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COMPARATIVE ANALYSIS OF THREE METHODS FOR INVERSE PROBLEMS APPLIED IN IMAGING.

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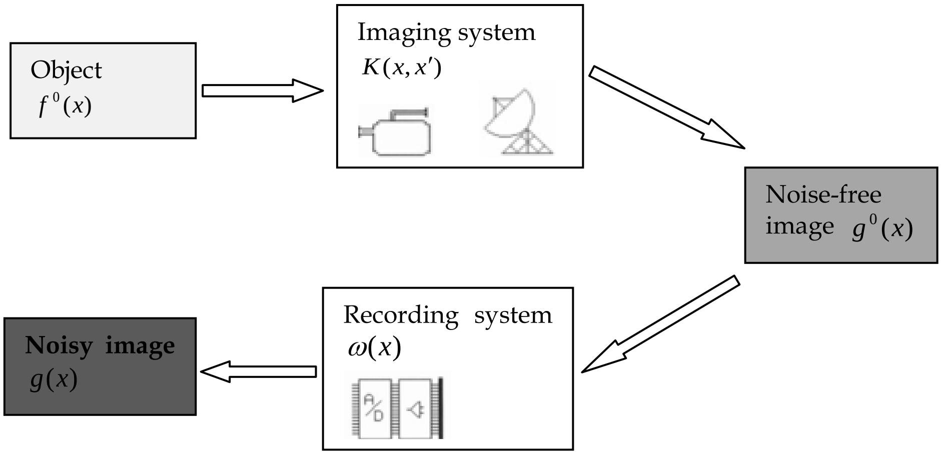
ABSTRACT
Nowadays, a core technology for solving image processing problems such as denoising and deblurring is regularization. Image processing is an interdisciplinary research area which has profound applications in many areas of science, technology, engineering and medicine. We plan to present a comparative analysis of three methods for commonly used regularization method namely total variation regularization. The three methods are the well-known Split Bregman algorithms, the Alternating Direction Method of Multipliers and the Rudin Osher Fatemi denoising model on the graph (ROF model on the graph) which is very recent. The analysis is performed through experimental results on multiple test images both synthetic and real. We observe that they are all comparable in many cases; however the ROF model on the graph achieves a prescribed tolerance in fewer iterations than the other methods.

INTRODUCTION

Nowadays, a core technology for solving image processing problems such as denoising and deblurring is reg-ularization of the corresponding inverse problem. Image processing is an interdisciplinary research area which has profound applications in many areas of science, technology, engineering and medicine. The main objective of this-this study is to present a comparative analysis of three methods for commonly used regularization method namely total variation regularization. The three methods are the well-known Split Bregman algorithms, the Alternating Direction Method of Multipliers and the Rudin Osher Fatemi denoising model on the graph (ROF model on the graph) which is very recent.



This figure shows a schema representation of the formation of the noisy image g as follows

$$g(x) = Kf^0(x) + \omega(x) \qquad (1)$$

where $f^0(x)$ denotes the object (original image to be reconstructed); $K(x,x')$ denotes the imaging system so that $g^0(x) = Kf^0(x)$ is the noise-free image and $\omega(x)$ is the noise term that comes from the recording system.

In this paper we treat the case where $K = I$ i.e., the denoising case.

To obtain an estimate f_* of f^0, the total variation regularization suggests to take the function f that is exact minimizer of the following optimization problem:

$$\inf\left\{\frac{1}{2}\|g-f\|_{L^2}^2 + \mu\|f\|_{BV};\ f \in BV\right\} \qquad (2)$$

where, μ is a positive regularization parameter. The three algorithms of our interest are implemented to solve the above problem in (2).

METHODOLOGY AND ALGORITHMS.

The analysis is performed through experimental results on multiple test images both synthetic and real. The test images we used are freely available on some well known online image databases. In this paper we illustrate the performance of algorithms on three 8 bits per pixel test images all of the same size 512×512. The reconstructed images are compared to the true images and algorithms are compared for convergence and speed. Below we give a very short description of each of the algorithms.

a. Algorithm 1 (Split Bregman)

Split Bregman method puts the problem (2) in the form

$$\begin{aligned} & f_* = \arg\min\ J(f) \\ & \textit{subject to } H(u) = 0, \end{aligned} \qquad (3)$$

where, $J(\cdot) = \|\cdot\|_{BV}$ and $H(\cdot) = \frac{1}{2}\|g-\cdot\|_{L^2}^2$ and use the following so called Bregman iterations:

$$\begin{aligned} & p^0 = 0 \\ & \textbf{While } \textit{"Not converged"} \textbf{ do} \\ & \qquad f^{k+1} = \arg\min J(f) - \langle p^k, f\rangle + \mu H(f) \\ & \qquad p^{k+1} = p^k - \mu\nabla H(f^{k+1}) \\ & \textbf{EndWhile} \end{aligned}$$

where $p^k \in \partial J(f^k)$.

