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COMPARATIVE TEST FOR MODEL BEST-FIT USING INFORMATION CRITERIA.

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ABSTRACT

This research work evaluated model fit through Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and Hannan-Quinn Information Criterion (HQC). Information criterion is an important aspect of scientific data analysis. This is due to the critical nature of selection of an appropriate approximating model to statistical inference. This research work examined AIC, BIC and HQC in terms of sample size, number of parameter estimates and also their measures of out of sample predicted accuracy (penalty term). Simulated data and secondary data gotten from CBN (Central Bank of Nigeria) yearly's bulletin were applied for this research. The data analyses were conducted with Statistical software R Studio. The data analysis indicated that HQC performs better than AIC and BIC for very small sample size while AIC performs better than BIC and HQC for sample size of 20 and above. The penalty terms of BIC and HQC increases with increasing sample size and number of parameter estimates, this indicates that, BIC and HQC penalizes the sample size and number of parameter estimates. The penalty terms of AIC remains constant with increasing number of sample size but increases with increasing number of parameter estimates, this indicates that, AIC penalizes the number of parameter estimates. The Bartlett's test of homogeneity of variances indicated that, there is no significant difference among the variances of the information criteria considered in this research work. Hence, we can conclude that HQC performs better than AIC and BIC for sample size less than 20 while AIC performs better than BIC and HQC for sample size of twenty and above.

I. Introduction

Statistical modeling is an important aspect of scientific data analysis. Statistical model as defined by Cambridge dictionary of Statistics is a description of the assumed structure of a set of observations that can range from a fairly imprecise verbal account to, more usually, formalized mathematical expression of the process assumed to have generated the data. Statistical model gives a better understanding of the data[4].

Konishi and Kitagawa (1974) identified three purposes of statistical model, which are, predictions, extraction of information and description of stochastic structures[1]. The purpose of statistical modeling is not to accurately describe current data or infer the true distribution but, rather to, predict future data as accurately as possible. Another important aspect considered is the extraction of information. Many conventional statistical inferences assumed that the true model that governs the object of modeling is a known entity or at least the true model exists. Statistical inferences have adopted the approach of estimation of parameters based on data, given that the true model exists, and that, these parameters are contained in the model. However, Statistical Models can be regarded as tools that can be used for extracting information and gaining knowledge about the set of observations. Statistical models are constructed based on prior knowledge of the analyst and expectation concerning the modeling objectives. The main aim of modeling is not whether a given statistical model accurately represents the true structure of phenomenon, but whether it is a suitable tool for extracting useful information from the data. Akaike and Kitagawa (1974) are of the view that, statistical modeling is not to estimate a perfect model, but rather to construct a good model as a tool for extraction of information according to the characteristics of the object and purpose of modeling. This implies that, results of inference and evaluation will vary according to specific models[1]. A good model generally yields good results; however, one cannot expect to obtain good results from an inappropriate model.

According to Akaike (1974), when a model is for the purpose of predictions, it should be evaluated in terms of the goodness of the results[1]. Furthermore, for general evaluation of a statistical model, Akaike is of the opinion that, the closeness between the predictive distribution defined by the model and the true distribution should be assessed, rather than, just minimizing the predictive error. Based on this concept, He proposed that evaluation of statistical models should be in terms of Kuller-Leibler information (K-L). Kuller-Liebler information as defined by Cambridge dictionary of statistics is a function, *I,* defined for two probability distributions, f(x) and g(x) and given by[1]:

$$I(f:g) = \int_0^\infty \ln\{\frac{f(x)}{g(x)}\}dx \quad (1)$$

This is essentially an asymmetric distance function of two distributions. This information criterion was derived from three fundamental concepts, which are; a predictive based viewpoint of modeling, evaluation of prediction accuracy in terms of distribution and evaluation of the Kullback-Leibler information. Kullback-Liebler distance as defined by Kullback (1959) is a directed distance between two model, say f(x) and g(x) [12]. According to Akaike (1983), the K-L distance between models is a fundamental quantity in science and information theory and is the logical basis for model selection in conjunction with likelihood inferences[2].

II. Literature Review

Over the past decades, many researchers have worked on Information Criterion (IC) model based selection procedures. Some of the researchers includes, Akaike (1973, 1981), Schwarz (1978), Koehler and Murphree (1988), Hannan and Quinn (1979), Sim and White (1996), Mills and Prasad (1992) and Granger et al (1995).

The merits of information criterion have been extensively debated mainly in the context of their asymptotic properties by some researchers. Information criteria were constructed as estimators of Kuller-Leibler information discrepancy between the true distribution and the statistical model.

A criterion is order consistent if, as the sample size increases, the criterion is minimized at the true order with probability that approaches unity. The AIC procedures have been criticized because of its inconsistency and overfitting of models.

Schwartz (1978) affirmed that, quantitatively, both BIC (also known as SIC or SBIC) and AIC gives a mathematical formulation of the principles of parsimony in model building. However, since BIC procedure differs from AIC only in that, the dimension is multiplied by $\frac{1}{2}\log n$, BIC procedure leans more than AIC towards lower-dimensional models (when there are 8 or more observations) and thus for large number of observations, the both procedures differs markedly from each other. Schwarz also concluded that if the assumptions of section two of his 1978 paper (Estimating the dimension of a model) are accepted, AIC could not be asymptotically optimal, as this would contradict any proof of its optimality[14].

Stone (1979) compared the AIC and BIC. He demonstrated that the comparison of the model selection criteria of the AIC and BIC is sensitive to the type of asymptotic analysis adopted. He suggested that AIC might not be asymptotically justifiable in any reasonable way and therefore, threw doubt on AIC's heuristic derivation. He is of the opinion that, Akaike (1978) should have defended his heuristic but rather Akaike chose to criticize the relevance of Schwarz's theorem, which he developed the "maximax type optimality" of AIC. Akaike showed that if $n\sigma^2 \rightarrow 1$, the selection of the model with the maximum posterior probability is asymptotically equivalent to the use of AIC. Stone is also of the opinion that, even if Akaike's "maximax type optimality" is rather contrived and specialized, it contains an interesting departure from the almost universal form of asymptotic studies, which is, in the context, to let $n \rightarrow \infty$ in a fixed model[16].

Shibata (1976) also studied the asymptotic properties of the AIC. He noticed that, although the estimator was not consistent, the probability of selecting too high order dies out quickly as the sample size increases. He did that by generating pseudo normal random numbers by summing 12 uniform random numbers which themselves were generated by mixed congruential method, varying the initial conditions for each 1000 uniform random numbers .However, Shibata's major concern was about the prediction error rather than the distribution of the order. Despite the inconsistency of the AIC approach, Shibata (1978) is of the view that, it is not necessarily wise to insist on the consistency of the selection procedure. Inconsistency does not imply a defect in the selection procedure but rather inevitable concomitant of balancing under fitting and over fitting risk[15].

Geweke and Meese (1981) showed that the AIC procedure is inconsistent and overfits models. They fitted regression models by simulating data of sample sizes 50, 100 and 200. Regression Models of different orders were fitted to each sample size respectively. They discovered that, among the criteria tested, SBIC lead to asymptotically efficient estimates of the parameters of the true model but AIC and other criteria investigated were inconsistent[6].

Koehler and Murphree (1988) compared the criteria on real data, their applications were on time series and their study was limited to AIC and BIC. In time series analysis, model identification and parameter estimation are very critical. Most analysts would agree that identification problem is the more difficult part, once the functional form of the model is specified, estimation of parameter model is usually straightforward. Koehler and Murphree (1988) fitted time series model using a set of randomly selected data from the series used in the Makridakis competition (1982). The AIC and BIC of the fitted models indicated different model orders in 27% of the cases. They also compared the forecasting accuracy of the models, the results indicated that BIC leads to lower order model for forecasting, hence, BIC was preferred to AIC.

Hossian M (2002) developed the modified version of AIC, which he called Modified Akaike Information Criterion (MAIC). His main interest was to examine the effect of making an interval restriction on the parameter of interest within the model selection framework. This was to ascertain whether such restrictions imposed on the parameter of interest could improve AIC in terms of accuracy of model selection. The MAIC was derived by imploring the general Kuhn-Tucker inequality constrained optimization framework considered by Gourierioux et al (1982) for a one sided hypothesis-testing problem. Hence, MAIC was derived and was discovered that, the MAIC possesses smaller penalties than the usual AIC, thus the MAIC provides more information than the AIC[8].

Wang and Liu (2005) compared the AIC and BIC in selection of stock recruitment relationships. They compared these criteria using simulated SR (stock recruitment) data. The simulated data were fitted to a SR statistical model. Their results indicated that, AIC and BIC are valid in selecting the most suitable SR relationships, however, as far as nested model are concerned, BIC is better when compared to AIC.

Karlsson et al (2017) worked on performance of model selection criteria when variables are ill conditioned, their application were limited to R^2 adjusted (coefficient of determination), AIC, BIC and HQC. In their application, they used multiple regression models to compare the various information criteria within the scope of their research work. They were of the opinion that, because some model selection criteria are based on asymptotic results, it is of interest to investigate these criteria for various sample size. They also compared these criteria as the estimated parameter increases. Their results indicated that, HQC outperforms AIC, BIC and R^2 adjusted under some specific circumstances, however, none of them performs satisfactorily when the degree of multicollinearity is high and the sample size is small[9].

III. Materials and Methods

We shall compare the under listed information criteria (AIC, BIC and HQC) by simulation of data with R Studio. R Studio is an integrated development environment for R, a programming language for statistical computing and graphics. R Studio will be used for data analysis. We shall simulate data with dependent variable Y and independent variables X_1, X_2, X_3 and X_4 for sample sizes 10, 15, 20, 25, 50, 100, 150 and 200 subsequently. Regression model will be fitted to the various set of simulated data. Regression model will also be fitted to a secondary data obtained from Central Bank of Nigeria (CBN) bulletin. The following information criteria; AIC, BIC and HQC will be calculated for each fitted regression model. In addition, because some model selection criteria are based on asymptotic results, it is also an interest to investigate the performance of these criteria for various sample sizes. Furthermore, these criteria will be investigated as the number of parameters increases. Bartlett test of homogeneity of variance will be conducted on the various information criteria to test for significant difference among their variances.

Multiple regression analysis

We consider the problem of regression when study variable depends on more than one explanatory or independent variables, called as multiple linear regression model. This model generalizes the simple linear regression in two ways. It allows the mean function E(y) to depend on more than one explanatory variable and to have shapes other than straight lines, although it does not allow for arbitrary shapes.

