GSJ: Volume 13, Issue 4, April 2025, Online: ISSN 2320-9186

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DEVELOPMENT OF LESSON EXEMPLAR – BASED DESIGN IN PRECALCULUS

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April 2025

APPROVAL SHEET

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ABSTRACT

LEUTERIO, JOSE JR. A. April 2025. **DEVELOPMENT OF LESSON EXEMPLAR BASED – DESIGN IN PRECALCULS**, MSE Major in Mathematics, Ilocos Sur Polytechnic State College, Graduate School, Sta. Maria, Ilocos Sur.

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CHAPTER I

INTRODUCTION

Background of the Study

Mathematics is the fundamental foundation upon which the pursuit of knowledge stands. It goes beyond specific disciplines; rather, it acts as a universal language that unifies and fills in gaps between various fields to drive innovation and progress. From its applications in daily life to its crucial role in shaping the future of technology and science, mathematics remains indispensable.

In engineering, mathematics serves as the backbone of all disciplines. It aids in analyzing, designing, and constructing systems, structures, and machines. According to Stewart (2019), crucial mathematical tools such as calculus, algebra, and differential equations enable engineers to create optimal and safe designs while addressing practical, real-world challenges. Similarly, in the sciences, mathematics plays a vital role in conducting experiments, evaluating complex data, and modeling natural phenomena. Devlin (2021) emphasizes that statistical analysis, geometry, and numerical modeling help establish and refine patterns, hypotheses, and predictions regarding the physical and biological world.

In the field of commerce and business, mathematics is essential for effective financial planning, market research, budgeting, and decision-making. Lay (2020) states that businesses rely on mathematical tools to analyze trends, forecast future performance, and allocate resources efficiently. From computing profit margins to developing pricing strategies and assessing risks, mathematics ensures accuracy and rationality in economic decisions.

Mathematics extends beyond these domains into information technology, where algorithms and data structures depend heavily on logic, a key mathematical branch. Hardy (2018) points out that mathematical models are crucial in predicting the behavior of diseases and improving medical treatments. Even the arts integrate mathematical principles, as seen in architectural designs, music theory, and digital animations. Additionally, Van Gerven et al. (2022) highlights that logical reasoning is a core element of computational thinking and problem-solving in information technology. They found a significant correlation between the development of logical thinking skills and enhanced computational problem-solving abilities. This suggests that improving logical reasoning directly impacts one's ability to tackle IT challenges effectively.

In medicine, Gaythorpe et al. (2024) emphasizes the essential role of mathematics, particularly in diagnostic imaging, understanding biological systems, and modeling disease spread. Their research shows that mathematical models are crucial in analyzing disease transmission patterns and simulating interventions to guide public health decisions. Additionally, Chukwu (2024) discusses the application of mathematical modeling in analyzing neurodegenerative diseases and mood disorders, further demonstrating the power of mathematics in unraveling complex biological processes and informing medical treatments

Furthermore, mathematics fosters critical thinking and logical reasoning, essential skills for addressing pressing global issues such as climate change, sustainable development, and technological advancements. In today's interconnected world, mathematical literacy is essential for navigating the complexities of a data-driven society. As highlighted by the Mathematical Association of America (2021), the importance of data literacy for all is paramount in a data-driven world. Similarly, the OECD (2021) defines mathematical literacy as an individual's capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts. These perspectives underscore the critical role of mathematical literacy in making informed decisions and judgments in our increasingly complex and data-centric society.

Ultimately, mathematics equips individuals and societies to explore possibilities. It acts as a universal framework that unites disparate disciplines and fosters a spirit of exploration and discovery. The National Academy of Engineering (2017) emphasizes that a strong grasp of mathematics is not just an academic necessity but a crucial tool in building a prosperous and sustainable future.

A study by Hanushek and Woessmann (2015) highlights the impact of cognitive skills on national economic success, with international student assessments in mathematics correlating significantly with economic development. Their extensive empirical evidence suggests that higher educational quality, particularly in mathematics, boosts economic growth. However, the Program for International Student Assessment (PISA) results indicate poor student performance in mathematics, with many failing to meet basic proficiency levels (OECD, 2019). This is particularly evident in developing countries where educational resources and quality education are lacking. The relationship between economic growth and the development of STEM (Science, Technology, Engineering, and Mathematics) fields is well-documented, as insufficient development in these areas can hinder innovation and productivity. According to the World Bank (2020), investments in STEM education are vital for economic growth, driving technological advances, and enhancing global competitiveness.

