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Conformal Lagrangian and Hamiltonian Mechanics Systems with Almost Three - Dimensional f-kenmotsu Manifolds

[1]Abdulaziz B. M.Hamed and[2] Ibrahim Yousif Ibrahim Abad alrhman

[1]Department of Mathematics & Statistics, Faculty of Science, Yobe State University, Damaturu, Nigeria

[1,2]Departmentof Mathematics, Faculty of Education, West Kordufan University–Sudan

Corresponding Author: aziz.hamed12@gmail.com

Abstract

The study concern with using dynamical formalism and equations to find produced dynamical Euler -Lagrangian Equations with Conformal Lagrangian and Hamiltonian Mechanics Systems with Almost three-dimensional f-kenmotsu Manifolds.However we gave some corollaries about related mechanical systems and equations.

Key words;f-Cosymplectic Manifolds, Almost conformal three-dimensional f-kenmotsu Manifolds, Euler-Lagrange Equations.

1 Introduction

Differential geometry is a branch of engineering concerned with the study of geometric shapes, in particular the curves, surfaces and envelopes of the families of curves and surfaces in the Euclidean and Chalcidice spheres. The first method of analysis is the differential calculus. The focus is especially on the differential characteristics of geometric shapes**.**Which are the immutable characteristics of motion.

The emergence of differential geometry has been closely related to the emergence and evolution of the concept of coordinates, tangles, curves, and spaces. Here, the topology, theLi groups and the so-called geometrical structures have replaced the curves and surfaces that were the basic themes of classical differential geometry. Differential geometry has an important application in many branches of mathematical science, the

basic and on top of classical mechanics (theory),such as, the theory of relativity and the theory of differential equations.
On the other hand, the equation of Euler- Lagrange is one of the most important applications of classical mechanics.In this paper we addressed the equations of Conformal Lagrangian and Hamiltonian Mechanics Systems with Almost three- dimensional f-kenmotsu Manifolds**.**

2.Preliminary

In this preliminary chapter, we recall basic definitions, results and formulas which we shall use in the subsequent chapters of the paper

Definition(Almost f −kenmotsu Manifolds) 2.1

Let M be areal $(2n + 1)$ −dimensional differentiable manifold endowed with an almost contact structure equipped with atriple (M, ϕ, ξ, η, g), where ϕ is a type of (1-1) tensor field, ξ is a vector field, η isa 1-form on M such that

$\eta(\xi) = 1 \quad , \quad \phi^2 = -1 + \eta \otimes \xi$ (1.1)

which implies

$\phi\xi = 0, \quad \eta \circ \phi = 0 \quad , rank\,(\phi) = n - 1.$ (1.2)

where $\emptyset$is a(1-1) tenser field η is a 1- form and the Riemannian metric$g$ .It well known that

$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \quad , \eta(X) = g(y)$ (1.3)

The fundamental 2- form of metric manifolds is defined by

$\phi(X, Y) = g(\phi X, Y)$ (1.4)

for any vector fields X, Y on M.

$\nabla_X \phi = f(g(\phi X, Y)\xi - \eta(Y)\phi X$ (1.5)

where$f \in C^\infty(M)$is such that $df \Lambda \eta = 0 \quad . If\ f = \alpha = constant\ \neq 0$ then the manifold is a f-Kenmotsu Manifold.

1-The 1-Kenmotsu Manifold is a Kenmotsu Manifold.

2-if $= 0$, then the manifolds cosymplectic .

