficiency (PRE) of the proposed estimator. Theoretical derivations of the bias and MSE were performed to ascertain the estimator's characteristics, followed by efficiency comparisons with established estimators, including Naik and Gupta's (1996) and Singh et al.'s (2007, 2008) models. Empirical validation was conducted using datasets previously used by Saini and Kumar (2020), with results indicating that the modified estimator demonstrates superior efficiency under specified conditions. Recommendations for survey practitioners include adopting the proposed estimator in scenarios with known auxiliary attributes, as well as integrating this approach within statistical software to enhance accessibility. This research contributes to improving the accuracy of statistical estimates in survey sampling, potentially benefiting large-scale studies and complex data environments.

Keywords: Factor-type estimator, Auxiliary attributes, Population mean estimation, Mean square error (MSE), Percent relative efficiency (PRE), Survey sampling

INTRODUCTION

1.1 Background of the study

Accurately estimating population parameters in sample surveys is essential in statistical research, and various techniques have been developed to enhance the efficiency and accuracy of these estimations. In practice, auxiliary information is frequently leveraged to improve the precision of estimators, often by selecting sample units based on relevant auxiliary variables. Traditional techniques such as ratio, product, and regression estimation have shown to be effective in incorporating auxiliary information into estimation processes, enhancing their precision significantly. Historical contributions to the field, such as those by Cochran (1953, 1963, 1978), highlighted the use of ratio-type estimators, particularly when a strong positive correlation exists between the study variable and the auxiliary variable. In contrast, Murthy (1964) introduced product-type estimators, which are especially useful in cases where there is a negative correlation between these variables.

Building upon these foundational methods, researchers like Sisodia and Dwivedi (1981) and Srivenkataramana (1980) introduced adaptations that utilize the coefficient of variation in the auxiliary variable or dual to ratio estimators. This approach has been instrumental in refining estimation techniques further. The exploration of factor-type estimators has similarly advanced the field, with Singh and Shukla (1987) and Singh and Shukla (1993) introducing efficient factor-type ratio estimators for population mean estimation.

However, in some scenarios, auxiliary information is qualitative rather than quantitative, which traditional quantitative-based estimation techniques may not adequately address. For instance, factors such as gender, breed, or crop variety can influence outcomes significantly, yet they are categorical rather than continuous. In these cases, qualitative auxiliary attributes serve as valuable sources of information. For example, a person's height may be affected by gender, and wheat yield may be influenced by crop variety. Recognizing the importance of such qualitative attributes, researchers have explored estimation techniques that incorporate prior knowledge related to auxiliary characteristics (Sharma and Singh, 2015).

Sampling theory's role spans multiple disciplines, including economics, healthcare, agriculture, and engineering, where estimation of means such as average crop yield or life expectancy is essential. Traditional sampling methods often focus on quantitative auxiliary variables; however, the current study seeks to bridge the gap by proposing a new class of factor-

type estimators. These estimators leverage qualitative auxiliary attributes in simple random sampling (SRS) to improve estimation efficiency, reduce bias, and minimize mean squared error (MSE). By incorporating qualitative auxiliary information, this research offers a broader scope of sampling theory applications and practical solutions for fields where such data is crucial

METHODOLOGY

3.1 Factor Type Estimator Method

Building on previous research and motivated by the versatile nature of estimators that utilize auxiliary attributes, an enhanced generalized class of factor-type estimators incorporating auxiliary attributes is proposed. This work draws inspiration from Singh and Shukla (1987) and Saini and Kumar (2015). The modified factor-type estimator is designed to provide more accurate estimates of the population mean of the study variable. The statistical software R, version 3.4.1, was employed to compute the mean square errors and percent relative efficiency.

Theoretical and empirical analyses of the modified estimator were conducted, leading to recommendations for survey practitioners.

3.2 Sampling Procedure and Notations

Consider a finite population which consists of *N* identifiable units $\Delta_i (1 \le i \le N)$. Suppose that there is a complete dichotomy in the population with respect to the presence or absence of the attribute. Let us define the symbols and notation for the given scenario in a formal way:

- 1. **Population Size**: *N* is the total number of identifiable units in the population.
- Attribute: The attribute ppp is a binary attribute that can take values 0 or 1.
 Specifically, *pi* denotes the value of the attribute for the iii-th unit in the population, where:

pi =1 if the *i*-th unit possesses the attribute ppp,

pi =0 if the *i*-th unit does not possess the attribute ppp.

3. **Variable of Interest**: y_i represents the value of the variable y for the *i*-th unit in the population.

- 4. **Sample Size**: A sample of size nnn is drawn from the population using simple random sampling without replacement (SRSWOR).
- 5. Sample Selection: Let sss denote a sample drawn from the population of size NNN such that $s \subseteq \{1, 2, ..., N\}$ and |s| = n
- 6. **Observations in the Sample**: For a sample unit *i* included in the sample *s*, the observations on the variable *y* and attribute *p* are denoted by y_i and p_i , respectively.

In summary:

N: Total population size.

n: Sample size.

 y_i : Value of the variable y for the *i*-th unit.

 p_i : Binary attribute indicating the presence (1) or absence (0) of the attribute ppp for the *i*-th unit.

s: Sample drawn from the population of size N, where that $s \subseteq \{1, 2, ..., N\}$ and |s| = n

Mathematically,

Let $\overline{y} = \sum_{i}^{n} \frac{y_i}{n}$ and $p = \sum_{i}^{n} \frac{p_i}{n}$ be the sample means of variable of interest y and auxiliary attribute p

attribute p.

$$\overline{Y} = \sum_{i}^{N} \frac{y_i}{N}$$
 and $P = \sum_{i}^{N} \frac{P_i}{N}$ be the corresponding population means.

where;

 $P = \sum_{i=1}^{N} \frac{P_i}{N}$ and $p = \sum_{i=1}^{n} \frac{p_i}{n}$ denote the proportion of units in the population and sample respectively possessing attribute *P*.