Let y denotes the dependent (or study) variable that is linearly related to k independent (or explanatory) variables $X_1, X_2,\ldots,X_k$ through the parameters $\beta_0, \beta_1, \beta_2,\ldots, \beta_k$ and we write

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k + \varepsilon. \quad (2)$$

This is called as the multiple linear regression model. The parameters $\beta_0, \beta_1, \beta_2,\ldots, \beta_k$ are the regression coefficients associated with $X_1, X_2,\ldots,X_k$ respectively and ε is the random error component reflecting the difference between the observed and fitted linear relationship. There can be various reasons for such difference, e.g., joint effect of those variables not included in the model, random factors that cannot be accounted in the model etc.

Note that the j^{th} regression coefficient β_j represents the expected change in y per unit change in j^{th} independent variable X_j. Assuming $E(\varepsilon) = 0$

$$\beta_j = \frac{\delta E(y)}{\delta X_j} \quad (3)$$

Akaike Information Criterion

The Akaike Information Criterion is an estimator of out of sample prediction error and thereby relative quality of statistical model, relative to each of the other models. Thus AIC provides a means for model selection. AIC was founded on information theory. When a statistical model is used to represent the process that generated the data, the representation will almost never be exact, so some information will be lost by using the model to represent the process. AIC estimates the relative amount of information loss by a given model, the less information a model loses, the higher the quality of that model.

Suppose that we have a statistical model of some data. Let k be the number of estimated parameters in the model. Let $\hat{L}$ be the maximum value of the likelihood function of the model. Then the AIC value of the model is given by;

$$AIC = 2k - 2\ln(\hat{L}) \quad (4)$$

Given a set of candidate models for the data, the preferred model is the one with the minimum AIC value. Thus, AIC rewards goodness of fit (as assessed by the likelihood function), but it also includes a penalty that is an increasing function of the number of estimated parameters.

Bayesian Information Criterion

In statistics, the Bayesian information criterion (BIC) or Schwarz information criterion (also SIC, SBC, SBIC) is a criterion for model selection among a finite set of models; the model with the lowest BIC is preferred. It is based, in part, on the likelihood function and it is closely related to the Akaike information criterion (AIC). When fitting models, it is possible to increase the likelihood by adding parameters, but doing so may result in overfitting. BIC attempts to resolve this problem by introducing a penalty term for the number of parameters in the model. Gideon E. Schwarz developed the BIC, which was published in a 1978 paper where he gave a Bayesian argument for adopting it.

Suppose we have a statistical model of some data. The BIC is defined by;

$$BIC = \ln(n)k - 2\ln(\hat{L}) \quad (6)$$

where;

$\hat{L}$ = the maximized value of the likelihood function of the model

x = the observed data

n = number of data points in x, the number of observation or the sample size.

k= the number of parameters estimated by the model.

Hannan-Quinn Information Criterion

In statistics, the Hannan–Quinn information criterion (HQC) is a criterion for model selection. It is an alternative to Akaike information criterion (AIC) and Bayesian information criterion (BIC). It is given as:

$$HQC = -2l_{max} + 2k(\ln(\ln(n))) \quad (7)$$

Where:

l_{max} = the log likelihood

k= the number of parameters estimated by the model.

Bartlett Test

Bartlett's test is used to test the null hypothesis, H_0 that all k population variances are equal against the alternative that at least two are different. If there are k samples with n_i samples variances s_i^2, then Bartlett's test statistic is given by;

$$\chi^2 = 2.3026 \frac{((N-K)\log_{10}(s_p^2) - \sum_{i=1}^{k}(n_i-1)\log_{10}(s_i^2)}{1+\frac{1}{3(k-1)}(\sum_{i=1}^{k}\left(\frac{1}{n_i-1}\right) - \frac{1}{N-K})} \quad (8)$$

where:

$N = \sum_{i=1}^{k} n_i$

$s_p^2 = \frac{1}{N-k}\sum_{i=1}^{k}(n_i-1)s_i^2$ is the pooled estimate for the variance.

IV. Results

Results of analysis for sample size n= 10

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  34.3541     3.9241   8.755 0.000322 ***
x1           -0.7814     0.2660  -2.938 0.032345 *
x2           -1.1771     0.3003  -3.919 0.011192 *
x3            0.1193     0.1668   0.715 0.506563
x4           -0.5624     0.1275  -4.412 0.006944 **
---
Signif. codes:‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.3841 on 5 degrees of freedom
Multiple R-squared:  0.8916,   Adjusted R-squared:  0.805
F-statistic: 10.29 on 4 and 5 DF,  p-value: 0.01248
```

The above regression model is given as:

y = $34.3541 - 0.7814x1 - 1.1771x2 + 0.1193x3 - 0.5624x4$

Table 1: The information criteria of the above model is given in the table below

Information Criteria	AIC	BIC	HQC
Value	14.30838	16.12389	12.91677

The above table indicates that HQC with the least information criteria performs better than AIC and BIC.

Results of analysis for sample size n=15

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  18.2080     3.3518   5.432 0.000288 ***
x1            0.2485     0.2449   1.015 0.334089
x2            0.6841     0.3252   2.103 0.061730 .
x3            0.1151     0.2768   0.416 0.686216
x4           -0.5788     0.2327  -2.487 0.032155 *
---
Signif. codes:  ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.103 on 10 degrees of freedom
Multiple R-squared:  0.616,   Adjusted R-squared:  0.4624
F-statistic:  4.01 on 4 and 10 DF,  p-value: 0.03407
```

The above regression model is given as:

$$y = 18.2080 + 0.2485x1 + 0.6841\,x2 + 0.1151x3 - 0.5788x4$$

Table 2: The information criteria of the above model is given in the table below

Information Criteria	AIC	BIC	HQC
Value	54.66672	58.91502	54.62146

The above table indicates that HQC with the least information criteria performs better than BIC and AIC.

Results of analysis for sample size n = 20

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  20.0757     0.1401 143.259   <2e-16 ***
x1           -0.2244     0.1670  -1.343   0.1992
x2           -0.1995     0.1671  -1.194   0.2511
x3            0.4399     0.1514   2.906   0.0109 *
x4           -0.3184     0.1632  -1.951   0.0700 .
---
Signif. codes:‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.5721 on 15 degrees of freedom
Multiple R-squared:  0.5753,  Adjusted R-squared:  0.462
F-statistic: 5.079 on 4 and 15 DF,  p-value: 0.008636
```

The above regression model is given as:

$$y = 20.0757 - 0.2244x1 - 0.1995x2 + 0.4399x3 - 0.3184x4$$

Table 3: The information criteria of the above model is given in the table below

Information Criteria	AIC	BIC	HQC
Value	40.66689	46.64128	41.83315

The above table indicates that AIC with the least information criteria performs better than BIC and

HQC.

Results of analysis for sample size n=25

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.3281     0.1222   2.685  0.01422 *
x1            0.0940     0.1387   0.678  0.50572
x2            0.4666     0.1378   3.385  0.00294 **
x3            0.1821     0.1449   1.257  0.22319
x4           -0.3009     0.1421  -2.118  0.04690 *
---
Signif. codes:  '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6009 on 20 degrees of freedom
Multiple R-squared:  0.524,	Adjusted R-squared:  0.4288
F-statistic: 5.504 on 4 and 20 DF,  p-value: 0.003727
```

The above regression model is given as:
$y = 0.3281 + 0.0940x1 + 0.4666x2 + \ 0.1821x3 - 0.3009x4$

Table 4: The information criteria of the above model is given in the table below

Information Criteria	AIC	BIC	HQC
Value	54.96148	61.05586	56.6518

The above table indicates that AIC with the least information criteria performs better than BIC and HQC.

Results of analysis for sample size n=25 and number of parameter estimates = 5

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.5017     0.1620   3.097  0.00546 **
x1           -0.3095     0.1630  -1.899  0.07145 .
x2            0.2127     0.1546   1.376  0.18332
x3           -0.6993     0.1486  -4.707  0.00012 ***
---
Signif. codes:  '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.7881 on 21 degrees of freedom
Multiple R-squared:  0.593,	Adjusted R-squared:  0.5348
F-statistic:  10.2 on 3 and 21 DF,  p-value: 0.0002388
```

The regression model of the above model is :
$y = 0.5017 - 0.3095x1 + 0.2127x2 - 0.6993x3$

Table 5: The information criteria of the above model is given in the table below

Information Criteria	AIC	BIC	HQC
Value	64.68201	70.77639	66.37233

The above table indicates that AIC with the least information criteria performs better than BIC and HQC.

Results of analysis for sample size n=25 and number of parameter estimate = 4

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.3453     0.1319   2.618  0.01570 *
x1            0.1356     0.1447   0.937  0.35898
x2            0.4325     0.1475   2.931  0.00773 **
---
Signif. codes: ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.651 on 22 degrees of freedom
Multiple R-squared:  0.3855,   Adjusted R-squared:  0.3296
F-statistic: 6.899 on 2 and 22 DF,  p-value: 0.004722
```

The above regression model is given by:

$y = 0.3453 + 0.1356x1 + 0.4325x2$

Table 6: The information criteria of the above model is given in the table below

Information Criteria	AIC	BIC	HQC
Value	54.28917	59.16467	55.64143

The above table indicates that AIC with the least information criteria performs better than BIC and HQC.

Results of analysis for sample size n=25 and number of parameter estimates = 3

```
Coefficients:

            Estimate Std. Error t value Pr(>|t|)

(Intercept)   0.3784     0.1516   2.497   0.0201 *

x1            0.3033     0.1533   1.979   0.0600 .

---

Signif. codes:‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.7508 on 23 degrees of freedom

Multiple R-squared:  0.1455,  Adjusted R-squared:  0.1083

F-statistic: 3.915 on 1 and 23 DF,  p-value: 0.05995

The above regression model is given by:

□=0.3784+0.3033□1
```

Table 7: The information criteria of the above model is given in the table below

Information Criteria	AIC	BIC	HQC
Value	60.53103	64.18766	61.54522

The above table indicates that AIC with the least information criteria performs better than BIC and HQC.