3-an$f$-Kenmotsu Manifold is called regular if $f^2 + \tilde{f} \neq 0$

Proposition2.2

(i) $f \wedge g = -g \wedge f$

(ii) $(f \wedge g)(x) = f(x)g - g(x)f$

(iii) $\left(dx^i \Lambda dx^j\right)\left(\frac{\partial}{\partial x^k}\right) = \frac{\partial x^i}{\partial x^k} dx^j - \frac{\partial x^j}{\partial x^k} dx^i$

(iv) $\frac{\partial x^j}{\partial x^i} = 0 \quad , \quad \frac{\partial x^i}{\partial x^i} = 1$ (1.6)

DefinitionConformal 3 – dimensional Almost$f$ −kenmotsu Manifolds2.3

Letthree-dimensional manifold $M = f(x, y, z) \in R^3, z \neq 0$;where (x, y, z) are the standard coordinates in R^3: The vector fields

$e_1 = \sin^2 z \frac{\partial}{\partial x} \quad , \quad e_2 = \sin^2 z \frac{\partial}{\partial y} \quad , = \sin z \frac{\partial}{\partial z}$ (1.7)

are$\{e_1, e_2, e_3\}$ linearly independent at each point of M: Let g be the Riemannian metricdefined by

$g(e_1, e_3) = g(e_2, e_3) = g(e_1, e_2) = 0$
$g(e_1, e_1) = g(e_2, e_2) = g(e_3, e_3) = 1$
and given by the tensor product

$$g = \sin^4 z\,(\mathrm{dx} \otimes \mathrm{dx} + \mathrm{dy} \otimes \mathrm{dx}) + \frac{1}{\sin^2 z}\mathrm{dz} \otimes \mathrm{dz} \quad (1.8)$$

Let η be the 1-form defined by $\eta(X) = g(X, e_3)$ for any $Z \in \aleph(M)$.

Proposition 2.4

Let ϕ bethe (1,1) tensorfield defined by

$$\phi(e_1) = -e_2 \quad , \quad \phi(e_2) = e_1 \quad , \phi(e_3) = 0 \quad (1.9)$$

Thenusing the linearity of ϕ and g we have
$\eta(e_3) = 1$; $\phi^2(X) = -X + \eta(X)e_3$;
$g(\phi Z; \phi W) = g(X; Y) - \eta(X)\eta(Y)$;
for any $Z, W \in \phi(M)$: Thus for $e_3 = \xi$, (ϕ, ξ, η, g) defines an almost contactmetric structure on M: Now, by direct computations we obtain
$[e_1, e_2] = 0 \quad ; [e_1, e_3] = -2Ze_1 \quad , \qquad [e_2, e_3] = -2Ze_1$

Proposition 3.3

the following expressions are given

$$\phi\left(\sin^2 z\frac{\partial}{\partial x}\right) = \sin^2 z\frac{\partial}{\partial y} \quad , \phi\left(\sin^2 z\frac{\partial}{\partial y}\right) = -\sin^2 z\frac{\partial}{\partial x} \quad , \quad \phi\left(\sin z\frac{\partial}{\partial z}\right) = 0 \quad (1.10)$$

The dual form ϕ^* of the aboveϕ is as follows

$$\phi^*(\sin^2 z\,dx) = \sin^2 z\,dy \ , \phi^*(\sin^2 z\,dy) = -\sin^2 z\,dx \ , \phi^*(\sin z\,dz) = 0 \quad (1.11)$$

Proposition 2.5

The vector fields

$$e_1 = \frac{\partial}{\partial x} \quad , \qquad e_2 = \frac{\partial}{\partial y} \qquad and \ \ e_3 = \frac{\partial}{\partial z}$$

If $\phi$is defined a $\mathcal{M}$then$\phi^2 = \phi \circ \phi = -1 \quad or\ 0$

Proof:

$$\phi^2(e_1) = \phi\big(\phi(e_1)\big) = \phi(-e_2) = -\phi(e_2) = -e_1 = -1$$
$$\phi^2(e_2) = \phi\big(\phi(e_2)\big) = \phi(e_1) = \phi(e_1) = -e_2 = -1$$
$$\phi^2(e_3) = \phi\big(\phi(e_3)\big) = \phi(0) = \phi(0) = 0$$

As can ϕ^2 is -1 (complex) or 0

4.Conformal Lagrangian Mechanics Systems with Almost three-dimensional f-kenmotsu Manifolds