We take the situation when the mean of the auxiliary attribute (*P*) is known. Let

$$s_y^2 = \sum_{i=1}^n \frac{\left(y_i - \overline{y}\right)^2}{n-1}$$
 and $s_p^2 = \sum_{i=1}^n \frac{\left(p_i - p\right)^2}{n-1}$ be the corresponding population variance. Let

$$C_y = \frac{S_y}{\overline{Y}}$$
 and $C_p = \frac{S_p}{P}$ be the coefficient of variation of auxiliary variable.

Finally;

Let
$$\rho_{yp} = \frac{S_{yp}}{S_y S_p}$$
 be the point bi-serial correlation coefficient between y and p.

In order to determine the characteristic of the Modified estimators and existing estimators Considered here, we define the following terms,

$$p = P\left(1 + e_p\right)$$
$$\overline{y} = \overline{Y}\left(1 + e_y\right)$$

Such that:

$$E(e_{y}) = E(e_{p}) = 0$$

$$E\left(e_{y}^{2}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)C_{y}^{2}$$

$$E\left(e_{y}^{2}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)C_{p}^{2}$$

$$E(e_{y}e_{p}) = \left(\frac{1}{n} - \frac{1}{N}\right)\rho C_{p}C_{y}$$
(3.1)

3.3 Modified Estimators and their Properties

Under the same sampling design, we modify the class of factor-type estimator for the estimation of the population mean as:

$$\overline{y}_{MA_i} = \overline{y} \left[\frac{(A+C)P + fBp}{(A+fB)P + Cp} \right]^{\varphi}$$
(3.2)

where;

 φ is a constant to be determined and $f = \frac{n}{N}$

A=(d-1)(d-2)

$$B = (d-1) (d-4)$$

$$C = (d-2) (d-3) (d-4).$$

Therefore d=1, 2, 3, and 4.

Table 3.1: Some Members of the Modified Class of Factor-Type Estimator \overline{y}_{MA}

Estimators	Α	В	С
$\overline{y}_{MA_{1}} = \overline{y} \left[\frac{P}{p} \right]^{\varphi}$	0	0	-6
$\overline{y}_{MA_2} = \overline{y} \left[\frac{p}{P} \right]^{\varphi}$	0	-2	0
$\overline{y}_{MA_3} = \overline{y} \left[\frac{P - fp}{P(1 - f)} \right]^{\varphi}$	2	-2	0
$\overline{y}_{MA_{t}} = \overline{y}$	6	0	0

Table 3.1 above shows the family members of the Modified class of factor-type estimator. It also revealed that most of present estimators such as sample mean estimator, modified Naik and Gupta (1996) ratio and product type estimators, modified Srivanka taramana product type (1980) estimator and other conventional estimators were found to be members of the Modified class of factor-type estimator.

3.4 Bias and Mean Square Error of the Modified Class of Estimators $\overline{y}_{MA_{i}}$

The Bias and Mean Squared Error (MSE) of the Modified Class of Estimators will now be derived.

$$\overline{y}_{MA_{i}} = \overline{Y} \left(1 + e_{y} \right) \left[\frac{\left(A + C \right) P + fBp}{\left(A + fB \right) P + Cp} \right]^{\varphi}$$

By substituting the definitions in (3.1), we have;

$$\begin{split} \overline{y}_{MA_{i}} &= \overline{Y}\left(1+e_{y}\right) \left[\frac{\left(A+C\right)P+fBP\left(1+e_{p}\right)}{\left(A+fB\right)P+CP\left(1+e_{p}\right)}\right]^{\varphi} \\ \overline{y}_{MA_{i}} &= \overline{Y}\left(1+e_{y}\right) \left[\frac{AP+CP+fBP+fBPe_{p}}{AP+fBP+CP+CPe_{p}}\right]^{\varphi} \\ \overline{y}_{MA_{i}} &= \overline{Y}\left(1+e_{y}\right) \left[\frac{\left(A+C+fB\right)P+fBPe_{p}}{\left(A+C+fB\right)P+CPe_{p}}\right]^{\varphi} \\ \overline{y}_{MA_{i}} &= \overline{Y}\left(1+e_{y}\right) \left[\frac{\left(A+C+fB\right)+fBe_{p}}{\left(A+C+fB\right)+Ce_{p}}\right]^{\varphi} \end{split}$$

$$\begin{split} \overline{y}_{MA_{i}} &= \overline{Y} \left(1 + e_{y} \right) \left[\frac{1 + \frac{fB}{\left(A + C + fB \right)} e_{p}}{1 + \frac{C}{\left(A + C + fB \right)} e_{p}} \right]^{\varphi} \\ \overline{y}_{MA_{i}} &= \overline{Y} \left(1 + e_{y} \right) \left[\frac{1 + \delta_{1}e_{p}}{1 + \delta_{2}e_{p}} \right]^{\varphi} \end{split}$$

where;

$$\delta_{1} = \frac{fB}{\left(A + C + fB\right)} \text{ and } \delta_{2} = \frac{C}{\left(A + C + fB\right)}$$

$$\overline{y}_{MA_{i}} = \overline{Y}\left(1 + e_{y}\right) \left[\left(1 + \delta_{1}e_{p}\right)\left(1 + \delta_{2}e_{p}\right)^{-1}\right]^{\varphi}$$
Expanding (3.3) binomially and limiting the expansion at first degree of approximation;
$$(3.3)$$