b. Algorithm 2 (ADMM)

The Alternating Direction Method of Multipliers (ADMM) puts the problem (2) in the form

$$\begin{aligned} &\min H(f) + J(f) \\ &\text{subject to } Mf - h = 0, \end{aligned} \tag{4}$$

for

$$H(f) = \frac{1}{2}\|g - f\|_{L^2}^2 \quad \text{and} \quad J(h) = \mu\|h\|_{BV} = \mu\|Mf\|_{BV},$$

where $M \in R^{(n-1,n)}$ is the difference matrix given by $M_{ij} = \begin{cases} 1 & \text{if } j = i+1 \\ -1 & \text{if } j = 1 \\ 0 & \text{otherwise.} \end{cases}$

Then perform the following iterations

$$\begin{aligned} f^{k+1} &= \left(I + \rho M^T M\right)^{-1}\left(g + \rho M^T (h^k - u^K)\right) \\ h^{k+1} &= S_{\frac{\mu}{\rho}}\left(Mf^{k+1} + u^K\right) = sign\left(Mf^{k+1} + u^K\right)\max\left(0,\ \left|Mf^{k+1} + u^K\right| - \frac{\mu}{\rho}\right) \\ u^{k+1} &= u^k + Mf^{k+1} - h^{k+1} \end{aligned}$$

where $\rho > 0$ is the augmented Lagrangian parameter and u is the dual variable.

c. Algorithm 3 (ROF Graph)

The ROF-Graph takes advantage of the fact that an image can be considered as a grid of points (pixels). The vertices of the graph are pixels and edges are pairs of connected pixels. The algorithm constructs f_* reiteratively using an operator that decreases the distance from transformed elements to g as follows:

Step 1. *Take* $f_0 = 0$ *or choose any* $f \in \mu B_{l^\infty(S_E)}$

Step 2. *Calculate* $f = Tf_0 = 0$.
If $f = f_0$ *then take* $\tilde{h} = div(f_0)$,
otherwise go to **Step 3**.

Step 3. *Put* $f_0 = f$ *and go to* **Step 2**

Here $T = T_M T_{M-1} \ldots T_2 T_1$ where $\forall k,\ T_k : \mu B_{l^\infty(S_E)} \to \mu B_{l^\infty(S_E)}$ is defined as follows:

$$(T_k f)(e) = \begin{cases} \psi(e) & \text{if } e = e_k \\ f(e) & \text{if } e \neq e_k \end{cases}$$

where $\psi(e) = \begin{cases} Kf(e_k) & \text{if } Kf(e_k) \in [-\mu,\ +\mu] \\ -\mu & \text{if } Kf(e_k) < -\mu \\ +\mu & \text{if } Kf(e_k) > +\mu \end{cases}$ and $Kf(e_k) = \frac{1}{2}\left\{\left[g(v_j) - div\ f(v_j) - f(e_k)\right] - \left[g(v_i) - div\ f(v_i) - f(e_k)\right]\right\}$

with $e_k = (v_i, v_j)$ for some i and j.

Experimental results

The figures below show experimental results for three test images. For each image we shoe the original image, the noisy image and the reconstructed images using ROF-Graph, Split Bregman and ADMM respectively and the figure at the bottom right shows the convergence history for all three algorithms.