The National Council of Teachers of Mathematics (2000) emphasizes that precalculus is not optional but a prerequisite for Differential and Integral Calculus and other advanced mathematics courses. Precalculus includes concepts such as functions, complex numbers, conic sections, and vectors—key components of trigonometry. Mastering these concepts is crucial for STEM students. However, due to the complexity of precalculus, students require effective teaching strategies to enhance comprehension and retention. Blanton et al. (2021) stress the importance of a strong mathematical foundation, noting that early preparation in algebra and precalculus is critical for success in advanced STEM courses. Zelkowski (2019) discusses innovative teaching methods, such as the use of technology to simulate complex problems, flipped classrooms, and differentiated instruction to meet diverse learning needs.

Research suggests that innovative teaching strategies significantly improve student engagement and comprehension in precalculus. Boaler (2019) and Peters-Burton & Johnson (2020) highlight the benefits of active learning techniques, which involve engaging students in problem-solving and discussions. Freeman et al. (2014) found that these methods enhance performance in science, engineering, and mathematics courses. Problem-based learning (PBL) and flipped classrooms are particularly effective, as PBL presents real-world problems for students to solve, fostering critical thinking and application of knowledge. Meanwhile, flipped classrooms allow students to engage with instructional content outside of class, freeing up classroom time for interactive problem-solving.

Agarwal and Bain (2019) emphasize that technology-enhanced teaching caters to different learning styles and enables personalized instruction. Geiger (2020) highlights how technology facilitates interactive learning, benefiting mathematics education. The integration of digital tools, such as dynamic geometry software and online collaborative platforms, provides engaging ways for students to explore mathematical concepts deeply.

The evolving pedagogy of precalculus aligns with broader educational trends favoring student engagement and active learning. Educators must implement effective strategies to address challenges in teaching complex mathematical concepts, leading to improved student outcomes.

The National Council of Teachers of Mathematics (2000) underscores the importance of precalculus in preparing students for advanced courses and STEM careers. Recent studies reinforce the value of lesson exemplar-based instructional strategies in mathematics education. Park et al. (2019) found that lesson exemplar-based designs enhance students' comprehension of complex mathematical concepts by providing clear examples and step-by-step solutions. These strategies improve problem-solving skills and conceptual knowledge.

Fuchs et al. (2020) provide evidence that lesson-exemplar-based instructional designs not only enhance academic performance but also promote deeper conceptual understanding and long-term retention of mathematical principles. These designs support diverse learners, making them particularly effective in inclusive classrooms.

Despite the importance of precalculus, there is a shortage of validated lesson exemplar-based designs proven to improve student understanding and participation. Kirschner, Sweller, and Clark (2006) argue that guided instruction is more effective for complex tasks, and well-structured lessons ensure transferable learning. They caution against minimally guided instruction, emphasizing the need for validated instructional materials. Smith and Jones

(2015) found that limited access to textbooks and supplementary resources affects mathematics instruction, particularly in precalculus. The National Mathematics Foundation (2018) similarly reports an unequal distribution of resources, highlighting the lack of teaching materials for advanced courses.

In the Philippines, a shortage of validated lesson plans remains a challenge, particularly in precalculus. Many teachers struggle to access high-quality lesson plans tailored to their needs, weakening instructional effectiveness. According to a report by the Department of Education Tambayan, efforts are being made to standardize and improve lesson planning, but challenges persist due to time constraints and varying teacher expectations. Addressing this issue requires the immediate development and validation of high-quality lesson exemplar-based designs to enhance the teaching of complex subjects like precalculus.

Currently, in the Division of Candon City, no approved model of lesson exemplar-based design exists for precalculus. Recognizing this gap, the researcher aims to develop and validate instructional materials for STEM Senior High School teachers to improve precalculus instruction effectively.

Thus, this study seeks to determine students' proficiency levels in precalculus and address instructional gaps through the development and validation of lesson exemplar-based designs. By systematically designing, implementing, and evaluating these materials, the study aims to equip educators with evidence-based strategies to enhance student understanding, engagement, and achievement in precalculus. Additionally, the study intends to create lesson plans incorporating best practices in mathematics education and provide widely adaptable instructional materials to improve student learning outcomes. Ultimately, this research aims to contribute to the advancement of precalculus instruction and support the academic success of STEM students. It can also serve as a benchmark for future studies and enrich the body of literature on mathematics education.