Results of analysis for sample size n=50

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 21.374334   1.784438  11.978 1.36e-15 ***
x1          -0.014870   0.149989  -0.099  0.92146
x2           0.266867   0.150383   1.775  0.08273 .
x3          -0.002828   0.136114  -0.021  0.98352
x4          -0.466848   0.159976  -2.918  0.00548 **
---
Signif. codes:  '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.9897 on 45 degrees of freedom
Multiple R-squared:  0.1854,   Adjusted R-squared:  0.113
F-statistic:  2.56 on 4 and 45 DF,  p-value: 0.05131
```

The above regression model is given by:

$$y = 21.374334 - 0.014870x1 + 0.266867x2 - 0.002828x3 - 0.466848x4$$

Table 8: The information criteria of the above model is given in the table below

Information Criteria	AIC	BIC	HQC
Value	147.587	159.0591	151.9556

The above table indicates that AIC with the least information criteria performs better than BIC and HQC.

Results of analysis for sample size n=100

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 24.21016    1.10612  21.888  < 2e-16 ***
x1          -0.31903    0.09523  -3.350 0.001160 **
x2           0.21446    0.10033   2.138 0.035119 *
x3          -0.35641    0.09145  -3.897 0.000181 ***
x4           0.12384    0.10151   1.220 0.225485
---
Signif. codes:'***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8783 on 95 degrees of freedom
Multiple R-squared:  0.2517,   Adjusted R-squared:  0.2202
F-statistic: 7.987 on 4 and 95 DF,  p-value: 1.355e-05
```

The above regression model is given by:

$$y = 24.21016 - 0.31903x1 + 0.21446x2 - 0.35641x3 + 0.12384x4$$

Table 9: The information criteria of the above model is given in the table below

Information Criteria	AIC	BIC	HQC
Value	264.6941	280.3251	272.0329

The above table indicates that AIC with the least information criteria performs better than BIC and HQC.

Results of analysis for sample size n=150

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 20.05682    0.06556 305.935  < 2e-16 ***
```

```
x1           0.09287    0.07251   1.281  0.20233
x2          -0.28525    0.08750  -3.260  0.00139 **
x3           0.21119    0.07478   2.824  0.00541 **
x4           0.13851    0.07792   1.778  0.07757 .
---
Signif. codes:'***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8003 on 145 degrees of freedom
Multiple R-squared:  0.1588,   Adjusted R-squared:  0.1356
F-statistic: 6.845 on 4 and 145 DF,  p-value: 4.481e-05
```

The above regression model is given by;

$$y = 20.05682 + 0.09287x1 - 0.28525x2 + 0.21119x3 + 0.13851x4$$

Table 10: The information criteria of the above model is given in the table below

Information Criteria	AIC	BIC	HQC
Value	365.7491	383.8129	373.0878

The above table indicates that AIC with the least information criteria performs better than BIC and HQC.

Results of analysis for sample size n=200

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.06977    0.06240  -1.118  0.26493
x1          -0.19948    0.06742  -2.959  0.00347 **
x2          -0.11256    0.06262  -1.798  0.07379 .
x3          -0.12162    0.06287  -1.934  0.05451 .
x4           0.12723    0.06745   1.886  0.06075 .
---
Signif. codes:'***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8727 on 195 degrees of freedom
Multiple R-squared:  0.08598, Adjusted R-squared:  0.06723
F-statistic: 4.586 on 4 and 195 DF,  p-value: 0.001463
```

The above regression model is given by:

$$y = -0.06977 - 0.199948x1 - 0.11256x2 - 0.12162x3 + 0.12723x4$$

Table 11: The information criteria of the above model is given in the table below

Information Criteria	AIC	BIC	HQC
Value	520.0262	539.8161	528.0348

The above table indicates that AIC with the least information criteria performs better than BIC and HQC.

Results of analysis for sample size n=200 and number of parameter estimates = 5

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.06107    0.06263  -0.975  0.33074
x1          -0.19568    0.06782  -2.885  0.00435 **
x2          -0.10104    0.06273  -1.611  0.10884
x3          -0.12377    0.06327  -1.956  0.05185 .
---
```

```
Signif. codes:'***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8783 on 196 degrees of freedom
Multiple R-squared:  0.06931, Adjusted R-squared:  0.05506
F-statistic: 4.865 on 3 and 196 DF,  p-value: 0.002755
```

The above regression model is given by:

$$y = -0.06107 - 0.19568x1 - 0.10104x2 - 0.12377x3$$

Table 12: The information criteria of the above model is given in the table below

Information Criteria	AIC	BIC	HQC
Value	521.6425	538.1341	528.3164

The above table indicates that AIC with the least information criteria performs better than BIC and HQC.

Results of analysis for sample size n= 200 and number of parameter estimates = 4

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.05135    0.06288  -0.817  0.41518
x1          -0.19567    0.06831  -2.864  0.00463 **
x2          -0.11536    0.06274  -1.839  0.06748 .
---
Signif. codes:'***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8846 on 197 degrees of freedom
Multiple R-squared:  0.05113, Adjusted R-squared:  0.0415
F-statistic: 5.308 on 2 and 197 DF,  p-value: 0.005684
```

The above regression model is given by:

$$y = -0.05135 - 0.19567x1 - 0.11536x2$$

Table 13: The information criteria of the above model is given in the table below

Information Criteria	AIC	BIC	HQC
Value	523.5099	536.7032	528.8491

The above table indicates that AIC with the least information criteria performs better than BIC and HQC.

Results of analysis for sample size n=200 and number of parameter estimates = 3

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.04029    0.06297  -0.640  0.52300
x1          -0.18278    0.06836  -2.674  0.00812 **
---
Signif. codes:'***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8899 on 198 degrees of freedom
Multiple R-squared:  0.03485, Adjusted R-squared:  0.02998
F-statistic:  7.15 on 1 and 198 DF,  p-value: 0.008123
```

The above regression model is given by:

$$y = -0.04029 - 0.18278x1$$

Table 14: The information criteria of the above model is given in the table below

Information Criteria	AIC	BIC	HQC
Value	524.9126	534.8076	528.917

The above table indicates that AIC with the least information criteria performs better than BIC and HQC

Results of Analysis for CBN data

```
Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)            7.0199   261.5782   0.027   0.9789
RecurrentExpenditure  23.4195     1.6807  13.934 9.29e-12 ***
CapitalExpenditure    -4.6311     2.4111  -1.921   0.0691 .
DomesticDebts          4.0695     2.1112   1.928   0.0682 .
ExternalDebts         -2.0476     0.3321  -6.166 5.04e-06 ***
---
Signif. codes:  '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 844 on 20 degrees of freedom
Multiple R-squared:  0.9895,   Adjusted R-squared:  0.9874
F-statistic: 471.6 on 4 and 20 DF,  p-value: < 2.2e-16
```

The above regression model is given by:

$$GDP = 7.0199 + 234195 RecurrentExpenditure - 4.6311 CapitalExpenditure + 4.0695 DomesticDebts - 2.0476 ExternalDebts$$

Table 15: The information criteria of the above model is given in the table below

Information Criteria	AIC	BIC	HQC
Value	414.2788	421.5921	416.3072

The above table indicates that AIC with the least information criteria performs better than BIC and HQC.

Table 16: Table of summary for information criterion according to samples sizes (Small Samples)

Sample size	AIC	BIC	HQC
10	14.30838	16.12389	12.91677
15	54.66672	58.91502	54.62146
20	40.66683	46.64128	41.83315
25	54.96148	61.05586	56.6518

Table 16 above indicate that HQC performs better than AIC and BIC for samples size less than 20 while AIC performs better than BIC and HQC for sample size 20 and above.

Tables 17: Table of summary for information criterion according to samples sizes (Large Samples)

Sample size	AIC	BIC	HQC
50	147.587	159.0591	151.9556
100	264.6941	280.3251	272.0329
150	365.7491	383.8129	373.0878
200	520.0262	539.8161	528.0348

Table 17 above indicates that, AIC have the least value, this implies that AIC performs better than, BIC and HQC across various sample sizes.

Table 18: Table of summary for information criterion according to number of parameters estimates for n=25

Number of parameter estimates	AIC	BIC	HQC
3	60.53103	64.18766	61.54522
4	54.28917	59.16467	55.64143
5	64.68201	70.77639	66.37233
6	54.96148	61.05586	56.65180

Table 19: Table of summary for information criterion according to number of parameters estimates for n=200

Number of parameter estimates	AIC	BIC	HQC
3	524.9126	534.8076	528.917
4	523.5099	536.7032	528.8491
5	521.6425	538.1341	528.3164
6	520.0262	539.8161	528.0348

Tables 18 and 19 indicate that, AIC have the least value, this implies that AIC performs better than, BIC and HQC across various number of parameter estimates.

Table 20: Table of summary of penalty terms for information criterion for various sample sizes (Small Samples)

Sample Size	AIC	BIC	HQC
10	12	13.81551	10.0089
15	12	16.2483	11.95475
20	12	17.97439	13.16626
25	12	19.31325	14.02839

Table 21: Table of summary of penalty terms for information criterion for various sample sizes (Large Samples)

Sample Size	AIC	BIC	HQC
50	12	23.47214	16.36866
100	12	27.63102	18.32616
150	12	30.06381	19.33875
200	12	31.78999	20.00867

Tables 20 and 21 indicate that, the penalty terms of AIC remains constant across various sample size. This implies that, AIC do not penalize sample size whereas BIC and HQC penalizes sample size.

Table 22: Table of summary of penalty terms for various number of parameter when sample size is 25

Number of parameter estimates	AIC	BIC	HQC
3	6	23.47214	16.36866
4	8	27.63102	18.32616
5	10	30.06381	19.33875
6	12	31.78999	20.00867

Table 23: Table of summary of penalty terms for various number of parameter estimates when sample size is 200

Number of parameter estimates	AIC	BIC	HQC
3	6	15.89495	10.00434
4	8	21.19327	13.33911
5	10	26.49159	16.67389
6	12	31.7899	20.00867

Tables 22 and 23 indicate that, the penalty terms of AIC, BIC and HQC increases as the number of parameter estimates increases. This implies that, AIC, BIC and HQC penalize the number of parameter estimates.

Bartlett Test of Homogeneity of variances

Decision Criteria

Level of Significance (α): 0.05 (5%)

Decision Rule: Reject $\mathbf{H_0}$ if p-value ≤ 0.05 and accept $\mathbf{H_0}$ if p-value > 0.05.

Hypothesis

H_o: There is no significant difference among the variances of the various information criteria.

H_1: There is significant difference among the variances of the various information criteria

Bartlett Test for Small sample

Table 24

Information Criteria	AIC	BIC	HQC
Variances	364.7216	428.7	406.2874