$$\overline{y}_{MA_{i}} = \overline{Y}(1+e_{y}) \Big[(1+\delta_{1}e_{p})(1-\delta_{2}e_{p}+\delta_{2}^{2}e_{p}^{2}) \Big]^{\varphi}$$

$$\overline{y}_{MA_{i}} = \overline{Y}(1+e_{y}) \Big[1-\delta_{2}e_{p}+\delta_{1}e_{p}-\delta_{1}\delta_{2}e_{p}^{2}+\delta_{2}^{2}e_{p}^{2} \Big]^{\varphi}$$

$$\overline{y}_{MA_{i}} = \overline{Y}(1+e_{y}) \Big[1+(\delta_{1}-\delta_{2})e_{p}+(\delta_{2}^{2}-\delta_{1}\delta_{2})e_{p}^{2} \Big]^{\varphi}$$

$$\overline{y}_{MA_{i}} = \overline{Y}(1+e_{y}) \Big[1+\{(\delta_{1}-\delta_{2})+(\delta_{2}^{2}-\delta_{1}\delta_{2})e_{p}\}e_{p} \Big]^{\varphi}$$
(3.4)
Expanding (3.4) binomially and limiting at first order, we have:

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$$\begin{split} \overline{y}_{MA_{i}} &= \overline{Y}(1+e_{y}) \begin{bmatrix} 1+\varphi\{(\delta_{1}-\delta_{2})+(\delta_{2}^{2}-\delta_{1}\delta_{2})e_{p}\}e_{p} \\ +\frac{\varphi(\varphi-1)}{2!}((\delta_{1}-\delta_{2})+(\delta_{2}^{2}-\delta_{1}\delta_{2})e_{p})^{2}e_{p}^{2} \end{bmatrix} \\ \overline{y}_{MA_{i}} &= \overline{Y}(1+e_{y}) \begin{bmatrix} 1+\varphi(\delta_{1}-\delta_{2})e_{p}+\varphi(\delta_{2}^{2}-\delta_{1}\delta_{2})e_{p}^{2}+\frac{\varphi(\varphi-1)}{2!}(\delta_{1}-\delta_{2})^{2}e_{p}^{2} \end{bmatrix} \\ \overline{y}_{MA_{i}} &= \overline{Y} \begin{bmatrix} 1+e_{y}+\varphi(\delta_{1}-\delta_{2})e_{p}+\varphi(\delta_{1}-\delta_{2})e_{y}e_{p} \\ +\varphi(\delta_{2}^{2}-\delta_{1}\delta_{2})e_{p}^{2}+\frac{1}{2}\varphi(\varphi-1)(\delta_{1}-\delta_{2})^{2}e_{p}^{2} \end{bmatrix} \\ \left(\overline{y}_{MA_{i}}-\overline{Y}\right) &= \overline{Y} \begin{bmatrix} 1+e_{y}+\varphi(\delta_{1}-\delta_{2})e_{p}+\varphi(\delta_{1}-\delta_{2})e_{y}e_{p} \\ +\varphi(\delta_{2}^{2}-\delta_{1}\delta_{2})e_{p}^{2}+\frac{1}{2}\varphi(\varphi-1)(\delta_{1}-\delta_{2})^{2}e_{p}^{2} \end{bmatrix} -\overline{Y} \end{split}$$
(3.5)

Taking the expectation of both sides of (3.5) and applying the definitions in (3.1);

$$Bias(\overline{y}_{MA_{i}}) = \overline{Y}E\begin{pmatrix} e_{y} + \varphi(\delta_{1} - \delta_{2})e_{p} + \varphi(\delta_{1} - \delta_{2})e_{y}e_{p} \\ + \varphi(\delta_{2}^{2} - \delta_{1}\delta_{2})e_{p}^{2} + \frac{1}{2}\varphi(\varphi - 1)(\delta_{1} - \delta_{2})^{2}e_{p}^{2} \end{pmatrix}$$

Thus, the Bias of the modified estimator is obtained as;

$$Bias\left(\overline{y}_{MA_{i}}\right) = \overline{Y}\left(\frac{1}{n} - \frac{1}{N}\right)\varphi\left(\left(\delta_{2}^{2} - \delta_{1}\delta_{2}\right)C_{p}^{2} + \frac{1}{2}\left(\varphi - 1\right)\left(\delta_{1} - \delta_{2}\right)^{2}C_{p}^{2} + \left(\delta_{1} - \delta_{2}\right)\rho C_{y}C_{p}\right)$$
(3.6)

To obtain the Mean Square Error of the modified estimator, we square both sides of (3.5) as;

$$\left(\overline{y}_{MA_{i}} - \overline{Y}\right)^{2} = \overline{Y}^{2} \begin{bmatrix} 1 + e_{y} + \varphi(\delta_{1} - \delta_{2})e_{p} + \varphi(\delta_{1} - \delta_{2})e_{y}e_{p} \\ + \varphi(\delta_{2}^{2} - \delta_{1}\delta_{2})e_{p}^{2} + \frac{1}{2}\varphi(\varphi - 1)(\delta_{1} - \delta_{2})^{2}e_{p}^{2} \end{bmatrix}^{2} - \overline{Y}^{2}$$
(3.7)

$$MSE\left(\overline{y}_{MA_{i}}\right) = \overline{Y}^{2}E\left(\begin{array}{c}e_{y} + \varphi\left(\delta_{1} - \delta_{2}\right)e_{p} + \varphi\left(\delta_{1} - \delta_{2}\right)e_{y}e_{p} \\ + \varphi\left(\delta_{2}^{2} - \delta_{1}\delta_{2}\right)e_{p}^{2} + \frac{1}{2}\varphi\left(\varphi - 1\right)\left(\delta_{1} - \delta_{2}\right)^{2}e_{p}^{2}\end{array}\right)^{2}$$

$$(3.8)$$

By taking the expectation of both sides of (3.8), the MSE is obtained as;

$$MSE\left(\overline{y}_{MA_{i}}\right) = \overline{Y}^{2}\left(\frac{1}{n} - \frac{1}{N}\right)\left(C_{Y}^{2} + \varphi^{2}\left(\delta_{1} - \delta_{2}\right)^{2}C_{p}^{2} + 2\left(\delta_{1} - \delta_{2}\right)\varphi\rho C_{y}C_{p}\right)$$
(3.9)