Framework of the Study

The following theories and concepts provide a clearer perspective of this study.

Theoretical Framework

Constructivist Learning Theory:

Constructivist Learning Theory suggests that students can create meaning and knowledge by interacting and engaging with the real world around them. According to Jonassen (1999), he emphasized that building students' understanding of precalculus concepts involves engaging them in solving authentic mathematical problems, linking new ideas with existing knowledge structures, and reflecting on their thinking processes. As cited by Fosnot and Perry (2019), they discuss the role of constructivist learning environments in fostering deep understanding and critical thinking in mathematics. Similarly, Niemi, Kumpulainen, and Lipponen (2020) demonstrated that constructivist strategies, such as problem-based learning and collaborative inquiry, significantly enhance students' conceptual understanding and engagement in pre-calculus.

Based on these principles, the lesson plan will also be based on the following constructivist norms aimed at encouraging students' engagement, a deep understanding of the material, and meaningful learning:

Prior Knowledge and Schema Activation: Constructivist approaches in education involve the activation of students' preexisting knowledge and experiences to facilitate the building of new knowledge. By establishing connections between novel information and students' existing knowledge, instructors can enhance the development of profound comprehension and foster meaningful learning encounters.

Authentic Tasks and Real-World Contexts: Constructivist learning settings provide students with authentic work and real-life situations that are important and relevant to their lives. Through participating in activities that simulate real-world problem-solving scenarios, students can see the significance of their learning and develop a deeper understanding of concepts.

Hands-On Exploration and Discovery: Constructivism emphasizes hands-on exploration and discovery as central to the learning process. Students are encouraged to actively explore materials, manipulate objects, and conduct experiments to construct their understanding of concepts. This approach fosters engagement, curiosity, and a sense of ownership over learning.

Scaffolding and Social Interaction: Constructivist pedagogy involves providing scaffolding and support to help students build their understanding gradually. Educators guide students through the learning process, offering support as needed while gradually fading assistance to promote independence. Additionally, social interaction with peers and teachers is integral to constructivist learning, as students engage in collaborative discussions, share ideas, and negotiate meaning together.

Reflection and Metacognition: Constructivist approaches encourage reflection and metacognition, enabling students to monitor their learning process and think critically about their thinking. By reflecting on their experiences, students can identify misconceptions, make connections between concepts, and develop strategies for problem-solving and self-regulation.

Constructive Feedback and Assessment: Constructivist classrooms provide constructive feedback and assessments that focus on the process of learning rather than just the final product. Feedback is aimed at helping students understand their strengths and areas for improvement, guiding them in refining their understanding and skills over time.

Principles of Experiential Learning:

According to Kolb's 1984 theory of experiential learning, lesson plans and their content must be based on Concrete experiences, reflective observation, abstract conceptualization, and active experimentation must be done. As a critical initiative, students will then be able to take part in hands-on experiences, practical and real-world applications, and engage in the reflective and feedback process as per Kolb’s cycle in the development of internal understanding of the pre-calculus concepts.

Furthermore, as cited by Poore, Cullen, and Schaar (2014), applying Kolb's Experiential Learning Theory to enhance simulation-based interprofessional education. The research emphasizes that to improve learning outcomes, educational activities must incorporate Kolb’s four-stage learning cycle: concrete experiences, reflective observation, abstract conceptualization, and active experimentation. By engaging in hands-on simulations, reflective practice, conceptual discussions, and experimental activities, students can develop a deeper understanding and practical skills in a collaborative educational setting.

Additionally, Kolb and Kolb (2017), provide an in-depth exploration of how Experiential Learning Theory (ELT) can be applied to higher education. They offer a comprehensive framework for integrating ELT into curriculum design and instructional practices, emphasizing the importance of engaging students through concrete experiences, reflective observation, abstract conceptualization, and active experimentation. This approach aims to foster deeper learning and practical skill development in higher education settings.