In this section, we obtain conformalLagrangian Mechanics Systemsfor classical mechanics structured on momentum space (M, ϕ, ξ, η, g) that is three- dimensional tangent bundle of an m-dimensional configuration manifold $\mathcal{M}$

Definition 4.1

Let map L: T$\mathcal{M} \to \mathcal{M}$ such that

$\mathrm{L} = \mathrm{T} - \mathrm{P}$ (4.1)

The Lagrangianfunction ,where we find that

T= Kinetic energy , P = Potential energy

Definition 4.2A given configuration manifold. If $\mathcal{M}$ is an three -dimensionalconfiguration manifold and $\mathrm{L}: \mathrm{T}\mathcal{M} \longrightarrow \mathcal{M}$is a regular Lagrangian function, then there is aunique vector field ξ_{L} on $\mathrm{T}\mathcal{M}$ such that dynamical equations

$\mathrm{i}_{\emptyset_{\mathrm{L}}} \phi_{\mathrm{L}} = \mathrm{dE}_{\mathrm{L}}$ (4.2)

where$\phi_{\mathrm{L}}$is the symplectic form and E_{L} is the energy associated to L

Let $\emptyset$be an almost complex structure on the $\mathrm{T}\mathcal{M}$ and $(\mathrm{x}, \mathrm{y}, \mathrm{z})$ its complexcoordinates.. Assume to be semispray to the vector field ξ given as

$$\xi = \mathrm{x}\frac{\partial}{\partial \mathrm{x}} + \mathrm{y}\frac{\partial}{\partial \mathrm{y}} + \mathrm{z}\frac{\partial}{\partial \mathrm{z}} \quad , \quad \mathrm{x} = \dot{\mathrm{x}} \ , \quad \mathrm{y} = \dot{\mathrm{y}} \ , \ \mathrm{z} = \dot{\mathrm{z}} \qquad (4.3)$$

By Liouville vector field Conformal 3 – dimensional Almostf −kenmotsu Manifoldsspace form $(M^3, \phi. \xi, \eta, g)$, we call the vector fielddetermined by $\mathrm{V} = \phi\xi$ and calculated by

$$\phi\xi = \phi\left(\mathrm{x}\frac{\partial}{\partial \mathrm{x}} + \mathrm{y}\frac{\partial}{\partial \mathrm{y}} + \mathrm{z}\frac{\partial}{\partial \mathrm{z}}\right) = x\phi\left(\frac{\partial}{\partial \mathrm{x}}\right) + \mathrm{y}\phi\left(\frac{\partial}{\partial \mathrm{x}}\right) + z\phi\left(\frac{\partial}{\partial \mathrm{x}}\right)$$

$$\phi\xi = -\sin^2 z\frac{\partial}{\partial \mathrm{y}} + \sin^2 z\frac{\partial}{\partial \mathrm{x}} + \mathrm{z}(0) = -\sin^2 z\frac{\partial}{\partial \mathrm{y}} + \sin^2 z\frac{\partial}{\partial \mathrm{x}} + \mathrm{z}(0)$$

$$\phi\xi = -X\sin^2 z\frac{\partial}{\partial \mathrm{y}} + \mathrm{Y}\ \sin^2 z\frac{\partial}{\partial \mathrm{x}} \qquad (4.4)$$

is called the interior product with ϕ, or sometimes the insertion operator, or contraction by ϕ.The exterior vertical derivation d_ϕ is defined by

$$\mathrm{d}_\phi = [\mathrm{i}_\phi, \mathrm{d}] = \mathrm{i}_\phi \mathrm{d} - \mathrm{d}\mathrm{i}_\phi \qquad (4.5)$$

whered is the usual exterior derivation. For almost product structure ϕ determined bythe closed Conformal 3 – dimensional Almost$\boldsymbol{f}$ −kenmotsu Manifoldsform is the closed 2-form given by

$$\phi_{\mathrm{L}} = -\mathrm{d}\mathrm{d}_\phi \mathrm{L}$$

Such that

$$\mathrm{d}_\phi = -X\sin^2 z\frac{\partial}{\partial \mathrm{y}} + \mathrm{Y}\sin^2 z\frac{\partial}{\partial \mathrm{x}}$$