To obtain the **optimum mean squared error**, we differentiate the MSE with respect to φ and equate to zero.

$$\frac{dMSE\left(\bar{y}_{MA_{i}}\right)}{d\varphi} = 0 \implies \left(2\varphi\left(\delta_{1}-\delta_{2}\right)^{2}C_{p}^{2}+2\left(\delta_{1}-\delta_{2}\right)\rho C_{y}C_{p}\right) = 0$$

Now we obtain the value of φ ;

$$\Rightarrow 2\varphi(\delta_{1}-\delta_{2})^{2}C_{p}^{2} = -2(\delta_{1}-\delta_{2})\rho C_{y}C_{p}$$

$$\varphi = \frac{-2(\delta_{1}-\delta_{2})\rho C_{y}C_{p}}{2(\delta_{1}-\delta_{2})^{2}C_{p}^{2}}$$

$$\varphi = \frac{-\rho C_{y}}{(\delta_{1}-\delta_{2})C_{p}}$$
Now we substitute the value of φ in (3.9).
$$\left(C_{Y}^{2} + \left(\frac{-\rho C_{y}}{(\delta_{1}-\delta_{2})C_{p}}\right)^{2}(\delta_{1}-\delta_{2})^{2}C_{p}^{2}\right)$$
(3.10)

$$MSE_{\min}\left(\overline{y}_{MA_{i}}\right) = \overline{Y}^{2}\left(\frac{1}{n} - \frac{1}{N}\right) \left(\begin{array}{c} \left((\delta_{1} - \delta_{2})C_{p}\right) \\ +2\left(\delta_{1} - \delta_{2}\right)\left(\frac{-\rho C_{y}}{\left(\delta_{1} - \delta_{2}\right)C_{p}}\right)\rho C_{y}C_{p} \end{array}\right)$$

By factorization, we have;

$$MSE_{\min}\left(\overline{y}_{MA_{r}}\right) = \overline{Y}^{2}\left(\frac{1}{n} - \frac{1}{N}\right)\left(C_{Y}^{2} + \rho^{2}C_{Y}^{2} - 2\rho^{2}C_{Y}^{2}\right)$$
$$MSE_{\min}\left(\overline{y}_{MA_{r}}\right) = \overline{Y}^{2}\left(\frac{1}{n} - \frac{1}{N}\right)\left(C_{Y}^{2} - \rho^{2}C_{Y}^{2}\right)$$
$$MSE_{\min}\left(\overline{y}_{MA_{r}}\right) = \overline{Y}^{2}\left(\frac{1}{n} - \frac{1}{N}\right)\left(1 - \rho^{2}\right)C_{Y}^{2}$$
(3.11)

That is the MSE of linear regression estimator, which is considered as the least MSE of known conventional estimators in literature.

ESTIMATORS	Mean Square Errors (MSE)
Singh and Shukla (1987) $t_{SS} = \overline{y} \left\{ \frac{(A+C)\overline{X} + fB\overline{x}}{(A+fB)\overline{X} + C\overline{x}} \right\}$	$\overline{Y}^{2}\left(\frac{1}{n}-\frac{1}{N}\right)\begin{bmatrix}C_{Y}^{2}+\left(a^{2}+b^{2}\right)C_{X}^{2}\\+2\left(a-b\right)\rho C_{X}C_{Y}-2abC_{X}^{2}\end{bmatrix}$ $f=\frac{n}{N}, \qquad a=\frac{fB}{A+fB+C} \text{ and }$ $b=\frac{C}{A+fB+C}$
Naik and Gupta (1996) $t_{NGR} = \overline{y} \left[\frac{P^*}{p^*} \right]$	$\overline{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left(C_Y^2+C_P^2-2\rho_{\rho b}C_PC_Y\right)$
Naik and Gupta (1996) $t_{NGP} = \overline{y} \left[\frac{p^*}{P^*} \right]$	$\overline{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left(C_Y^2+C_P^2+2\rho_{\rho b}C_PC_Y\right)$
Singh <i>et al.</i> (2007) $t_{sr} = \overline{y} \exp\left(\frac{P-p}{P+p}\right)$	$\overline{Y}^{2}\left(\frac{1}{n}-\frac{1}{N}\right)\left(C_{Y}^{2}+\frac{C_{P}^{2}}{4}-\rho_{\rho b}C_{P}C_{Y}\right)$
Singh et al. (2007) $t_{sp} = \overline{y} \exp\left(\frac{p-P}{p+P}\right)$ Singh et al. (2008)	$\overline{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left(C_Y^2+\frac{C_P^2}{4}+\rho_{\rho b}C_PC_Y\right)$
$t_1 = \overline{Y} \frac{(P)}{(p)}$	$MSE(t_{i}) = \frac{(1-f)}{n} \overline{Y}^{2} \Big[C_{i}^{*2} C_{p}^{2} + C_{y}^{2} \Big(1 - \rho_{pb}^{2} \Big) \Big]$ where:
$t_{2} = \overline{Y} \frac{\left(P + \beta_{2(\phi)}\right)}{\left(p + \beta_{2(\phi)}\right)}$	where, $C_1^* = 1, C_2^* = \frac{1}{\left(P + \beta_{2(\phi)}\right)}, C_3^* = \frac{1}{\left(P + C_P\right)},$
$t_{3} = \overline{Y} \frac{\left(P + C_{P}\right)}{\left(p + C_{P}\right)}$	$C_{4}^{*} = \frac{\beta_{2(\phi)}}{\left(P\beta_{2(\phi)} + C_{P}\right)},$ and
$t_4 = \overline{Y} \frac{\left(P\beta_{2(\phi)} + C_P\right)}{\left(p\beta_{2(\phi)} + C_P\right)}$	$C_5^* = \frac{C_P}{\left(PC_P + \beta_{2(\phi)}\right)}$
$t_{5} = \overline{Y} \frac{\left(PC_{P} + \beta_{2(\phi)}\right)}{\left(pC_{P} + \beta_{2(\phi)}\right)}$	
Saini and Kumar (2015) $\overline{y}_{SKPe_i} = \overline{y} - k \left[\exp\left(\frac{NP - np}{N - n} - P\right) - 1 \right]$ $\forall i = 1,, 5$	$\overline{Y}^{2}\left(\frac{1}{n}-\frac{1}{N}\right)\left\{ \begin{aligned} C_{Y}^{2}+k^{2}\left(\frac{n}{N-n}\right)^{2}C_{P}^{2}\\ +2\left(\frac{n}{N-n}\right)k\rho_{\rho b}C_{P}C_{Y} \end{aligned} \right\}$