Differentiated Instruction:

The design of the lesson model should also cater to the different needs and learning preferences of all students. As cited by Pozas and Schneider (2019), highlight the importance of differentiated assessment, which makes assessments of the varied strengths and learning methods of students, thus promoting more effective learning outcomes. This approach includes the use of ongoing assessments to adjust instructional strategies dynamically, ensuring that each student's needs are met throughout the course. Furthermore, Van Geel et al. (2019), discuss the necessity of ongoing training for teachers to effectively implement DI, demonstrating that successful DI requires continuous assessment and adaptation to student feedback. Their study indicates that well-implemented DI can significantly enhance student engagement and learning by addressing individual differences in readiness and interest. Lastly, Moallemi (2023), examines the relationship between DI and student engagement at the university level, finding that DI can significantly enhance student engagement and motivation. The study suggests that when teachers respond to individual learner differences, students feel valued and are more likely to engage deeply with the material.

Formative Assessment Practices:

According to Black and Wiliam (1998), formative assessment is paramount in the lesson plan's exemplar-based design it utilizes various ongoing formative assessments like quizzes, peer evaluations, and self-assessments to offer learners feedback about their progress and comprehension. As cited by Lipnevich et al. (2016), investigate the cognitive and emotional responses of students to feedback within the context of formative assessment. They highlight the significant impact that these responses can have on students' learning behaviors and overall outcomes. The research underscores that students' reactions to feedback, whether cognitive (such as understanding and processing the feedback) or emotional (such as feelings of motivation or frustration), play a crucial role in how effectively they engage with and benefit from the learning process. These findings point to the importance of providing clear, constructive feedback to foster positive learning experiences and outcomes. Furthermore, Van der Kleij et al. (2015) conducted a comprehensive systematic review examining the impact of formative assessment on learning outcomes. Their analysis underscores the importance of effective feedback mechanisms as an essential factor in enhancing student engagement and academic achievement. By synthesizing evidence from various studies, the authors illustrate how timely and constructive feedback can significantly influence students' motivation and their ability to assimilate and apply new knowledge. The review highlights that formative assessment when implemented with robust feedback strategies, serves not only as a tool for measuring student progress but also as a catalyst for fostering an interactive and dynamic learning environment.

Technology Integration:

According to Mishra & Koehler (2006), the use of technology-enhanced learning tools and resources significantly improves student engagement and outcomes. As cited by Zheng, Warschauer, Lin, and Chang (2016), supports this view, demonstrating that technology integration in classrooms fosters active learning and enhances academic performance. The design of these lessons incorporates online resources, graphing calculators, and dynamic interactive simulations, providing numerous opportunities for student participation. An exemplar-based pre-calculus lesson plan designed in this manner allows for visualization, exploration, and experimentation, thereby enabling students to develop a deeper understanding of mathematical concepts and achieve proficiency.