$$\mathrm{d}_\phi \mathrm{L} = \left(-X\sin^2 z\frac{\partial}{\partial \mathrm{y}} + \mathrm{Y}\sin^2 z\frac{\partial}{\partial \mathrm{x}}\right)L = -\mathrm{X}\sin^2 z\frac{\partial \mathrm{L}}{\partial \mathrm{y}} + \mathrm{Y}\sin^2 z\frac{\partial \mathrm{L}}{\partial \mathrm{x}}$$

Thus we get

$$\phi_{\mathrm{L}} = -\mathrm{d}(\mathrm{d}_\phi \mathrm{L}) = -\mathrm{d}\left(-\mathrm{X}\sin^2 z\frac{\partial \mathrm{L}}{\partial \mathrm{y}} + \mathrm{Y}\sin^2 z\frac{\partial \mathrm{L}}{\partial \mathrm{x}}\right) (4.6)$$

$$\phi_{\mathrm{L}} = \mathrm{X}\sin^2 z\frac{\partial^2 \mathrm{L}}{\partial \mathrm{x}\,\partial \mathrm{y}}\mathrm{dx}\wedge\mathrm{dx} + \mathrm{X}\sin^2 z\frac{\partial^2 \mathrm{L}}{\partial \mathrm{y}\,\partial \mathrm{y}}\mathrm{dy}\wedge\mathrm{dy} - \mathrm{Y}\sin^2 z\frac{\partial^2 \mathrm{L}}{\partial \mathrm{x}\,\partial \mathrm{x}}\mathrm{dx}\wedge\mathrm{dy} - \mathrm{Y}\sin^2 z\frac{\partial^2 \mathrm{L}}{\partial \mathrm{y}\,\partial \mathrm{y}}\mathrm{dy}\wedge\mathrm{dy} + \mathrm{Z}\sin^2 z\frac{\partial^2 \mathrm{L}}{\partial \mathrm{x}\,\partial \mathrm{z}}\mathrm{dx}\wedge\mathrm{dz} - \mathrm{Z}\sin^2 z\frac{\partial^2 \mathrm{L}}{\partial \mathrm{y}\,\partial \mathrm{z}}\mathrm{dy}\wedge\mathrm{dz}$$

(4.7)

Because of the closed Conformal 3 – dimensional Almost$\boldsymbol{f}$ −kenmotsu Manifoldsform ϕ_L on 3- Almost f-Cosymplectic Manifoldsspace form $(M^3, \phi. \xi, \eta, g)$ is para-symplectic structure, one may obtain

$$E_L = -X\sin^2 z\frac{\partial L}{\partial Y} + Y\sin^2 z\frac{\partial L}{\partial x} - L \qquad (4.8)$$

Considering (0,1) we calculate

$$dE_L = d\left(-X\sin^2 z\frac{\partial L}{\partial Y} + Y\sin^2 z\frac{\partial L}{\partial x} - L\right)$$

$$dE_L = -X\sin^2 z\frac{\partial^2 L}{\partial x\,\partial y}dx - Y\sin^2 z\frac{\partial^2 L}{\partial y\,\partial y}dx + X\sin^2 z\frac{\partial^2 L}{\partial x\,\partial x}dy - Y\sin^2 z\frac{\partial^2 L}{\partial y\,\partial x}dy + \frac{\partial L}{\partial x}dx - Z\sin^2 z\frac{\partial^2 L}{\partial x\,\partial z}dx - Z\sin^2 z\frac{\partial^2 L}{\partial y\,\partial z}dy - \frac{\partial L}{\partial y}dy \qquad (4.9)$$