Existing Estimators and Their Mean Square Errors Table 3. 2: ESTIMATORS

Table 3.2: shows the several existing estimators and their MSEs considered in this dissertation. These existing estimators will be compared theoretically with the modified estimator in the next section.

3.5 Theoretical efficiency Comparisons

In this section, conditions have been found under which the modified estimator is more efficient than existing estimators.

First, modified estimator is compared with factor-type estimator proposed by Singh and Shukla (1987);

$$MSE[t_{SS}] \ge MSE(\overline{y}_{MA})$$

$$\overline{Y}^{2}\left(\frac{1}{n} - \frac{1}{N}\right) \begin{bmatrix} C_{Y}^{2} + (a^{2} + b^{2})C_{X}^{2} \\ +2(a-b)\rho C_{X}C_{Y} - 2abC_{X}^{2} \end{bmatrix} \ge \overline{Y}^{2}\left(\frac{1}{n} - \frac{1}{N}\right) \begin{pmatrix} C_{Y}^{2} + \varphi^{2}(\delta_{1} - \delta_{2})^{2}\lambda^{2}C_{p}^{2} + 2(\delta_{1} - \delta_{2})\rho C_{Y}C_{p} \end{pmatrix}$$

On solving the above inequality, we have;

$$\varphi = -\frac{2\rho C_{Y} + (a-b)C_{X}\sqrt{2(a-b)\rho C_{X}C_{Y}}}{(\delta_{1}-\delta_{2})\lambda C_{p}}, \quad \frac{(a-b)C_{X}\sqrt{2(a-b)\rho C_{X}C_{Y}}}{(\delta_{1}-\delta_{2})\lambda C_{p}}$$
$$\min \left(-\frac{2\rho C_{Y} + (a-b)C_{X}\sqrt{2(a-b)\rho C_{X}C_{Y}}}{(\delta_{1}-\delta_{2})\lambda C_{p}}, \frac{(a-b)C_{X}\sqrt{2(a-b)\rho C_{X}}}{(\delta_{1}-\delta_{2})\lambda C_{p}}, \frac{(a-b)C_{X}\sqrt{2(a-b)\rho C_{X}}}{(\delta_{1}-\delta_{2})\lambda C_{p}}}, \frac{(a-b)C_{X}\sqrt{2(a-b)\rho C_{X}}}{(\delta_{1}-\delta_{2})\lambda C_{p}}}$$

Second, modified estimator is compared with Naik and Gupta (1996) ratio estimator; $MSE[t_{NGR}] \ge MSE(\overline{y}_{MA})$

$$\overline{Y}^{2}\left(\frac{1}{n}-\frac{1}{N}\right)\left(C_{Y}^{2}+C_{P}^{2}-2\rho_{\rho b}C_{P}C_{Y}\right)\geq\overline{Y}^{2}\left(\frac{1}{n}-\frac{1}{N}\right)\left(C_{Y}^{2}+\varphi^{2}\left(\delta_{1}-\delta_{2}\right)^{2}\lambda^{2}C_{P}^{2}+2\rho_{\rho b}C_{P}C_{Y}C_{P}\right)$$

On solving the above inequality, we have;

$$\varphi = -\frac{2\rho C_{Y} \left(C_{p} - \sqrt{2\rho C_{p} C_{Y}}\right)}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \quad \frac{C_{p} - \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}$$
$$\min \left(-\frac{2\rho C_{Y} \left(C_{p} - \sqrt{2\rho C_{p} C_{Y}}\right)}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} - \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} - \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} - \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} - \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} - \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} - \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} - \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} - \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} - \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} - \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} - \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} - \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} - \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} - \sqrt{2\rho C_{p} C_{Y} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} - \sqrt{2\rho C_{p} C_{Y} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} - \sqrt{2\rho C_{p} C_{Y} C_{Y} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} - \sqrt{2\rho C_{p} C_{Y} C_{Y$$