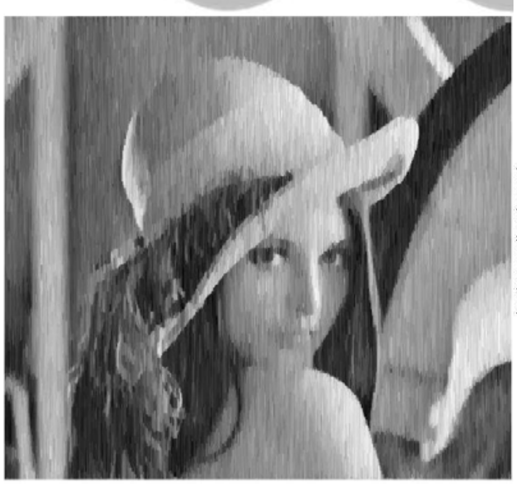
a) Test Image 1: Lena

Original image

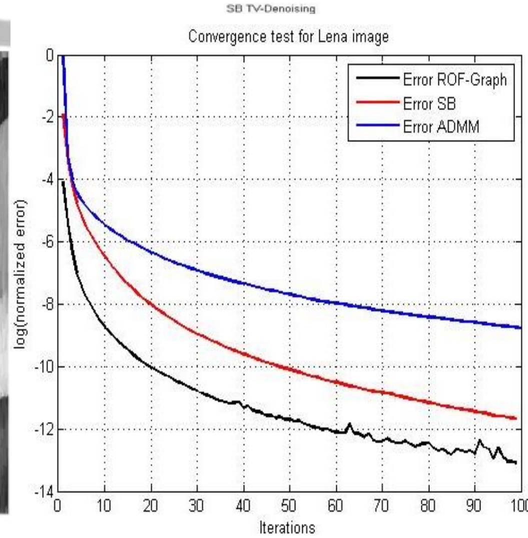
Observed noisy image

ROFGraph TV-Denoising

SB TV-Denoising

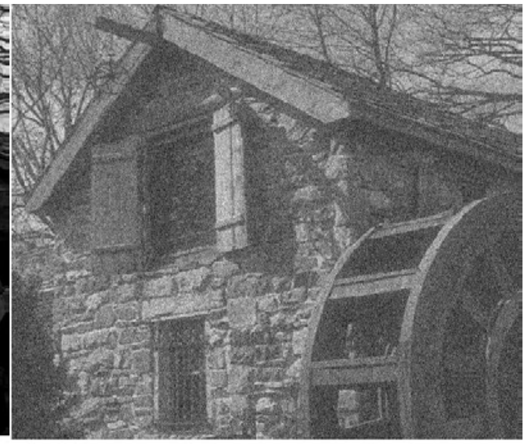


ADMM TV-Denoising



b) Test Image 2: House

Original image

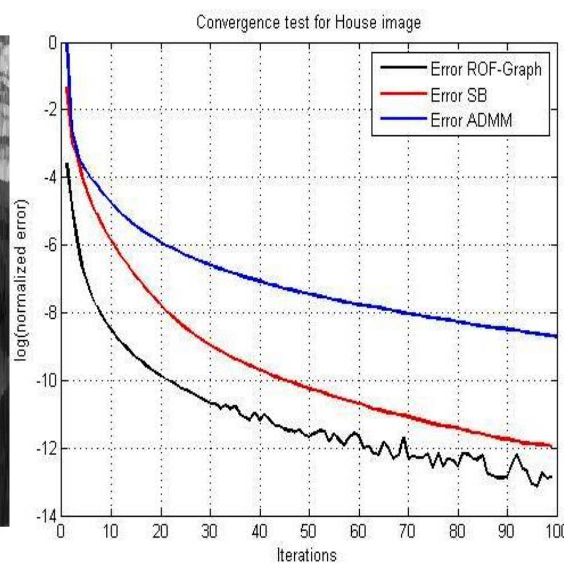


Observed noisy image

ROFGraph TV-Denoising

SB TV-Denoising

ADMM TV-Denoising



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c) Test Image 3: Pentagone

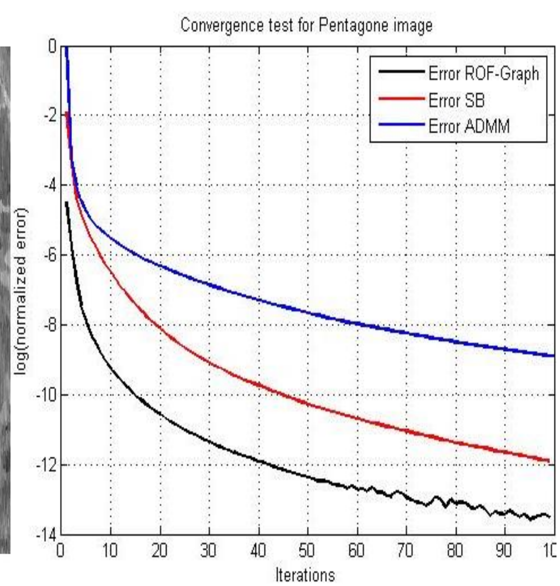
Original image

Observed noisy image

ROFGraph TV-Denoising

SB TV-Denoising

ADMM TV-Denoising



Relative errors comparison.

The table below shows the relative errors between the original image and reconstructed image for each method and each testing image.

Images / Methods	Lena	House	Pentagone
ROF-Graph	2.0601×10^{-6}	2.57415×10^{-6}	1.37305×10^{-6}
S.Bregman	8.45849×10^{-6}	6.43004×10^{-6}	6.73468×10^{-6}
ADMM	1.55788×10^{-4}	2.64989×10^{-4}	1.36008×0^{-4}

Conclusion

We have observed that in many cases, all three algorithms are comparable. However in general, as seen on the convergence history plots, the ROF-Graph algorithm will achieve a prescribed tolerance in fewer that both Split Bregman and ADMM algorithms.

Inthis work, we have studied and analyses for comparison purposes two well known algorithms (Split Bregman and ADMM) with a recent algorithm named ROF-Graph that solves the model taking advantage of insights from that fact that a digital image can be as a function defined on a graph. All three algorithms were compared with respect to their performance in solving the total variation model for denoising. Results show that our algorithm compares very well the two and can even outperform them by achieving a desired result using fewer iterations. The results hence show a very good indication that our algorithm can be used to solve other ℓ^1 regularized problems appearing in machine learning and statistics.

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