Taking care of $i_\xi\phi_L = dE_L$,

$$X\sin^2 z\frac{\partial^2 L}{\partial x\,\partial y}dx\wedge dx + X\sin^2 z\frac{\partial^2 L}{\partial y\,\partial y}dy\wedge dy - Y\sin^2 z\frac{\partial^2 L}{\partial x\,\partial x}dx\wedge dy - Y\sin^2 z\frac{\partial^2 L}{\partial y\,\partial y}dy\wedge dy + Z\sin^2 z\frac{\partial^2 L}{\partial x\,\partial z}dx\wedge dz - Z\sin^2 z\frac{\partial^2 L}{\partial y\,\partial z}dy\wedge dz = X\sin^2 z\frac{\partial^2 L}{\partial x\,\partial y}dx - Y\sin^2 z\frac{\partial^2 L}{\partial y\,\partial y}dx + X\sin^2 z\frac{\partial^2 L}{\partial x\,\partial x}dy - Y\sin^2 z\frac{\partial^2 L}{\partial y\,\partial x}dy + \frac{\partial L}{\partial x}dx - Z\sin^2 z\frac{\partial^2 L}{\partial x\,\partial z}dx - Z\sin^2 z\frac{\partial^2 L}{\partial y\,\partial z}dy - \frac{\partial L}{\partial y}dy \qquad (4.10)$$

we have

$$\sin^2 z\frac{\partial^2 L}{\partial x\,\partial y}dx + Y\sin^2 z\frac{\partial^2 L}{\partial y\,\partial y}dx + Z\sin^2 z\frac{\partial^2 L}{\partial y\,\partial z}dx - \frac{\partial L}{\partial x}dx + X\sin^2 z\frac{\partial^2 L}{\partial x\,\partial x}dy + Y\sin^2 z\frac{\partial^2 L}{\partial y\,\partial x}dy + Z\sin^2 z\frac{\partial^2 L}{\partial x\,\partial z}dy - \frac{\partial L}{\partial y}dy = 0 \qquad (4.11)$$

$$\frac{\partial L}{\partial y}\sin^2 z\left(X\frac{\partial}{\partial x}dx + y\frac{\partial}{\partial y}dx + Z\frac{\partial}{\partial z}dx\right) - \frac{\partial L}{\partial x}dx + \sin^2 z\frac{\partial L}{\partial x}\left(X\frac{\partial}{\partial x}dy + Y\frac{\partial}{\partial y}dy + Z\frac{\partial}{\partial z}dy\right) + \frac{\partial L}{\partial y}dy = 0 \qquad (4.12)$$

$$\frac{\partial L}{\partial y}\left(X\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + Z\frac{\partial}{\partial z}\right)dx - \frac{\partial L}{\partial x}dx + \frac{\partial L}{\partial x}\left(X\frac{\partial}{\partial x} + Y\frac{\partial}{\partial y} + Z\frac{\partial}{\partial z}\right)dy + \frac{\partial L}{\partial y}dy = 0$$

$$\left[\frac{\partial L}{\partial y}\sin^2 z\left(X\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + Z\frac{\partial}{\partial z}\right) + \frac{\partial L}{\partial x}\right]dx + \left[\frac{\partial L}{\partial x}\sin^2 z\left(X\frac{\partial}{\partial x} + Y\frac{\partial}{\partial y} + Z\frac{\partial}{\partial z}\right) - \frac{\partial L}{\partial y}\right]dy = 0 \qquad (4.13)$$

If the curve $\alpha : I \subset R \to M^3$ be integral curve of ξ,

$$\alpha = \frac{\partial}{\partial t} = X\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + Z\frac{\partial}{\partial z} \qquad (4.14)$$

Which satisfies

$$\left[\sin^2 z\frac{\partial}{\partial t}\frac{\partial L}{\partial y} + \frac{\partial L}{\partial x}\right]dx + \left[\sin^2 z\frac{\partial}{\partial t}\frac{\partial L}{\partial x} - \frac{\partial L}{\partial y}\right]dy = 0$$

it follows equations

$$\sin^2 z\frac{\partial}{\partial t}\frac{\partial L}{\partial y} + \frac{\partial L}{\partial x} = 0\ , \quad \sin^2 z\frac{\partial}{\partial t}\frac{\partial L}{\partial x} - \frac{\partial L}{\partial y} = 0 \qquad (4.15)$$

so-called Euler-Lagrange equations whose solutions are the paths of the semispray ξ on Conformal 3 – dimensional Almost $\boldsymbol{f}$ −kenmotsu