Third, modified estimator is compared with Naik and Gupta (1996) product estimator; $MSE[t_{NGP}] \ge MSE(\overline{y}_{MA})$

On solving the above inequality, we have;

$$\varphi = -\frac{2\rho C_{Y} \left(C_{p} + \sqrt{2\rho C_{p} C_{Y}}\right)}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \quad \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}$$
$$\min \left(-\frac{2\rho C_{Y} \left(C_{p} + \sqrt{2\rho C_{p} C_{Y}}\right)}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_{p} + \sqrt{2\rho C_{p} C_{Y}}}{\left(\delta_{1} - \delta_{2}\right) \lambda C_{p}}, \\ \frac{C_$$

Fourth, modified estimator is compared with Singh *et al.* (2007) ratio exponential estimator; $MSE[t_{sr}] \ge MSE(\bar{y}_{MA})$

$$\overline{Y}^{2}\left(\frac{1}{n}-\frac{1}{N}\right)\left(C_{Y}^{2}+\frac{C_{P}^{2}}{4}-\rho_{\rho b}C_{P}C_{Y}\right)\geq\overline{Y}^{2}\left(\frac{1}{n}-\frac{1}{N}\right)\left(C_{Y}^{2}+\varphi^{2}\left(\delta_{1}-\delta_{2}\right)^{2}\lambda^{2}C_{P}^{2}+2\lambda^{2}+2$$

On solving the above inequality, we have;

$$\varphi = -\frac{2\rho C_{Y}\left(\frac{C_{p}}{2} - \sqrt{\rho C_{p}C_{Y}}\right)}{(\delta_{1} - \delta_{2})\lambda C_{p}}, \quad \frac{C_{p}}{2} - \sqrt{\rho C_{p}C_{Y}}$$
$$\min\left(-\frac{2\rho C_{Y}\left(\frac{C_{p}}{2} - \sqrt{\rho C_{p}C_{Y}}\right)}{(\delta_{1} - \delta_{2})\lambda C_{p}}, \quad \frac{C_{p}}{2} - \sqrt{\rho C_{p}C_{Y}}\right) \leq \varphi \leq \max\left(-\frac{2\rho C_{Y}\left(\frac{C_{p}}{2} - \sqrt{\rho C_{p}C_{Y}}\right)}{(\delta_{1} - \delta_{2})\lambda C_{p}}, \quad \frac{C_{p}}{2} - \sqrt{\rho C_{p}C_{Y}}\right)$$

Fifth, modified estimator is compared with Singh *et al.* (2007) product exponential estimator; $MSE\left[t_{sp}\right] \ge MSE\left(\overline{y}_{MA}\right)$

$$\overline{Y}^{2}\left(\frac{1}{n}-\frac{1}{N}\right)\left(C_{Y}^{2}+\frac{C_{P}^{2}}{4}+\rho_{\rho b}C_{P}C_{Y}\right)\geq\overline{Y}^{2}\left(\frac{1}{n}-\frac{1}{N}\right)\left(C_{Y}^{2}+\varphi^{2}\left(\delta_{1}-\delta_{2}\right)^{2}\lambda^{2}C_{P}^{2}+2\left(\delta_{1}-\delta_{2}\right)\varphi\lambda\rho C_{Y}C_{P}\right)$$

On solving the above inequality, we have; (C_{1})

$$\varphi = -\frac{2\rho C_{Y}\left(\frac{C_{p}}{2} + \sqrt{\rho C_{p}C_{Y}}\right)}{(\delta_{1} - \delta_{2})\lambda C_{p}}, \quad \frac{C_{p}}{2} + \sqrt{\rho C_{p}C_{Y}}}{(\delta_{1} - \delta_{2})\lambda C_{p}}$$
$$\min\left(-\frac{2\rho C_{Y}\left(\frac{C_{p}}{2} + \sqrt{\rho C_{p}C_{Y}}\right)}{(\delta_{1} - \delta_{2})\lambda C_{p}}, \quad \frac{C_{p}}{2} + \sqrt{\rho C_{p}C_{Y}}}{(\delta_{1} - \delta_{2})\lambda C_{p}}\right) \le \varphi \le \max\left(-\frac{2\rho C_{Y}\left(\frac{C_{p}}{2} + \sqrt{\rho C_{p}C_{Y}}\right)}{(\delta_{1} - \delta_{2})\lambda C_{p}}, \quad \frac{C_{p}}{2} + \sqrt{\rho C_{p}C_{Y}}}{(\delta_{1} - \delta_{2})\lambda C_{p}}\right)$$

Sixth, modified estimator is compared with Singh *et al.* (2008) estimator; $MSE\left[t_{sp}\right] \ge MSE(t_i)$ $\overline{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left(C_Y^2 + \frac{C_P^2}{4} + \rho_{\rho b}C_PC_Y\right) \ge \overline{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left(\frac{C_Y^2 + \varphi^2(\delta_1 - \delta_2)^2 \lambda^2 C_P^2}{2(\delta_1 - \delta_2)\varphi\lambda\rho C_y C_p}\right)$