```
Bartlett test of homogeneity of variances

data:  list(AIC, BIC, HQC)
Bartlett's K-squared = 0.017542, df = 2, p-value = 0.9913
```

Conclusion: The p-value (0.9913) is greater than the level of significance (0.05), hence, we accept H_0 at 5% level of significance. This implies that, there is no significant difference among the variances of the various information criteria.

Bartlett Test for Large Sample

Table 25

Information Criteria	AIC	BIC	HQC
Variances	24935.65	26048.16	25375.93

```
Bartlett test of homogeneity of variances

data:  list(AIC, BIC, HQC)
Bartlett's K-squared = 0.0012625, df = 2, p-value = 0.9994
```

Conclusion: The p-value (0.9994) is greater than the level of significance (0.05), hence, we accept H_0 at 5% level of significance. This implies that, there is no significant difference among the variances of the various information criteria.

V. Discussion

In model fitting, we try to determine how well our model generalizes to a new sample of data. We evaluate the fit of our model using the same sample of data that was used to fit the model. Due to the fact that, we are using same sample of data to evaluate our model fit, we often have an issue of selection bias or getting inflated sense of how well the model can predict an out of sample data which often leads to an over fitted model. Over fitted model occurs when a Statistical model cannot adequately capture the underlying structure of the data or fail to predict future observations reliably. The criteria captured within the scope of this research have to apply a correction to correct the selection bias. Hence, all the criteria aim to come out with out of sample predicted accuracy. Each of the criteria obtains a measure of out of sample prediction in different ways. Each of the criteria often evaluates the fit of the model on the data, and then subtracts the penalty term, which accounts for the selection bias. Table 16 indicates that HQC performs better than AIC and HQC for sample size less than 20 while AIC performs better than BIC and HQC for sample size 20 and above. Tables 17, 18 and 19 indicate that, AIC have the least information criteria, hence AIC performs better than BIC and HQC for sample size above 20 and number of parameter estimates increases respectively. The penalty terms of AIC remains constant as the sample size increases, whereas, it varies as the number of parameter estimates increases as can be seen in tables 20, 21, 22 and 23 above. The penalty terms of BIC and HQC increases with increasing sample size and number of parameter estimates, this indicates that, BIC and HQC penalizes the sample size and number of parameter estimates. The penalty terms of AIC remains constant with increasing sample size but increases with increasing number of parameter estimates, this indicates that, AIC penalizes the number of parameter estimates. The Bartlett test of homogeneity of variances indicates that there is no significant difference among the information criteria.

VI. Conclusion

Due to the outcome from the results of the data analysis, it can be concluded that, HQC performs better than AIC and BIC for sample size less than 20 while AIC performs better than BIC and HQC for sample size 20 and above.

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APPENDIX 1

	y	x1	x2	x3	x4
1	19.55595	10.073457	4.714477	3.984026	3.471026
2	19.71104	10.618270	4.492061	2.761579	3.423818
3	18.90541	9.762103	5.055238	3.979242	3.293155
4	19.44579	9.705019	4.308550	3.990108	5.113810
5	20.41578	9.812329	3.743645	2.612833	4.256953
6	19.76431	9.807524	4.766182	3.031735	2.827963
7	18.04422	8.314505	5.825379	1.968793	5.687089
8	18.49996	11.442655	3.288485	2.179550	5.347880
9	17.67636	7.763368	6.419298	3.843443	6.326348
10	19.77596	9.170616	4.370102	1.943787	4.135630

The above table is simulated data for n=10

APPENDIX 2

	y	x1	x2	x3	x4
1	19.02725	1.32934138	-0.8682382	12.800967	-1.075786229
2	17.89068	-2.23818490	2.1699866	12.713322	2.127178617
3	23.31575	3.57206533	0.9426923	13.362704	-3.304515449
4	19.87242	0.79501243	-0.7822654	12.816003	0.636461179
5	19.85312	0.43826551	0.4549128	11.661059	-0.318621941
6	19.95186	0.07862887	-0.1229100	12.143447	0.542634411
7	21.96561	-1.10891022	0.8283260	13.687609	-1.199509839
8	20.90143	-0.22191788	1.0467021	10.908245	0.364508171
9	19.23266	-0.71661103	-0.6044748	12.092754	-0.807783387
10	22.44050	-1.11786128	2.2166658	11.545235	-2.445709323
11	18.85366	1.62546868	-1.0949574	9.766537	0.028800092
12	20.19312	-0.55426779	-0.2487501	11.363213	-0.174299479
13	19.46467	1.80310770	0.5919864	13.822018	-0.008824302
14	21.08741	0.69447144	0.2879795	13.338399	-0.353443580
15	18.68505	-0.61300732	-0.6681350	11.053390	-2.235967619

The above table is simulated data for n=15

APPENDIX 3

	y	x1	x2	x3	x4
1	19.83894	0.056037391	0.06363888	-0.18097342	-0.12039920
2	19.54387	-0.842400588	0.92615376	-0.67391265	0.95718440
3	20.69597	0.400102614	-0.85248224	-1.31928979	-0.90625895
4	20.96245	-0.517721761	-0.18123065	-0.20677612	1.27965304
5	20.78531	2.110346398	-1.69350309	2.79873319	0.80331697
6	19.25406	0.295658761	-0.74988918	-0.08226149	0.93307717
7	19.91959	-0.980584860	-0.11981597	-0.44445507	0.89046914
8	19.73389	0.003815244	-0.18744117	0.26895841	0.08497723
9	20.87435	-0.767932648	-0.77387213	-0.46890532	-0.78566745
10	20.12225	0.104887473	0.12949153	0.07251728	-0.14368885
11	19.24437	-0.323969706	1.07829917	-1.11918998	-1.13346757
12	20.09397	0.339840903	0.20454609	-0.23424233	0.28147259
13	20.70018	-0.869142121	-0.80784551	-1.04860665	-0.12025920
14	20.15250	-0.322901378	0.54497092	0.24855539	-0.54706832
15	20.31796	0.327485785	0.28802313	0.29512991	-0.14804531
16	19.37995	-0.273805962	-0.55314872	-0.57988153	0.90993786
17	21.49264	-1.460126023	-1.23034648	0.88571290	-1.88781719
18	18.95624	-1.881680071	-1.98262699	-2.03010749	0.66359127
19	18.27019	1.726708467	1.46522814	-1.70930154	0.27068913
20	19.96770	-0.170967254	-0.11273615	0.15859278	0.08516663

The above table is simulated data for sample size n=20

APPENDIX 4

	y	x1	x2	x3	x4
1	-0.7190437	-0.5884860	-0.67597445	0.155764612	0.7168431
2	-0.1324739	0.7258685	-0.45618534	0.142321076	0.7118540
3	-0.1739087	-0.3117654	-0.16529635	0.195430459	-0.2436556
4	0.9808166	-0.9359990	0.68409224	-0.148969015	1.3238529
5	0.7644795	1.9455689	-1.46640006	-0.007578066	-1.9554742
6	0.9582863	-1.4415792	-1.84911021	1.942490401	0.1474273
7	1.6639553	1.2903777	2.08516262	2.074948189	0.3141569
8	0.4051948	-0.4004388	-0.23480663	0.241047350	0.4284206
9	-0.6898578	-0.2681876	-0.89972755	-0.461556854	-0.7849931
10	0.8092122	-0.6715532	-0.35780194	0.846386138	0.1660018
11	-1.7883957	-1.6375231	-1.71243768	-0.128012954	2.1060763
12	1.6488253	2.3668532	2.14109741	-2.219332154	0.8235143
13	0.1113842	-0.6273120	-0.09618183	-0.202616111	0.6414847
14	0.9693636	-1.2442771	1.05856964	0.613691866	-1.0837288
15	0.4806047	0.4962845	0.42498861	-0.185873507	-0.6158707
16	0.7555994	-0.7863486	0.67577454	-1.190769424	-0.9556446
17	0.3454941	-0.3459162	-0.37247609	0.369673396	-0.2080625
18	-0.4488670	-0.4958938	-0.11469685	0.400258019	0.2976837
19	-0.3770344	-0.2013543	0.55661533	0.467721715	0.3020897
20	0.7403429	1.0673890	0.74929544	-0.247542984	-1.0382880
21	-0.3617953	-0.2717944	0.09976601	-0.268587419	0.2615902
22	0.5701690	-0.8316584	-0.19807225	-1.031295409	-0.2796659
23	0.2075217	0.4542412	-0.28576784	-0.491363555	-0.2553590
24	1.2246082	-0.8546109	0.68380285	-0.784493543	-1.3255274
25	0.5002116	0.2216825	0.33834768	-0.222053190	-0.4798214

The above table is simulated data for sample size n=25

APPENDIX 5

	y	x1	x2	x3	x4
1	20.34114	9.494588	1.656995023	3.2087820	3.472575
2	19.29512	11.036903	2.452290695	2.9999300	5.581707
3	19.80482	9.293502	2.493255747	4.6791832	4.069919
4	21.00326	10.540055	2.984849436	1.9682123	2.963523
5	20.55097	10.732732	1.821060139	3.7440886	3.898245
6	18.83108	10.194502	0.939050965	3.8789895	3.181122
7	19.17232	10.914020	2.437150018	2.1462208	3.655298
8	17.64637	12.364695	-0.538640862	0.7805552	2.763801