Conceptual Framework

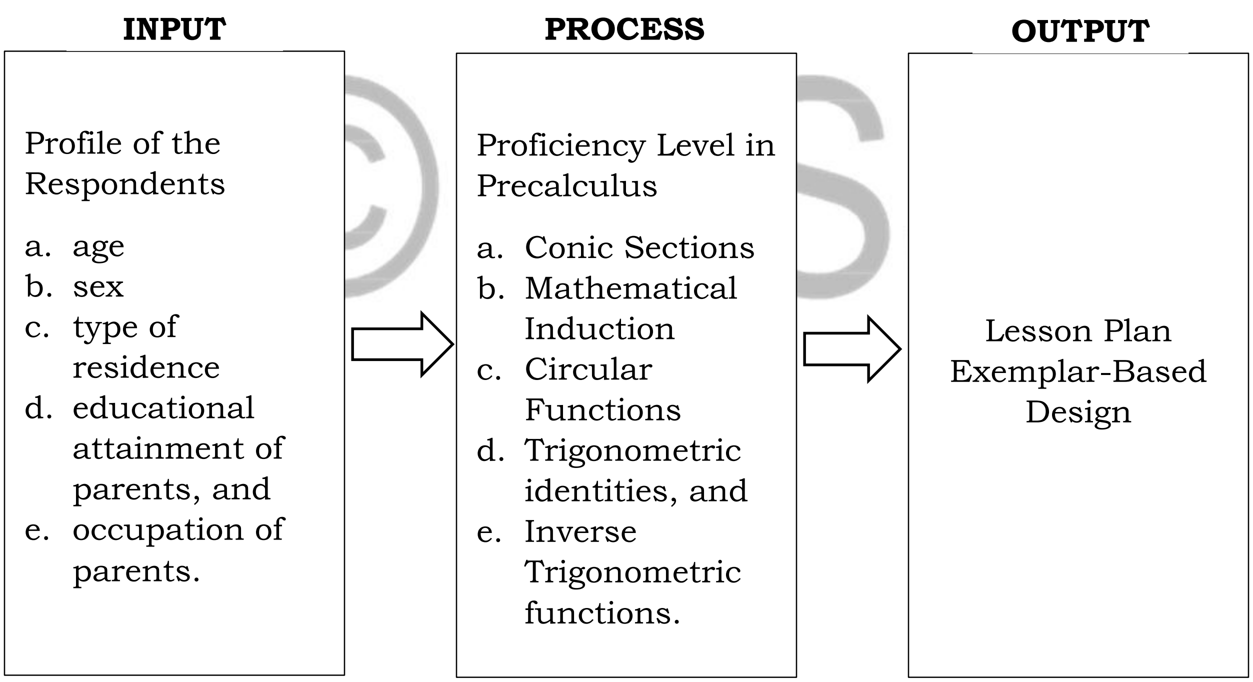

Figure 1. *Research Paradigm*

The figure above shows an Input – Process-Output (IPO) model reflects the research paradigm. The input involves the profile of the respondents in terms of their age, sex, type of residence, educational attainment of parents, and occupation of parents and their level of proficiency along Conic Sections, Mathematical Induction, Circular Functions, Trigonometric Identities, and Inverse Trigonometric Functions.

Moreover, the process includes analysis of the profile of the respondents, level of proficiency, correlation between the profile of the respondents and their level of proficiency, the level of difficulty of the topics covered in Precalculus, and the formulation and validation of lesson plan exemplar-based design.

Lastly, the output will present a validated and final copy of a Lesson Plan Exemplar-Based Design containing lessons and activities that address the different topics in the subject Precalculus.

Statement of the Problem

This study assessed the level of proficiency of the Grade 11 STEM students of Candon National High School for the Second Semester, School Year 2024 – 2025 along the subject Precalculus that led to the formulation and validation of the Lesson Plan Exemplar – Based Design.

1. What is the profile of the respondents along the following variables:
   a. age
   b. sex
   c. type of residence
   d. educational attainment of parents, and
   e. occupation of parents?

2. What is the level of proficiency of Grade 11 STEM students in Precalculus along:
    a. Conic Sections;
    b. Mathematical Induction;
    c. Circular Functions;
    d. Trigonometric identities and
    e. Inverse Trigonometric functions?
3. What is the level of difficulty of the students on the various topics covered in Precalculus?
4. Is there a significant relationship between the profile and the level of proficiency of the students?
5. What instructional material can be formulated that can be widely adopted by educators to improve student learning outcomes in precalculus?
6. What is the level of validity of the exemplar lesson–based design in precalculus?

Hypothesis

There is no significant relationship between the profile and the level of proficiency of the students in Precalculus.

Scope and Limitation of the Study

This study was limited to the proficiency of the Grade 11 STEM students of Candon National High School, Senior High School for School Year 2024 – 2025.

The profile of the respondents, level of proficiency among Conic sections, Mathematical Induction, Circular Functions, Trigonometric Identities, and Inverse Trigonometric Functions were gathered through a questionnaire. Other areas of concern dealt with the relationship between the profile and proficiency level of the students, and the level of difficulty on the various topics covered in Precalculus which was the basis in the formulation and validation of the Lesson Plan Exemplar – based design. The respondents of this study were the Grade 11 STEM students from the four sections of Candon National High School, Senior High School for the School Year 2024 – 2025.

To gather data, the researcher used a two-part questionnaire. Part 1 composed the profile of the respondents, while Part 2 was a 50-item test questionnaire constructed by the researcher and was validated by Precalculus and Mathematics Teachers. The items were equally divided into 5 main topics covered in Precalculus based on the Most Essential Learning Competencies by DepEd. The items were constructed based on the Table of Specifications by DepEd. Items 1 – 15 were taken from the topic Conic Sections; items 16 – 20 were from Mathematical Induction; items 21 – 30 were from Circular Functions; items 31 – 45 were from Trigonometric Identities and items 46 – 50 were from Inverse Trigonometric Functions.

Importance of the Study

This study was conducted to benefit the following:

Learners. Through this study, the learners were provided with learning opportunities and experiences developing their mathematics skills particularly the areas covered in Precalculus.

Mathematics Teachers. The results of this study helped them develop pedagogical interventions and differentiated instructional materials that may yield significant results in the performance of the students.