Manifoldsspace form $(M^3, \phi. \xi, \eta, g)$. Finally one may say that the triple $(M^3, \phi. \xi, \eta, g)$ is mechanical system on Conformal 3 – dimensional Almost$\boldsymbol{f}$ −kenmotsu Manifolds$(M^3, \phi. \xi, \eta, g)$.

5.Conformal Humiliation Mechanics Systems with Almost three-dimensional $\boldsymbol{f}$-kenmotsu Manifolds

In this section, we obtain conformalHumiliation Mechanics Systemsfor classical mechanics structured on momentum space $(M^3, \phi, \xi, \eta, g)$ that is three- dimensional tangent bundle of an m-dimensional configuration manifold$\mathcal{M}$

Definition 5.1

Let $X_H is$ unique vector field on H is Hamiltonian function and ϕ^* is dual of ϕ and ω from on T^*M and

$$i_{X_H} = dH \qquad (5.1)$$

Is called Hamiltonian dynamical equation

Definition 5.2

The kinetic energy $T = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$ and $P = m_i gh$ is Potential energy .then $L = T + P$ is called the Hamiltonian function.

Theorem 3. 3

Hamiltonian equations for Classical Mechanics are obtained on the distributions $\mathrm{HT}^*\mathcal{M}$ and $\mathrm{V\,T}^*\mathcal{M}$ of $\mathrm{T}^*\mathcal{M}$ is

$$\frac{\mathrm{dx}}{\mathrm{dt}} = -\sin^2 z \frac{\partial \mathrm{H}}{\partial \mathrm{y}} \quad , \frac{\mathrm{dy}}{\mathrm{dt}} = \sin^2 z \frac{\partial \mathrm{H}}{\partial \mathrm{x}} \qquad (5.2)$$

Proof

Suppose that an almost real structure, a Liouville form and a1-form on $\mathrm{T}^*\mathcal{M}$ are shown by Φ^*, λ and ω, respectively. Then we have

$$\omega = \frac{1}{2}(\mathrm{Xdx} + \mathrm{Ydy} + \mathrm{Zdz}) \qquad (5.3)$$

1-form on

Let ϕ^* be an almost product structure defined by λLiouville form determined by

$$\lambda = \phi^*(\omega) = \phi^*\left[\frac{1}{2}(\mathrm{Xdx} + \mathrm{Ydy} + \mathrm{Zdz})\right] \qquad (5.4)$$

$$\lambda = \phi^*(\omega) = \frac{1}{2}[\mathrm{X}\phi^*(\mathrm{dx}) + \mathrm{Y}\phi^*(\mathrm{dy}) + \mathrm{Z}\phi^*(\mathrm{dz})]$$

$$\lambda = \phi^*(\omega) = \frac{1}{2}[\mathrm{X}(-\sin^2 z\,\mathrm{dy}) + \mathrm{Y}(\sin^2 z\,\mathrm{dx}) + \mathrm{Z}(0)]$$

$$\lambda = \phi^*(\omega) = \frac{1}{2}[-\mathrm{X}\sin^2 z\,\mathrm{dy} + \mathrm{Y}\sin^2 z\,\mathrm{dx}] \qquad (5.5)$$

differential of λ

$$\varphi = -\mathrm{d}\lambda = -\mathrm{d}\left(\frac{1}{2}[-\mathrm{X}\sin^2 z\,\mathrm{dy} + \mathrm{Y}\sin^2 z\,\mathrm{dx}]\right)$$

It is known that ifφ is a closed 2- form on T^*M^3, then φ_H is also a Symplectic structure onT^*M^3