9	19.63849	9.613759	3.343835313	1.3241938	4.178700
10	21.03558	10.966395	2.876455857	4.1869656	3.165691
11	19.36074	9.012284	2.374159827	3.8925717	4.702361
12	20.19823	8.721187	0.707910333	4.2121527	4.450163
13	21.02362	8.978849	2.754423622	1.8909319	3.904599
14	21.01209	8.687178	2.366600611	4.1764166	4.863457
15	20.79301	10.781972	2.542631356	2.2878970	4.354857
16	18.82050	9.502512	2.022888685	2.7404211	2.803983
17	18.29399	9.614804	3.338597572	4.8825026	4.401071
18	17.40132	7.665224	4.016068267	4.4725952	6.141400
19	20.55996	9.631135	1.463718768	3.3539238	3.564840
20	18.63177	9.933208	1.113981050	2.6354571	5.343970
21	19.49362	9.377092	1.464260102	2.8900109	2.977722
22	20.77044	11.128844	1.605660278	2.3204017	5.174064
23	18.81751	11.493342	3.338293603	4.4083062	4.499233
24	18.01012	11.499320	-0.009133805	3.7748244	5.856298
25	19.37621	11.738337	1.283878303	3.7164289	5.677464
26	21.21882	9.058095	3.072994951	4.2135683	3.562743
27	19.59379	10.231602	2.522811259	2.3322109	4.051644
28	20.12317	9.877166	1.848895665	2.8564922	4.059692
29	21.29490	11.490822	2.730857876	4.4227211	4.663380
30	19.96520	10.093117	2.288337625	2.6448127	4.026420
31	18.07519	11.818598	2.320047364	1.3046977	5.110987
32	20.60678	9.425332	2.149040667	3.2759480	3.404176
33	18.91789	9.972692	2.178693956	3.0298386	2.769757
34	21.69127	9.970318	2.959174402	4.3654805	2.331438
35	19.50004	9.890336	1.191487833	2.2083311	3.694525
36	19.07011	8.995109	0.939233465	3.6759188	3.179323
37	19.23719	9.749481	1.307886998	3.6562269	4.389302
38	20.15344	9.596157	1.582091325	3.4156865	3.943023
39	20.17857	10.167209	1.970036498	3.0573570	4.169160
40	21.06224	8.016404	4.462899713	0.6559154	4.785005
41	18.70453	8.766545	0.707193805	1.8301318	4.556788
42	20.19296	9.367152	2.631666646	3.5117005	3.626918
43	19.48856	8.868779	4.012428317	1.6198439	5.500111
44	19.38650	9.346370	2.578678452	3.0852990	5.301356
45	21.55454	9.675789	0.581177901	1.6582008	3.196733
46	20.57318	11.070829	0.891523175	1.9548760	3.478879
47	20.70522	10.920408	1.930585533	3.5961742	3.297920
48	21.08363	10.514970	0.716746205	2.1161420	2.987768
49	20.44247	10.281418	2.635114258	2.6502174	4.599672
50	20.24777	10.325292	2.101736475	2.6475618	4.081639

The above table is the simulated data for sample size n=50

APPENDIX 6

	y	x1	x2	x3	x4
1	20.14543	0.524513684	-0.271150756	11.388171	-0.767944198
2	20.32730	0.286249253	0.118550540	12.986049	-0.992407907
3	19.46495	0.567465304	-0.188491223	12.282884	-0.493780993
4	20.08118	0.791752996	0.116422781	11.880987	0.841747209
5	19.31537	-0.363759201	0.170534568	11.488771	0.531989441
6	17.56124	2.608951608	-0.552961961	12.922462	-2.447937250
7	19.56806	0.584027024	-0.560255293	11.504432	0.317311328
8	19.57943	-0.107574325	0.465362124	11.491890	-0.032849375
9	21.43664	1.836147357	1.799622896	10.289514	0.854228204
10	18.26189	-1.528270258	-0.308186160	13.724271	1.331593942
11	19.70173	-0.574502571	0.815124921	12.461696	1.157320194
12	20.22632	-1.356745144	0.536091414	13.185903	-0.764505556
13	19.98757	-1.220877304	0.851320011	13.140377	-0.842528678
14	20.68206	0.751559521	-0.427528061	11.396358	0.505078690
15	21.82662	-1.859971310	1.732156957	10.117774	0.251155926
16	19.87585	0.615851267	-0.562337966	12.645017	0.248273446
17	19.03917	-0.322194212	0.861247184	11.527276	-1.296848471
18	18.46302	-1.180219228	-0.813081388	11.246201	-1.729636235
19	20.49391	-0.005412360	-0.006851357	11.605422	-0.389433767
20	20.78981	-0.008687313	-0.437643868	10.687398	1.327674601
21	19.67415	0.792914776	0.065979053	12.734224	-0.314700669
22	20.08642	0.258919720	-0.300695869	11.690081	-0.006894027
23	19.50416	-0.540061512	0.606698979	12.166898	0.596946035
24	18.60622	-1.607592824	0.431420908	13.605784	-0.076290466
25	17.45168	1.486507076	-2.136980623	9.904524	-1.455695768
26	21.92101	-1.680647926	-2.290569741	9.726555	-0.455541947
27	19.02675	1.233727922	-0.969518554	13.632030	0.433359814
28	19.79148	-0.255632856	0.302746441	12.457260	-1.973516077
29	18.55898	-1.674961802	0.404145396	13.398385	0.977090526
30	20.07462	-0.612635016	-1.554279923	13.491041	0.512552775
31	18.24626	-0.856185921	1.989570558	12.550543	-2.085094534
32	18.75803	1.588688587	-0.890720407	12.377528	1.552763347
33	19.75124	0.541271490	0.639583675	11.300443	0.319546723
34	19.51155	-0.018145762	-0.553750146	12.337177	0.455055342
35	19.21053	1.114342608	-1.114449575	13.060370	0.445015259
36	19.95238	-0.240665803	0.164834714	12.033350	-0.260479999
37	21.04747	1.676125452	0.017977165	13.781868	-0.202369317
38	19.46734	-0.509242557	-0.838316881	11.574161	-0.738353526
39	19.81422	0.390717389	0.753088178	11.156235	-0.169541076
40	18.78053	0.781464333	0.236338178	13.078298	-0.759145170
41	21.18455	-0.027313025	-0.656355837	13.188025	0.183815913
42	19.67710	0.288392103	-0.400302473	12.250042	0.400961926
43	19.09871	-0.128898614	0.334645709	10.400730	-1.304000769
44	18.94802	-1.214097357	0.862698908	13.188422	-0.629569888
45	19.63841	0.369698593	0.357244476	12.379600	0.146994780
46	20.59799	0.358772628	-0.495979572	11.334730	0.195507501
47	19.72176	0.581193831	-0.286032985	11.560565	0.385014603
48	22.09942	-1.704620927	-1.087073854	11.512271	-2.054304706
49	20.14086	-0.161751477	-0.156977091	12.147421	0.069120104
50	20.81777	0.529561881	0.018817463	12.763470	-0.606891015
51	20.47309	-0.486633921	-0.016001584	11.503038	-0.369524282
52	18.78312	0.376314448	-0.973643845	11.264266	-1.547316700
53	20.39521	-0.540611905	0.397171667	11.419003	-0.405486639
54	21.94835	-1.537429018	1.650839033	12.649680	-1.879444239
55	20.87231	-0.112724244	-0.069507531	11.439247	-1.168210823
56	20.00184	-0.018334067	0.004341898	11.986659	-0.015114814
57	20.94631	0.951645612	0.919916298	12.493759	0.813590742
58	20.63607	-0.319609611	-0.637083121	11.744299	0.584299779
59	19.71473	0.396951781	0.302175771	12.252713	-0.306610097
60	17.95213	2.140191168	-0.375181207	11.125767	1.957770781
61	18.36960	-1.663929485	-1.142384756	13.640090	0.414474578

APPENDIX 6 (Cont.)

62	20.62775	-0.324231070	0.636547582	11.557842	-0.468611247
63	19.40775	0.749937841	0.254287399	12.088510	-0.760516553
64	20.04766	-0.037299128	-0.036402387	12.057548	0.011426431
65	19.61475	0.267840514	0.311197928	12.420535	-0.271440000
66	20.53259	0.550256437	-0.592146502	11.524964	-0.362459438
67	20.38214	-0.591713998	-0.995226291	10.712589	0.070677337
68	21.66355	-1.368349967	1.131228796	12.252007	-1.644858229
69	20.78755	-0.887113312	-1.052499259	11.162666	0.640321252
70	21.16005	-0.557374613	-1.582972001	10.550012	0.708827593
71	19.15001	0.347153617	0.787459136	11.573165	-0.752231858
72	19.10936	-0.971013825	-1.027253903	12.863422	-0.592084541
73	20.59588	-0.346907146	0.340977329	11.803820	-0.624762930
74	19.86833	-0.267881500	0.761163551	12.700357	0.343916246
75	20.77528	-0.435377886	1.192013582	13.172420	0.215586947
76	21.70369	0.542631588	1.713953073	10.582582	1.042989109
77	18.94761	0.954002470	0.803469091	12.305846	1.180212469
78	19.79954	-0.522629533	0.301079590	12.430234	0.385511181
79	17.78350	2.221467663	-2.299883177	14.305788	0.083721042
80	19.51184	0.363797761	-0.451729154	12.479257	-0.094392210
81	20.44365	-0.306653647	-0.108207621	12.801155	0.479153022
82	18.22362	0.885827194	-1.380714528	13.767604	-0.240559271
83	20.20938	-0.700645704	0.648995463	12.664175	-0.223729197
84	20.10753	1.039616006	1.116536547	10.910156	0.297434581
85	20.81626	-1.560605319	-2.158844728	13.228516	2.617962374
86	20.79571	0.120974224	0.133650585	11.220367	-0.277435600
87	19.88137	-1.015440915	-0.015631058	12.656050	-0.777745488
88	21.02335	-0.395498856	-1.559561641	10.305351	-0.260626293
89	20.39309	-0.108384626	-1.048883179	11.179964	0.763901161
90	20.01738	0.338441938	0.073153915	12.354976	-0.034977429
91	19.26929	0.411183183	0.648573650	12.729015	-0.070128804
92	19.10819	0.469332480	0.244636844	12.774364	-0.574573607
93	20.98944	-0.623699673	-0.037353497	12.945487	0.345428715
94	19.74099	-0.601233529	-0.324322946	11.622909	0.468526471
95	21.30739	0.501315325	1.507108485	10.320876	-0.337248157
96	20.53777	-0.613640591	-0.132281124	11.386712	-0.033649192
