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DEMONSTRATION

OF

The Christian Goldbach Conjecture:

Any even integer greater than 3 can be written as the sum of two prime numbers.

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I.Introduction

In mathematics, we callguess (1), a hypothesis that can be verified to be true on many examples and which has never been proven. In mathematics there are several conjectures including the Goldbach conjecture discovered by the mathematician Christian Goldbach (2). In a letter addressed toLeonhard Euler (3) in 1742 Goldbach submitted his conjecture. Many mathematicians have tried and are still trying to explain it, but no one has yet succeeded.

II.Goldbach's hypothesis

Goldbach's conjecture is the mathematical assertion which is stated as follows:

Any even integer greater than 3 can be written as the sum of twoprime numbers (4)

Formulated in 1742 by Christian Goldbach, it is one of the oldest unresolved problems of number theory (5) and mathematics.

III.Origin

On June 7, 1742, the mathematicianChristian Goldbach (1) written to the Swiss mathematician Leonhard Euler (2) aletter (a) at the end of which he proposes the following conjecture:

Any number strictly greater than 2 can be written as a sum of three prime numbers.

(Goldbach admitted 1 as a prime number; the modern conjecture excludes 1, and therefore replaces 2 by 5) (6)

In his response dated June 30, 1742, Euler reminds Goldbach that this statement stems from a previous statement (7) which Goldbach had already communicated to him:

Any even number can be written as the sum of two prime numbers.

(As before, "number" is to be taken in the sense of "integer strictly greater than 0" and the modern conjecture replaces 0 by 2.)

IV.Digital checks

In 2014, the digital verifications published lead to the following conclusions:

- Goldbach's conjecture holds for all even integers up to**4.10^{18}**(Tomás Oliveira e Silva, Siegfried Herzog and Silvio Pardi) (8)

v.Demonstration of the Conjecture

Hypothesis A: "Any even whole number greater than Three (3), can be written as the sum of two prime numbers"

That's to say :∀n ≥ 2, n∈NOT,∃p-q∈P / 2n=p + q

- For n = 2 we have 2n = 2 x 2 = 4 = 2 + 2 and 2 is prime

  So hypothesis A is True

- **It remains to show that:**

Hypothesis B: "∀n > 2; not∈NOT,∃p-q∈P / 2n=p + q"

P: it is the set of prime numbers N: it is the set of natural numbers

Demonstration by the Absurd (9)

Suppose hypothesis B is false

SO :**∃n > 2, n∈NOT /∀p-q∈P/2n ≠ p+**q

Now in N, 2n ≠ p + q⇒2n > p + q or 2n < p + q

- **Case No. 1: 2 n > p + q**

(2n > p + q,∀p-q∈P)⇒ (0<p+q<2n,∀p-q∈P)

However, according to the properties of prime numbers, we have :(∀ n ∈ N n>1, ∃ p∈ P / n <p<2n) (((Postulate of Bertrand(10)(1845) demonstrated by Chebyshev(11)(1852) then by Paul Erdös (12)(1932)))(b)

So we can say that**:(∀n > 2, n∈NOT,∃R∈P / 2n<R<4n) and we have (2 n > p + q for∀p-q ∈P)**

If takes some: p=R then we have: (2n > R + q >R and 2n < R)<u>what is contradictory</u> and therefore wrong<u>hypothesis B is false</u>

- <u>**Case N° 2: 2n < p + q for∀p-q∈P**</u>
  - **So for p=q=2 so: 2n < 2+2⇒ 2n < 4 ⇒ (n<2 <u>what is</u> and on 'an > 2),<u>This contradictory</u>**
  - **For p=2 and∀q∈P**

    We therefore have: 2<2n<2+q for∀q ∈ P

So for q=3 we have: 2<2n<2+3⇒: 2<2n<5 ⇒ 2n=4 ⇒ n=2 and n>2<u>what is contradictory</u>

- **In the properties of prime numbers we have: (∀p-q∈P, p>2, q >2, p+q is even)**

So: (2<2n<p+q and p+q is even): (2<2n < p + q) ⇒ (2<n<p+q/2) ⇒ (1<n<p+q/2 for ∀ p, q ∈ P)

So for p=3 and q=3 we have: 2<n<3+3/2⇒ 1<n<6/2 ⇒ 1<n<3 ⇒ (n=2 and n >2)<u>which is contradictory</u>

<u>**So: in all cases, what we assumed is false therefore hypothesis B is false therefore hypothesis A is always varies**</u>

- <u>**Conclusion**</u>

- <u>**For n=2 we have hypothesis A is true**</u>
- <u>**For n > 2 we have hypothesis A is true**</u>

SO :∀n ≥ 2, n∈NOT,∃p-q∈P / 2n=p + q

VI. Bibliography and References

This document is partially taken from the Wikipedia article in English entitled "Goldbach's conjecture" except the demonstration that Is made by myself

(a): Letter from 1742 to Euler in which Goldbach introduces his conjecture, at the end of the addition in the margin: “Es scheinet wenigstens, daß eine jede Zahl, die größer ist als 2, ein aggregatum trium numerorum primorum sey”.