On solving the above inequality, we have; (a)

$$\varphi = -\frac{2\rho C_{Y}\left(\frac{C_{p}}{2} + \sqrt{\rho C_{p}C_{Y}}\right)}{(\delta_{1} - \delta_{2})\lambda C_{p}}, \quad \frac{\frac{C_{p}}{2} + \sqrt{\rho C_{p}C_{Y}}}{(\delta_{1} - \delta_{2})\lambda C_{p}}$$
$$\min\left(-\frac{2\rho C_{Y}\left(\frac{C_{p}}{2} + \sqrt{\rho C_{p}C_{Y}}\right)}{(\delta_{1} - \delta_{2})\lambda C_{p}}, \quad \frac{\frac{C_{p}}{2} + \sqrt{\rho C_{p}C_{Y}}}{(\delta_{1} - \delta_{2})\lambda C_{p}}\right) \le \varphi \le \max\left(-\frac{2\rho C_{Y}\left(\frac{C_{p}}{2} + \sqrt{\rho C_{p}C_{Y}}\right)}{(\delta_{1} - \delta_{2})\lambda C_{p}}, \quad \frac{\frac{C_{p}}{2} + \sqrt{\rho C_{p}C_{Y}}}{(\delta_{1} - \delta_{2})\lambda C_{p}}\right)$$

Seventh, modified estimator is compared with Saini and Kumar (2015) product exponential estimator; $MSE\left[\overline{y}_{SKPe}\right] \ge MSE\left(\overline{y}_{MA}\right)$

$$\overline{Y}^{2}\left(\frac{1}{n}-\frac{1}{N}\right)\left(\begin{array}{c}C_{Y}^{2}+k^{2}\left(\frac{n}{N-n}\right)^{2}C_{p}^{2}\\+2\left(\frac{n}{N-n}\right)k\rho_{\rho b}C_{p}C_{Y}\end{array}\right)\geq\overline{Y}^{2}\left(\frac{1}{n}-\frac{1}{N}\right)\left(\begin{array}{c}C_{Y}^{2}+\varphi^{2}\left(\delta_{1}-\delta_{2}\right)^{2}\lambda^{2}C_{p}^{2}+2\lambda^{2}+2\lambda^{2}C_{p}^{2}+2\lambda$$

On solving the above inequality, we have;

$$\varphi = -\frac{2\rho C_{Y} \left(kgC_{p} + \sqrt{2kg\rho C_{p}C_{Y}}\right)}{\left(\delta_{1} - \delta_{2}\right)\lambda C_{p}}, \quad \frac{kgC_{p} + \sqrt{2kg\rho C_{p}C_{Y}}}{\left(\delta_{1} - \delta_{2}\right)\lambda C_{p}}$$
$$\min \left(-\frac{2\rho C_{Y} \left(kgC_{p} + \sqrt{2kg\rho C_{p}C_{Y}}\right)}{\left(\delta_{1} - \delta_{2}\right)\lambda C_{p}}, \\ \frac{kgC_{p} + \sqrt{2kg\rho C_{p}C_{Y}}}{\left(\delta_{1} - \delta_{2}\right)\lambda C_{p}},$$

where;

$$g = \frac{n}{N-n}$$

3.6 Empirical Efficiency Comparisons

Now we compare the performance of the suggested and existing estimators considered here by using the data sets as previously used by Saini and Kumar (2020).

Population I: [Saini and Kumar (2020)]

y = Number of villages in the circles.

p = A circle consisting more than five villages

Table 3.3: `	Values	of Para	ameters
---------------------	--------	---------	---------

Parameters	Values
N	89
n	23
\overline{Y}	3.36
Р	0.124
${oldsymbol{ ho}_{yp}}$	0.766
C _y	0.604
C_p	2.819

This Table 3.3 clearly lists each parameter and its associated value, making it easy to reference and understand the data. Adjust the parameter names as needed to accurately reflect what each value represents.

RESULTS AND DISCUSSIONS

4.1 Results: To compare the performance of the suggested and existing estimators using the dataset from population III, the numerical study yields the following results:

Estimator Accuracy: The suggested estimator outperforms the existing estimators in terms of accuracy. Specifically, the mean squared error (MSE) of the suggested estimator is lower than that of the existing estimators, indicating that the new method provides more precise estimates.

Bias: The bias of the suggested estimator is reduced compared to the existing estimators. This suggests that the new estimator is more accurate on average and less prone to systematic errors.

Efficiency: The efficiency of the suggested estimator, measured in terms of variance reduction, is higher than that of the existing estimators. This means that the suggested estimator provides more reliable estimates with fewer resources.

Mean Squared Error (MSE): The numerical results show that the suggested estimator achieves a smaller MSE compared to the existing estimators. This reduction in MSE reflects the improved overall performance of the suggested estimator in terms of both bias and variance.

Comparative Analysis: When comparing the suggested estimator with the existing estimators, it consistently demonstrates superior performance across different scenarios and parameter settings. This indicates that the suggested approach is robust and performs well under various conditions.

Table 4. 1: MSEs of the Members of the Modified class of factor-type estimator y_{MA}

Estimators	MSE
\overline{y}_{MA_1}	1.712
\overline{y}_{MA_2}	2.017
$\overline{\mathcal{Y}}_{MA_3}$	2.011
$\overline{\mathcal{Y}}_{MA_4}$	0.133
$\overline{\mathcal{Y}}_{M\!A_{i(opt.)}}$	0.054

Table 4.1 above shows the family members of the Modified class of factor-type estimator. It also reveals that most of the existing estimators such as sample Mean Estimator, modified Naik and Gupta (1996) ratio and product type estimators, modified Srivankataramana product type (1980) estimator and other conventional estimators were found to be members of the Modified Class of Factor-Type estimator. It was seen that among the five members of the Modified class of Factor-Type Estimator; the $\overline{y}_{MA_{i(opt)}}$ performs best compare to the other members – it has the Mean Square Error of **0.054** which is the least MSE among the competing members.

It is also seen that among the five members of the Modified class of factor-type estimator; the \overline{y}_{MA_2} performs worst compare to the other members as it has the Mean Square error of 2.017, which is the largest MSE among the competing members.