97	19.65017	-0.251955420	-0.145056716	12.328019	0.121924403
98	18.81255	1.143500960	-1.186630252	11.881074	1.207806390
99	18.93691	-0.927338163	-0.668796034	11.196462	0.750154330
100	19.55455	0.907637133	0.466637928	12.502427	-0.755937209

The above table is simulated data for n=100

APPENDIX 7

	y	x1	x2	x3	x4
1	19.74816	-0.250236505	0.408005592	0.33102064	0.30098098
2	20.77428	-1.235614177	-0.845556682	0.92632393	-0.99968105
3	19.96162	-0.454872751	0.488543134	0.38455651	0.47925457
4	19.43885	0.584348742	-0.190602169	-0.20267043	0.55084154
5	20.26252	0.769158820	0.913511198	0.50303133	-0.76253677
6	20.14117	0.019009632	-0.182985363	0.12279853	0.16163113
7	19.92489	-0.090392062	0.046469106	0.03303940	-0.14183411
8	21.54215	-1.174011436	1.052858392	1.05460934	1.23127788
9	18.46817	-1.376146933	0.125771666	1.24663780	0.92695940
10	20.69822	0.673732250	0.164732618	0.23337593	0.65826649
11	19.82969	0.118570228	-0.162526059	0.17065070	-0.01186946
12	21.45666	1.600860771	-0.499165988	0.25502197	1.61339843
13	20.42233	0.300203143	0.726899994	-0.62779830	-0.82399239
14	19.63592	0.363341167	-0.048306911	-0.08981295	-0.35713620
15	19.99087	-0.229986456	-0.205248997	0.18310209	-0.14715199
16	20.76288	-0.899713872	-0.686853294	0.50609493	-0.82081439
17	21.13065	1.476577885	-1.041822737	0.46948786	1.45668898
18	20.27056	-0.262636141	0.195360940	0.04011039	0.31633015
19	19.83945	-0.587436762	-0.081724389	1.18222951	0.30528908
20	21.50685	1.901597974	-0.955391788	0.22420756	-2.04666295
21	20.03376	0.431252496	0.057977749	0.40623355	0.14609939
22	19.86100	-0.017109153	-0.815115618	-0.17236897	-0.82296404
23	18.39750	-1.263419370	-1.421677736	1.68308897	-0.73568907
24	20.45084	-0.759627189	-0.556332211	-0.55027618	-0.64559817
25	20.17879	-1.710327109	-0.254543466	1.42840630	-0.94550623
26	20.67632	-0.368726495	0.219649627	-0.53142012	0.42638948
27	19.23331	0.487364132	-0.251109886	-0.34870821	-0.70609515
28	18.78432	-1.264118943	0.129040249	-1.10446082	-0.62361475
29	20.69769	-0.488946013	0.014019621	-0.77267529	0.50875585
30	20.47519	0.679914882	0.653274707	0.52619256	0.66867141
31	19.66478	0.815123248	0.526460705	0.54276325	0.64611170
32	19.71713	-0.538417485	-0.011597367	0.62829271	-0.06932454
33	20.22091	-0.212203174	1.368311432	-1.57608919	0.19681756
34	20.51874	-0.422375980	-0.402263823	-0.45220786	0.25458666
35	21.23709	0.626439576	-0.953663538	0.91839702	0.82885019
36	19.60787	0.504867421	-0.129360410	-0.52796058	-0.02182335
37	18.09136	0.293289386	0.433683013	-0.42493962	-1.92265786
38	20.75703	-1.088091098	0.919513420	-1.04501789	0.51771294
39	20.41987	0.673726611	-0.733331544	-0.38798440	0.63523199
40	19.96438	0.158315879	0.081569418	-0.17212665	0.04821634
41	21.45726	1.441222148	-0.485131591	0.96442668	1.09322187
42	19.52512	-0.170667414	-0.307456820	0.62513095	-0.18583533
43	20.50527	0.986017196	-0.474681548	-0.93650956	-0.32477549
44	19.34032	-0.020826569	0.559599815	-0.66865176	-0.19599783
45	19.20865	-0.372954092	-1.086467849	-1.27724573	-0.30796146
46	19.88314	0.194359477	0.146513519	-0.01499767	0.19884845
47	20.84073	0.711464094	0.299162056	0.67388904	-0.72960120
48	21.97000	-1.671779921	-1.729035669	1.98903044	0.50876287
49	19.91177	0.152562917	-0.025368612	0.10466654	0.34000843
50	19.27420	-1.565755378	-0.111794630	0.32337859	-1.73397495
51	20.82823	-1.571526895	0.733003434	1.57101630	0.30060631
52	19.54057	-0.383340060	-0.159094963	-0.46405839	-0.05125453
53	20.69774	-0.368795160	-0.164755479	-0.24334217	1.29953509
54	19.73642	-0.002345501	-0.725087207	-0.79432009	0.68543221
55	19.54675	-1.451018700	0.165414979	-1.19243708	0.94966061
56	20.29328	0.113463753	0.429787664	0.38712834	-0.18984727
57	19.76699	-1.518962905	1.044117704	-1.58047519	-0.31485746
58	20.47497	0.954004677	-0.292079392	0.56455519	-0.89730820

APPENDIX 7 (Cont.)

59	18.72966	-1.291780989	0.741705406	-0.81926318	1.05370601
60	20.23729	1.076143769	0.301305563	-1.10947468	-0.49546057
61	19.63240	0.616439779	0.806776688	-0.69713593	-0.41424181
62	20.18714	0.169939226	0.404982533	-0.12818301	-0.38826797
63	19.78071	-0.085517727	0.283070085	-0.04614902	0.34866010
64	20.86874	-0.411107458	-0.806115330	-1.54701004	-0.42399173
65	19.53258	-0.175005453	-0.241756502	-0.47465584	0.57466624
66	19.99185	1.984746723	1.838492591	0.94974002	-1.82413832
67	19.25484	1.107965107	-0.494111164	0.03376604	1.11831796
68	19.68252	-0.884433500	-0.608646743	-0.45725795	0.79093301
69	19.74402	0.229631516	0.209603263	-0.04484241	0.26237431
70	19.52829	-0.367827953	-0.475138114	0.18574703	-0.45108738
71	21.03337	-1.077360632	-1.116137445	-0.02683327	1.12512464
72	20.10498	0.111271078	-1.238007145	1.03540563	1.24295305
73	19.36783	0.615736862	-0.103423207	-0.39866451	-0.49065579
74	19.32794	-1.884380740	0.894816221	1.45352368	-1.31872048
75	20.51101	0.554971580	0.571578332	0.15990102	1.04047136
76	19.08404	1.003096442	-0.624303123	-0.30111127	1.02922269
77	19.51040	0.413829253	0.190636044	-0.24060044	-0.42683132
78	19.51487	1.075587802	0.564879152	0.85576799	0.79694043
79	20.92842	-1.347779055	-1.765875085	0.15899385	2.10596887
80	21.17280	0.167665923	0.005694304	-0.31984248	1.56213745
81	20.02011	-0.246707511	0.425305032	-0.34121593	-0.25605204
82	19.09751	2.510878257	1.747054919	1.59734291	-1.93739911
83	20.27609	-0.171221783	0.270047456	0.07590477	-0.29792850
84	18.75384	0.804028551	0.809527351	1.06762551	-0.69390526
85	20.82229	0.131566903	0.462677893	0.79091117	-0.25284338
86	20.59347	0.445083680	0.602893710	0.59420919	-0.14506322
87	19.26077	1.532293304	0.894849142	-1.25424346	0.88411632
88	19.85372	0.885707406	1.221250957	-1.55988510	0.21522442
89	21.07435	-0.640660669	-0.875881188	-0.31662301	-1.04881908
90	20.11921	0.600143406	-0.598271249	-0.72584066	-0.02798854
91	19.51894	0.038429840	-0.112781952	0.60877444	0.27581620
92	21.40854	1.266633566	-1.236743136	-1.41104543	0.04334159
93	20.71243	-0.570250401	-0.258423018	-0.64843542	0.31473652
94	21.23856	-0.325144918	1.102800474	-0.98443015	-0.89802749
95	18.16536	0.931512355	-0.175127283	-1.49092449	-1.28534655
96	19.17530	1.496491205	1.019239196	-1.53733452	0.05792243
97	21.06402	0.607291510	0.160454287	-0.76356743	0.86883634
98	19.93459	0.205781953	-0.337542640	0.28355051	0.32898702
99	20.56971	1.028035547	-0.619072901	1.11189275	-0.82866568
100	21.80443	-0.509330889	1.824307566	1.71543761	-0.76775907
101	19.80483	-1.315034226	-0.567945680	1.50404480	0.82041623
102	20.95510	0.767990158	1.203221402	0.65302023	-2.12337924
103	19.38254	0.439207637	0.142483135	-0.44334532	-0.60130386
104	19.11432	0.295939775	1.339271234	-0.27373699	1.41075416
105	19.73433	0.185766793	-0.110526663	-0.26950750	0.09622895
106	19.97122	-0.246160338	-0.326367392	0.19985366	-0.30344583
107	20.80335	0.271786371	-0.909489190	0.84621052	0.33528405
108	19.14934	-0.945267835	1.298492166	-1.94913877	0.98276001
109	19.65572	0.341439042	-0.377837346	-0.23818245	-0.46086602
110	21.60168	1.558589034	-0.997457994	0.93544667	1.33427016
111	21.11955	0.572398089	-1.430413533	1.85561982	0.95842119
112	20.78296	0.351815310	-0.907591569	-0.91102058	-0.04098430
113	20.64294	-0.218310535	0.415940627	-0.70872485	-0.03123276
114	19.53111	0.113328135	-0.368511960	0.61833462	-0.02646581
115	20.45019	0.287806804	0.383451401	-0.01444909	-0.48564006
116	18.12238	-1.870971151	-0.242901191	-0.57764169	-1.78667860
117	20.71017	-0.697794941	-0.697214274	0.12581421	-0.72978220
118	17.79009	1.226984471	1.038828253	-2.28540809	1.59744885
119	21.08528	-1.265641610	-0.599077481	1.17194766	-0.48024281
120	19.47063	0.204630336	0.477903908	0.35199851	-0.47986688

APPENDIX 7 (Cont.)

121	19.64628	-0.302747887	-0.235506406	0.31747228	1.36147965
122	21.15836	-1.232729066	-0.488858813	-1.24008468	-0.10913903
123	20.56271	-0.737883710	-0.083397420	-0.56369270	-0.48646964
124	18.65382	0.736708005	1.025952963	1.03544905	-0.94370457
125	20.13424	-0.170157398	-0.361380381	-0.10248304	0.36666105
126	20.04908	0.440468951	0.067487867	0.52599059	0.01005803
127	19.92210	-0.072993746	0.075463977	-0.05923939	-0.20630904