(1): In mathematics, a conjecture is an assertion for which we do not yet know a demonstration, but which we strongly believe to be true (in the absence of a counterexample, or as a generalization of demonstrated results)

(2): Christian Goldbach: (born March 18, 1690 in Königsberg, Duchy of Prussia, died November 20, 1764) was a German mathematician. He is best known for the conjecture that bears his name.

(3): Leonhard Euler : born April 15, 1707 in Basel (Switzerland) and died September 7, 1783 (September 18 in the Gregorian calendar) in Saint Petersburg, he was a Swiss mathematician and physicist, who spent most of his life in the Russian Empire and Germany. He was notably a member of the Royal Prussian Academy of Sciences in Berlin.

(4): A prime number is a natural number which admits exactly two distinct integer and positive divisors which 1 and itself

(5): Traditionally, number theory is a branch of mathematics that deals with the properties of integers (whether natural or relative). More generally, the field of study of this theory concerns a large class of problems which arise naturally from the study of integers. Number theory occupies a special place in mathematics, both through its connections with many other fields, and through the fascination exerted by its theorems and its open problems, the statements of which are often easy to understand, even for non-mathematicians.

(6): But with this replacement, the modern guess is a bit stronger than the original

(7): In fact, the two conjectures are equivalent: if any even number greater than 2 can be written as the sum of three primes, one of them is necessarily 2, and then any even number greater than 0 can be written as the sum of two primes. Note that Euler presents his version to Goldbach as the one received from him: "...so Ew. vormals mit mir communicirt haben, dass nehmlich ein jeder numerus par eine summa duorum numerorum primorum sey...", Letter XLIV [archive].

(8):(en) Tomás Oliveira e Silva, Siegfried Herzog and Silvio Pardi, "Empirical verification of the even Goldbach conjecture and computation of prime gaps up to 4.1018", Math. Comp., vol. 83, 2014, p. 2033-2060 (DOI 10.1090/S0025-5718-2013-02787-1, read online [archive]).

(9): In mathematics, a reductio ad absurdum consists in demonstrating that the veracity of a hypothesis would lead to a contradiction, which leads to its rejection. Absurd reasoning is formal in mathematical terms and perfectly rigorous (at least in classical logic); it is absolutely not, as the ordinary meaning of the terms would suggest, meaningless reasoning. In practice, to prove the proposition "P", reasoning by reductio ad absurdum works by temporarily assuming that "not-P" is true, and then demonstrating that it leads to an impossible or contradictory conclusion. This then demonstrates that "P" is true.

(b): Inmathematics , THE**postulate ofBertrand** (10)affirms that between aentire and its double, there is always at least onePrime number .

More precisely, the usual statement is the following:For any integer n >1, there exists a prime number p such that: n<p<2n

Bertrand's postulate is also known as**Chebyshev's theorem)** , sincePafnuti Chebyshev (11) demonstrated this in1850

(10) :Joseph Louis Francois Bertrand , born March 11, 1822 in Paris and died April 3, 1900 in Paris 6th, was a French mathematician, economist and historian of science.

(11) :Pafnuti Lvovich Chebyshev (in Russian: Пафнутий Львович Чебышёв), born on May 4, 1821 (May 16 in the Gregorian calendar) in Okatovo, near Borovsk, and died on November 26, 1894 (December 8 in the Gregorian calendar) in Saint Petersburg, is a Russian mathematician. His name was first transcribed in French Tchebychef 1 and the form Tchebycheff is also used in French2. It is also transcribed Tschebyschef or Tschebyscheff (German forms), Chebyshov or Chebyshev3 (Anglo-Saxon forms). He is known for his work in the fields of probability, statistics, and number theory.

(12) : Paul Erdős, born Pál Erdős (/ p̍aː l ˈɛrdøːʃ/1) on March 26, 1913 in Budapest and died on September 20, 1996 in Warsaw, was a Hungarian mathematician. He is famous for his eccentricity, and the large number of his scientific publications and its collaborators. His abundant work gave rise to the concept of Erdős number representing the degree of separation (in terms of successive collaborations) between a given researcher and the Hungarian mathematician.

Done in Agadir on 08/28/2023 by Mr Senhaji Mostafa

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