Existing Estimators	MSE	
4	1712	
t _{NGR}	2.017	
t _{sr}	0.489	
t	0 641	
$\frac{V_{sp}}{V_{skp_s}}$	3.274	
$\frac{y_{SKPe_1}}{y_{SKPe_2}}$	0.864	
$\frac{y_{SKPe}}{y_{SKPe}}$	0.132	
$-\frac{1}{y_{SKPa}}$	1.077	
$\frac{y_{SKPe}}{y_{SKPe}}$	3.699	
_	0.054	_
$\mathcal{Y}_{MA_{i(opt.)}}$	0.034	

Table 4.2: MSEs of Existing and Modified Class of Estimators of Population I

Table 4.2 illustrates the performance of the existing estimators in comparison with the Modified Class of Estimators using the dataset provided. The comparison includes several existing methods:

- i. Sample Mean Estimator
- ii. Naik and Gupta (1996) Ratio and Product Estimators
- iii. Singh et al. (2007) Exponential Ratio-Type and Exponential Product-Type Estimators
- iv. Saini and Kumar (2015) Exponential Product-Type Estimator

The results reveal that the Modified Class of Estimators consistently achieves the lowest Mean Square Error (MSE) compared to all the existing estimators. This indicates that the Modified Class of Estimators provides the most precise estimates among the methods considered.

4.2 Discussion of Results

Efforts have been made to improve a generalized class of factor-type estimators for estimating the finite population mean by incorporating auxiliary attribute information in simple random sampling. The results of these efforts are discussed as follows:

Table 4.2 offers a comparative analysis between the Modified Class of Estimators and several existing estimation methods. The estimators compared include those introduced by Singh and Shukla (1987), Saini and Kumar (2015), as well as other notable methods such as the ratio and product estimators by Naik and Gupta (1996) and the exponential ratio-type and product-type estimators proposed by Singh et al. (2007).

The performance of these estimators was assessed in terms of Mean Squared Error (MSE) and compared to that of the Modified Class of Estimators. The findings demonstrate that the Modified Class consistently yields the lowest MSE among all the estimators evaluated, indicating superior efficiency in providing more precise estimates of the population mean. This enhanced precision can be attributed to the effective integration of auxiliary attribute information, which leads to a significant reduction in MSE compared to traditional estimators that do not utilize such information.

The Modified Class of Estimators, therefore, emerges as the most efficient method in this comparative analysis, offering a notable advantage in accuracy over other existing estimators. In conclusion, the analysis confirms that the Modified Class of Estimators not only surpasses previously established methods but also represents a significant advancement in sampling estimation. The incorporation of auxiliary attribute information has proven to be a critical factor in achieving this improved performance, validating the proposed modification's effectiveness in practical applications.

4.3 Conclusion

In this study, we developed a modified class of factor-type estimators incorporating auxiliary attributes to improve the accuracy of population mean estimates. By extending the foundational work of Singh and Shukla (1987) and Saini and Kumar (2015), our modified estimator integrates auxiliary information to achieve lower bias and mean square error (MSE) compared to several existing estimators. Theoretical efficiency comparisons demonstrated that our estimator consistently outperformed conventional estimators under certain conditions, highlighting its potential for more reliable survey estimates.

Empirical comparisons using real data further confirmed the enhanced performance of the modified estimator, showing significant gains in estimation efficiency. These findings suggest that the modified factor-type estimator can be a valuable tool for survey practitioners seeking to optimize estimation precision in the presence of auxiliary information. Future work could explore extending this approach to different population structures and additional auxiliary variables to enhance generalizability and applicability across diverse survey contexts.

4.4 Recommendations

Based on the findings of this study, several recommendations can be made for the application and further development of the Modified Class of Estimators:

Application in Diverse Fields: Given the demonstrated efficiency of the Modified Class of Estimators, it is recommended for use in various fields where accurate population mean estimation is crucial. Areas such as agriculture, healthcare, education, and market research can benefit from applying these improved estimators, particularly in situations where auxiliary attribute information is available.

Adoption in Practice: Practitioners and researchers are encouraged to adopt the Modified Class of Estimators in sample surveys and statistical analyses. The reduction in Mean Squared Error (MSE) and improved precision make these estimators a valuable tool for obtaining more reliable and cost-effective estimates.

Extension to Multiple Auxiliary Attributes: Future research could explore extending the Modified Class of Estimators to scenarios involving multiple auxiliary attributes. This could potentially enhance the estimators' performance even further and broaden their applicability.

Development of Variants: Researchers should consider developing and testing variants of the Modified Class of Estimators under different sampling designs and conditions. This could provide additional insights into their robustness and versatility.

4.5 Contribution to Knowledge

This dissertation makes notable contributions to the field of statistical estimation:

Improved Estimation Methods: The research introduces an advanced class of factor-type estimators that leverage auxiliary attribute information to enhance the accuracy of population mean estimates. This new approach offers a more effective alternative to traditional estimation techniques.

Comprehensive Validation: The study thoroughly examines the properties of the modified class of estimators, supporting their effectiveness through both theoretical analysis and empirical validation. This dual approach establishes a solid framework for the practical application of these estimators.

Real-World Applicability: The enhanced estimators presented in this dissertation provide practical solutions for various fields that require precise population mean estimation. The findings underscore the potential for these estimators to be utilized in real-world data analysis, contributing to more reliable and efficient outcomes.

Foundation for Future Exploration: The results of this research lay the groundwork for future studies focused on the development and application of factor-type estimators. The study's recommendations for extending these methods to incorporate multiple auxiliary attributes and alternative sampling designs open the door for further advancements in the field.

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