128	20.49221	0.345694480	-0.277194011	-0.80794312	-0.30666412
129	21.50305	-1.148176821	-1.509365668	0.08375650	1.51297722
130	19.39120	0.614844542	0.569686698	1.33136125	-0.02558329
131	17.86005	-2.143667108	-1.568375789	-0.78433141	-1.99868491
132	20.70368	-0.184658388	0.270328208	-0.61796484	-0.98189686
133	19.73563	-0.282355497	-0.138893142	-0.05117707	0.27794669
134	19.51780	0.996404365	1.413592280	-1.34252920	-0.80451197
135	18.52365	1.393740483	0.846611845	-0.99653481	1.08999461
136	20.24690	0.668150871	-0.808574166	0.77666830	-0.82778466
137	20.49803	0.855696991	-0.687613130	-1.74106822	-0.17467617
138	19.89722	-1.200983225	-1.179209808	1.20864603	-0.03959180
139	19.95002	-1.240141335	-1.149484313	-1.07062707	0.70327615
140	17.44948	-2.164798133	1.900408622	-2.31265332	1.13881557
141	21.38210	1.304601123	-1.435974188	1.27145680	0.75530839
142	19.34963	-0.777813978	-0.951319410	-0.86281712	-0.40177249
143	20.87315	-1.074695043	0.702315003	-0.72044086	0.84697633
144	20.97126	-0.082476477	-1.450602126	1.43569820	-0.24115014
145	20.65977	-0.886975739	0.540891019	0.01420138	0.90660033
146	19.34644	-0.622405971	0.666991772	0.56959730	0.35703584
147	21.08846	-0.410992123	-0.673026503	-0.42180120	-1.06423271
148	20.58803	-1.158807356	-0.595920730	-1.25978975	-0.45947413
149	20.25296	-0.806958092	-0.057891655	-0.65722418	1.03459919
150	19.89148	0.802117704	-0.445152958	-0.31660038	0.74101829

The above table is simulated data for n=150

APPENDIX 8

	y	x1	x2	x3	x4
1	-0.313999125	-0.185190222	-0.513907085	0.64081420	0.5372370596
2	0.072472351	-0.692750008	1.054755196	1.07502579	0.5004948256
3	-0.575392041	-1.532801965	0.908827125	1.09851740	1.1805587974
4	0.048027403	0.607913265	-0.599631771	0.69615853	-0.1819874681
5	-0.526584379	0.098208673	0.765271328	0.47447083	-0.6004721578
6	-0.522819859	-1.266037263	1.316865851	-1.16550209	0.6220195381
7	2.654936203	0.351578664	-2.671691975	-2.53425135	0.8599922896
8	0.146120951	1.514793201	-0.958281176	-1.77237899	-0.4488155513
9	-0.103542754	-0.088640413	0.295958250	-0.33306905	-0.0419494751
10	0.553090717	-0.960942596	-1.117715674	-1.03015743	0.6182103758
11	0.026866045	0.520600220	-0.408266235	-0.66062787	-1.0851973578
12	-0.693302465	0.037113809	-0.313623221	0.51167155	-0.7738687703
13	1.924606758	-0.341131550	-2.722690255	-2.41290650	1.2692081834
14	0.364492428	-0.067307093	0.539916085	0.55018426	-0.0693931010
15	0.796717345	0.959261384	-1.415642028	-2.27793536	1.1395186904
16	-0.822063781	0.572442288	-1.159432101	-1.10410104	0.4679127558
17	0.494730626	0.198147631	-0.447896296	0.49360159	-0.3338929148
18	0.568295588	-0.023454400	-0.653591181	-0.30972757	0.5807295808
19	0.861354138	0.813701734	-0.663634610	-0.78800318	-0.4097437849
20	-0.278865893	0.466983134	-0.200461786	0.41295097	-0.2679786079
21	-1.265811228	-0.893502705	0.466877562	0.65712640	1.3384279741
22	0.687779140	0.041323633	0.659459730	0.09998206	-0.6894156035
23	-0.396729428	0.373254121	-0.405076007	0.47601922	-0.0062967042
24	0.413211797	-0.303819634	-0.529993019	-0.22566299	0.5920542319
25	0.488758342	0.222833462	-0.355064565	0.50067928	-0.1402399225
26	0.079540437	0.152811542	0.205370775	-0.11668541	0.1695780525
27	1.354414189	-1.392498801	2.044662629	-0.03404836	2.1014215065
28	1.435837392	0.933630606	-0.755159091	-1.15285271	0.8599142583
29	1.866078283	0.950917326	-1.912319930	-1.45429384	-1.2430934057
30	1.095506379	0.297524603	-0.961914951	-0.63035891	1.0557850579
31	0.324085209	0.288065236	0.260196848	-0.31827991	0.3753205432
32	-0.223825480	-0.041072847	-0.669594663	0.78180012	1.0284631075
33	0.144419637	-0.998100602	-1.288576435	-1.38964699	-0.0359405225
34	0.194840597	0.126902206	0.187949686	-0.17646190	-0.0830902653
35	-1.653274190	1.076264162	1.646654663	-1.69665592	0.2138058001
36	0.043548022	0.017780036	-0.004208611	-0.03450515	0.0350668125
37	-0.606827628	-0.166230713	-0.864859592	0.95042776	0.5647743003
38	0.202926775	-0.573562164	0.067765992	0.40223394	-0.5730463424
39	0.566190566	0.138077901	-0.361393602	-0.50782589	-0.2781483382
40	2.044513175	1.399021484	1.668698291	1.78856016	1.7809459791
41	-1.466637039	1.194321646	1.443974273	1.50173023	-0.0270293808
42	-0.123607154	1.417773578	-0.025006155	-1.64773706	0.5191293942
43	1.812341240	-0.756301733	0.430687057	-0.50551198	-1.7449131742
44	-0.193402908	0.600061854	-0.457359063	-0.10036586	0.6212074825
45	-2.037557400	1.892344633	-2.618830567	-0.88518006	-2.4649067420
46	-1.182964364	-0.086427021	0.216267190	-0.65728574	-1.2361422485
47	0.029044485	0.841556949	1.208045127	0.32208694	1.2746535966
48	-0.021596028	0.278211079	0.406976165	0.45639199	0.0084442061
49	-1.443657197	0.509260979	0.305460867	-0.50193781	1.4019879559
50	-0.960968021	-1.014758995	0.669977815	0.19378259	-1.1434738831
51	0.884045623	0.036306811	0.901710284	-0.90814073	0.2461119325
52	-0.536138869	-0.602247930	-0.598563712	-0.15940140	0.5818117572
53	0.320288277	2.409531401	2.444032246	0.61107179	2.4032736184
54	-0.473313935	0.717097308	-0.889968035	1.07933216	0.2562002655
55	0.920209563	-0.930184938	-0.881082828	-0.48196158	0.8646947549
56	0.317488999	-1.272899733	-1.483089320	-1.45529036	-0.5297728816
57	0.682288302	0.597195477	0.437230685	-0.33626478	0.8015039548
58	0.128310697	-0.691827033	-0.568623122	-0.39932997	1.0635138069
59	-0.114125443	-0.132394193	-0.089008135	0.06653914	-0.1569736591
60	0.130999568	-0.165367371	0.111427663	0.13500471	-0.1070442217
61	-0.819686619	0.819833243	0.763910020	-0.63466623	-0.5978240583
62	-0.697419446	2.379605297	-2.289325921	2.10822306	1.1238876022

APPENDIX 8 (Cont.)

63	-0.688511431	-0.418813501	-0.476537907	0.01528514	-0.7967161654
64	1.078932099	-1.074926152	1.081643885	-0.99127912	-0.4335200154
65	0.774808487	-0.563600995	-0.994805891	-0.54006041	-0.9259005938
66	1.379596672	1.759972135	0.279262467	-0.79173958	-1.5825932361
67	-0.633557406	0.781731527	-0.752158697	0.04730008	-0.7907832413
68	1.968435884	-1.985887455	0.410622440	-0.95756905	1.7650040853
69	-1.071349313	0.893816048	1.033746471	-1.14076197	0.0101757844
70	0.677574751	-0.511390660	-0.800819436	-0.94785303	-0.5734401624
71	-0.378581006	-0.503149561	0.323501508	0.68272673	0.5135409519
72	-0.659687292	0.479912071	0.769800161	-0.72696385	-0.3335796976
73	-0.279543703	-0.282915929	0.087239713	-0.13870932	0.2635848305
74	0.064140720	-1.036943927	-0.670665039	-1.07049818	-0.4930822532
75	-0.577877092	2.163352066	0.714697446	-1.39065940	-1.6614625900
76	0.069970213	-0.527931668	-0.545188842	-0.33805997	-0.4460667734
77	0.223315631	-1.294216468	-0.365617452	0.56927425	-1.5711080931
78	0.394496348	0.407872363	-0.277334015	-0.37519484	-0.2022722274
79	-0.745667320	0.919082234	-0.940733363	0.33310937	-1.5788239666
80	-0.154033026	-0.893032663	0.423834360	1.30707097	0.0235705758
81	-0.049077831	0.410198052	1.030421674	-0.09717197	-1.0263414933
82	-1.719152199	0.521534714	-0.938359950	-1.63163704	-0.7484306462
83	-0.506145910	-0.220272872	-0.284845588	-0.96804357	0.2365935166
84	0.527765776	0.836298353	0.922736705	0.87750174	0.5178840629
85	-0.945414629	1.031450132	-0.407152982	0.99461268	0.3044011532
86	2.233032745	1.848860404	1.040352315	2.35760031	0.4840992642
87	-0.304184631	-0.711656489	0.662468111	-0.61473704	0.3637373368
88	-0.866605607	0.556835416	-0.660593776	-1.00774114	-0.4515107313
89	-0.023028134	0.246619015	-0.292494957	-0.24825573	-0.4605142931
90	0.435117046	-0.609888028	0.972642899	0.58024979	0.8255476669
91	-0.197473142	1.185009531	-0.598048996	0.44260998	1.5684182909
92	-0.550692256	0.941893365	0.766215123	-0.92242064	-0.5100367816
93	0.576591137	1.006755373	-1.715616134	-0.35318441	1.7457774305
94	-1.114325857	0.223301662	0.980220991	0.34486077	1.0688104437
95	0.026095975	-0.440842639	-1.377051034	-0.77974092	-1.1374835133
96	-0.535007123	-1.368887177	1.216472677	0.23936010	1.3481276619
97	0.261216332	0.039466841	-0.314869870	-0.33979800	0.0307459185
98	-1.888891853	-0.973549199	0.398209064	1.49858675	-1.2069029191
99	0.533567887	0.651344195	-0.192320340	0.20110245	0.6196929078