$\varphi = -\mathrm{d}\lambda = \sin^2 z\, \mathrm{dy}\wedge\mathrm{dx}.$ (5.6)

Let $(M^3, \phi.\xi, \eta, g)$three- dimensional f-kenmotsu Manifoldsform φ. Suppose that Hamiltonian vector field X_H associated to Hamiltonian energy H is given by

$$X_H = X\frac{\partial}{\partial x} + Y\frac{\partial}{\partial y} + Z\frac{\partial}{\partial z} \qquad (5.7)$$

Calculates a value X_H and φ

$$\varphi(Z_H) = (\mathrm{dy}\wedge\mathrm{dx})\left(X\frac{\partial}{\partial x} + Y\frac{\partial}{\partial y} + Z\frac{\partial}{\partial z}\right)$$

$$\varphi(Z_H) = \sin^2 z\,(\mathrm{dy}\wedge\mathrm{dx})\left(x\frac{\partial}{\partial x}\right) - \sin^2 z\,\mathrm{dy}\wedge\mathrm{dx}\left(y\frac{\partial}{\partial y}\right) + \sin^2 z\,\mathrm{dy}\wedge\mathrm{dx}\left(z\frac{\partial}{\partial z}\right)$$

$$\varphi(Z_H) = X\sin^2 z\frac{\partial}{\partial y} - Y\sin^2 z\frac{\partial}{\partial y} + Y\sin^2 z\frac{\partial}{\partial x} + X\sin^2 z\frac{\partial}{\partial x} - Z\sin^2 z\frac{\partial}{\partial x} + Z\sin^2 z\frac{\partial}{\partial y} \qquad (5.8)$$

Otherwise, one may calculate the differential of Hamiltonian energy as follows:

$$\mathrm{dH} = \frac{\partial H}{\partial x}\mathrm{dx} + \frac{\partial H}{\partial y}\mathrm{dy} + \frac{\partial H}{\partial z}\mathrm{dz} \qquad (5.9)$$

From (5.6) and (5.7) with respect to $i_{X_H}\varphi = \mathrm{dH}$,

$$X\sin^2 z\frac{\partial}{\partial y} - Y\sin^2 z\frac{\partial}{\partial y} + Y\sin^2 z\frac{\partial}{\partial x} + X\sin^2 z\frac{\partial}{\partial x} - Z\sin^2 z\frac{\partial}{\partial x} + Z\sin^2 z\frac{\partial}{\partial y} = \frac{\partial H}{\partial x}\mathrm{dx} + \frac{\partial H}{\partial y}\mathrm{dy} + \frac{\partial H}{\partial z}\mathrm{dz}$$

So we find Hamiltonian vector field on 3- dimensional f-kenmotsu Manifoldsspace be

$$X_H = \frac{\partial H}{\partial y}\frac{\partial}{\partial x} - \frac{\partial H}{\partial y}\frac{\partial}{\partial y} \qquad (5.10)$$

Suppose that the curve

$\alpha: I \subset R \to R_n^{2n}$

be an integral curve of Hamiltonian vector field Z_H, i.e.,

$Z_H(\alpha(t)) = \dot{\alpha} \quad , t \in I.$

In the local coordinates we have

$\alpha(t) = (x(t), y(t), z(t)),$

$$\dot{\alpha}(t) = \frac{dx}{dt}\frac{\partial}{\partial x} + \frac{dy}{dt}\frac{\partial}{\partial y} + \frac{dz}{dt}\frac{\partial}{\partial z} \qquad (5.11)$$

Now, by means of (5.8), from (5.10) and (5.11), we deduce the equations so-called Hamiltonian equations

$$\frac{dx}{dt} = \sin^2 z\frac{\partial H}{\partial y} \quad , \frac{dy}{dt} = -\sin^2 z\frac{\partial H}{\partial x} \qquad (5.12)$$

6. Conclusion:

Eventually, we could say the mechanical system $(M^3, \phi. \xi, \eta, g)$ triple on Conformal 3- three- dimensional f-kenmotsu Manifolds.

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