100	0.263094580	0.372386992	0.010441158	0.25170957	0.8799124329
101	-1.014408066	-0.410914948	-0.886892681	0.79826184	-0.7359196070
102	-0.610141584	-0.788685883	0.700074899	-0.75237347	0.3356502370
103	1.328030347	1.111894120	-1.222552672	0.60608756	-1.1829650638
104	-0.893838557	-0.179646580	-0.346552460	0.75252599	-0.4905169554
105	0.368022699	0.539392278	0.118700380	0.50357757	-0.3432197856
106	-0.459991888	0.391872354	-0.391757723	-0.45665623	0.0630328411
107	-0.451689048	-0.723089414	1.308316198	1.25616563	0.5349002226
108	0.328402564	0.047044398	-0.550939182	0.18956161	0.5177185315
109	0.369289201	-0.478323982	1.205033235	0.52348915	1.2803632569
110	0.482187204	0.109681994	0.833081933	-0.90589177	1.1805388567
111	0.701132960	-1.362966595	-0.865014921	-1.37358839	0.2692626877
112	0.633068247	-0.239178909	-1.049318120	-0.42210011	1.0550615021
113	-1.856131490	1.430019835	1.409668573	1.84699537	-0.1845142298
114	0.001300760	0.525789745	0.385935217	0.42074502	0.9608606276
115	-0.389289417	0.667082494	-0.620529926	0.49352001	-0.4503922309
116	-0.330114251	-0.773125813	-0.606542628	0.29520753	0.7859946531
117	-0.529180389	0.050368263	1.377139605	-1.36422281	-0.1969088702
118	0.170411431	-0.031296466	-1.786107184	2.11507196	-1.1509849291
119	-0.589561059	0.256690242	0.553323837	-0.41745101	-0.6659671057
120	-0.708919603	0.535300522	-0.727831379	0.73282468	-0.2088850692
121	-1.757352433	1.864915349	0.925738395	-1.40715795	-1.3180249328
122	-0.125605615	-0.624397399	0.677355083	0.71392679	-0.4656249110
123	-0.546572922	-1.514341611	-1.582883747	1.03626292	-1.2626071527
124	0.298586288	-2.979306215	3.326751375	2.43363479	-2.3066783219

APPENDIX 8 (Cont.)

125	1.077571616	-0.212534580	-0.437923195	-1.60434777	1.5362499742
126	0.140177834	0.024543963	0.042307473	0.08609551	0.3648216525
127	0.282114488	-0.666884619	-0.253701192	0.66646469	0.0580260406
128	0.975690266	-1.884974671	-0.335929029	0.27338507	1.8656519451
129	-0.501073997	0.458719713	-0.543807285	0.46558607	1.0575283918
130	-0.368458465	-0.596578907	0.494110239	0.05432816	0.6011149100
131	-0.560132188	-0.584367510	1.813033218	-1.16467298	-1.7561778905
132	-0.006147860	0.352327881	-0.985699943	-1.09742538	0.7222381967
133	-0.918997079	0.343568589	0.668514055	1.44636038	0.0445004145
134	-2.314989017	-0.727668984	-2.203594048	1.34273472	1.9649443756
135	-0.685042781	1.002470918	0.417628868	1.00322627	0.3125063850
136	1.131034447	-1.303067735	-0.366974819	1.75410718	0.1788920351
137	-0.032624143	0.058411355	-0.023767425	0.01282564	-0.0616105235
138	0.651995393	0.188022474	-0.593149348	0.29589142	0.6588821780
139	-1.289988582	-1.551218896	1.558692950	-0.99812561	-1.1990907592
140	0.563754347	-0.001642289	1.236303469	-0.94690840	0.9766104663
141	1.605598532	-0.256471526	-1.568236601	-1.52661231	0.5029243793
142	0.830318720	1.180060276	-1.194386648	-0.08022402	-1.2934261337
143	0.065382549	0.377189641	-0.504584489	0.32251366	-0.4145423419
144	-0.702785743	-0.840520848	-0.794461347	-0.74025925	0.4019627333
145	0.188996662	0.202752540	-0.203441496	0.10684967	-0.1761341576
146	0.287107622	-1.220292582	0.114961549	0.59614605	1.0650251177
147	-1.832257410	-1.003705267	0.089333994	-2.43652279	-0.3371394524
148	-0.300161173	-0.138552363	-0.133187516	-0.30983077	0.2075576441
149	-0.789396286	-0.917479863	1.255396170	1.18038832	0.7400694978
150	0.449805333	-0.213307223	0.493975555	0.50726647	0.1211316424
151	0.294199762	-0.584396441	-1.813226278	2.13041999	0.1817345419
152	0.445899139	-0.282263890	0.845125165	-0.52571820	0.6829104702
153	0.760341229	0.073187334	-0.565273919	0.07830451	-0.7645375188
154	-0.776115430	1.091229371	-1.106501748	-0.56293941	0.9595913407
155	0.733518828	-0.513604106	0.813012598	0.70038353	0.6292121771
156	-0.155489772	-0.163079447	0.463255651	-0.44460208	-0.1302996258
157	-0.435528941	-0.594589507	-0.076368483	-0.49593427	0.3280318222
158	0.902949324	1.217340305	-0.890233220	0.84558100	-0.8820290906
159	-0.091132592	0.708384025	-0.842639103	-0.74785227	-0.4238407419
160	0.207834280	-0.629681061	0.772216115	-0.71916489	0.6961621727
161	-0.451058638	0.234264127	-0.575630729	-0.08816061	-0.6444242102
162	0.183733682	-0.748766915	0.386076438	-0.51750835	-0.6913358004
163	0.143672673	-0.474793988	0.395888818	0.15788511	0.4524192614
164	1.568337527	-1.814952698	-1.800754863	0.27460706	-1.7948551197
165	-1.363217178	1.802832190	1.802894694	-0.28216805	1.7885362430
166	0.785201314	-1.344953567	-1.303011709	0.94164422	-1.1052154284
167	0.694441158	-1.020166106	-0.539207066	1.10406334	0.0273673219
168	-1.436222056	1.588106180	-2.008886304	-2.17707134	-0.0276797511
169	1.266418143	1.114412859	-0.141410103	1.00780571	0.7883193042
170	-0.006635536	-0.378853032	-0.447469958	-0.03685378	0.4647285465
171	0.378332544	-0.925441849	1.097160985	-0.94500533	-0.5580027728
172	-1.714949087	-1.118970705	-1.086437038	1.72805154	-0.0799279322
173	0.251387299	0.263170475	0.085599763	0.50958223	0.0008641616
174	-0.640764148	0.915643180	-0.906471940	0.92430349	-0.1060460508
175	-0.201537585	1.061319945	-1.490099335	-1.15487850	1.2862749239
176	0.213885741	0.329760157	0.306728950	0.07820209	-0.3205800272
177	-0.573887320	0.081233008	-0.520732314	0.15425308	-0.5528565581
178	0.521465243	-0.896014938	0.166790134	-0.64929288	0.7152447831
179	-0.971291914	1.465602587	-0.516405801	-0.81546943	1.6378996955
180	-0.604806143	0.244605107	0.456546546	0.56963137	-0.2032651143
181	-0.886318227	-0.394618548	-0.398520714	0.49081132	0.9120243346
182	-1.609274755	1.825277784	1.004152277	1.19635195	-1.6111908944
183	-0.741585298	0.762536545	-0.730286483	-0.50376260	0.5843497824
184	-0.093176858	-0.702239478	-0.549005578	-0.74716770	-0.0795520239
185	0.821838590	0.575158574	0.005382338	-1.18547464	0.5940317994
186	-1.404859926	-0.775242727	1.045022217	0.04467774	-1.4170784279

APPENDIX 8 (Cont.)

187	0.978003945	-0.348035161	-2.932696289	-0.91986403	-2.8035971466
188	-0.595612347	1.038107660	-0.268752459	-1.07736914	0.2097534308
189	-0.437445123	-0.238474047	0.063065935	0.03865286	0.5526380655
190	0.463865011	1.139975547	-0.389905662	-1.49668483	0.6255249990
191	3.105308928	-2.838372345	2.164904227	-3.08772568	0.3550033678
192	-0.333359786	-0.204207605	-0.295717406	-0.49654039	0.5210421131
193	-0.309866317	-0.459816487	0.377907352	-0.35758964	-0.3561259668
194	0.712446055	-0.589910454	-0.241860585	-0.52795494	-0.5948543005
195	-2.052671487	0.854735265	1.322034729	-1.53408262	1.4865651656
196	0.796016760	0.354053053	0.023814838	-0.10371849	-0.8876026499
197	-1.428523883	0.123166922	-0.774611868	0.18564257	-1.4953799980
198	-0.916801766	-0.750086183	-1.097576360	0.18235082	1.1264953333
199	-1.174508959	1.234001813	1.270863038	-0.31324389	-1.2777156698
200	-0.732074232	1.575953147	-0.814028901	2.66540579	-0.0162485932

The above table is simulated data for sample size n=200

APPENDIX 9

	Year	GDP	Recurrent Expenditure	Capital Expenditure	DomesticDebts	ExternalDebts
1	1981	144.83	4.58	6.57	11.19	2.33
2	1982	154.98	5.51	6.42	15.01	8.82
3	1983	163.00	4.75	4.89	22.22	10.58
4	1984	170.38	5.83	4.10	25.67	14.81
5	1985	192.27	7.58	5.46	27.95	17.30
6	1986	202.44	7.70	8.53	28.44	41.45
7	1987	249.44	15.65	6.37	36.79	100.79
8	1988	320.33	19.41	8.34	47.03	133.96
9	1989	419.20	25.99	15.03	47.05	240.39
10	1990	299.68	36.22	24.05	84.09	298.61
11	1991	596.04	38.24	28.34	116.20	328.45
12	1992	909.80	53.03	39.76	177.96	544.26
13	1993	1259.01	136.73	54.50	273.84	633.14
14	1994	1762.81	89.97	70.92	407.58	648.81
15	1995	2895.20	127.63	121.14	477.73	716.32
16	1996	3779.13	124.49	212.93	419.98	617.32
17	1997	4111.64	158.56	269.65	501.75	595.93
18	1998	4588.99	178.10	309.02	560.83	633.02
19	1999	5307.36	449.66	498.03	794.81	2577.37
20	2000	6897.48	461.60	239.45	898.25	3097.38
21	2001	8134.14	579.30	438.70	1016.97	3176.29
22	2002	11332.25	696.80	321.38	1166.00	3932.88
23	2003	17321.30	984.30	241.69	1329.68	4478.33
24	2004	22269.98	1110.64	351.25	1370.33	4890.27
25	2005	28662.47	1321.23	519.47	1525.91	2695.07

The above data is real life data gotten from CBN bulletin