Administrators. The administrators are provided with research-based feedback which may help them design programs or activities that improve teachers' proficiency and efficiency in teaching the subject and help the students as well to give better performance.

Curriculum Planners. This study may help them plan a better curriculum suitable to the existing needs of the teachers and students.

Division of Candon City. The results contributed to institutions' increasing collection of scholarly books which is important in research and development.

Researcher. The researcher may share the results with others which may help them develop better strategies and methodologies in teaching the subject.

Future Researchers. This study may serve as their base for conducting related studies.

Definition of Terms

To ensure understanding, the following terms are defined operationally:

Precalculus. It refers to the discipline to which the proficiency of the learners is assessed such as Conic Sections, Series and Mathematical Induction, Circular Functions, Trigonometric Identities, and Trigonometric Functions.

Profile of the Respondents. This pertains to respondents' personal information gathered from the questionnaire.

Age. This refers to the length of existence of the respondents by the time this study was conducted.

Sex. This describes whether the respondent is male or female.

Type of Residence. This describes whether the respondents live in rural or urban areas.

Educational Attainment of Parents. This pertains to the highest level of education attained by both parents.

Occupation of Parents. This includes the primary job of the respondents' parents.

Proficiency in Precalculus. This pertains to the ability of the students to perform various tasks based on topics such as Conic Sections, Series and

Mathematical Induction, Circular Functions, Trigonometric Identities, and Trigonometric Functions.

Lesson Plan Exemplar–Based Design. It is an instructional material with a set of activities on a particular lesson based on the Most Essential Learning Competency provided by the Department of Education.

Validity. This refers to the acceptability of the lesson plan exemplar-based design where it measures what is intended for.

Review of Related Literature

The following studies are believed to be related and relevant to the present study. The following studies were carefully and analytically examined to suit the concepts covered in this research.

Profile of the Respondents

The profile of respondents in research is crucial as it provides context and depth to the findings (Creswell, 2014). Key variables such as age, sex, type of residence, educational attainment of parents, and occupation of parents often influence the outcomes and interpretations of studies (Bryman, 2016).

Age

Different elements shape one's behavior and perception such as age which is an important demographic factor. According to Smith & Baltes (2018, they show that aging has a direct correlation with students' performance in school where those at older ages tend to perform better academically due to their maturity levels and life experiences. Moreover, Littrell et al. (2021), stated that younger students exhibit higher engagement and learning outcomes when using technology-enhanced learning tools, while older learners often face challenges due to lower digital proficiency and comfort levels. Similarly, Teo et al. (2020), emphasized that generational differences play a significant role in the effectiveness of technology integration in educational settings, with digital natives showing a stronger inclination towards and benefit from interactive and multimedia learning resources.

Sex

Extensive studies have been conducted on how gender affects academic and occupational achievements. According to the American Association of University Women (AAUW, 2019), they highlight sex differences in STEM courses as more boys often take part compared to girls. Similarly, McDaniel (2020), affirms that female students generally outperform males when it comes to verbal and written communication skills.

Type of Residence

The type of residence, whether it is urban or rural, determines how many resources and educational opportunities are available to the residents. According to Kirtz and Davies (2017), students in urban areas usually have better access to educational resources and extra-curricular activities compared to their counterparts in rural areas. Similarly, a recent study by the National Center for Education Statistics (NCES, 2016), indicates that different levels of education between the urban and rural areas often result in diverging educational achievements as well as occupational opportunities. They also revealed that students from urban backgrounds tend to score higher marks in standardized exams than their counterparts in rural areas.

Educational Attainment of Parents

Parents' level of education highly determines children's success at school. According to a study by Davis-Kean (2015), children whose parents were more educated managed to perform well academically. This is because they learn better when they are nurtured lovingly and are subjected to high standards set by their parents. Additionally, Haveman and Wolfe (2014) found that there was a positive relationship between parental schooling years and both the cognitive growth as well as total educational achievement of children.

Occupation of Parents

In addition to this, it is also important to note that the occupation of parents plays a crucial role in shaping children's learning and career objectives. According to Blau and Duncan (2019), kids whose parents are professionals or managers are more likely to proceed to higher studies and get employed in occupations like their own. This is confirmed by the Social Mobility Commission (2018), which highlights that children from well-off backgrounds have more resources as well as networking opportunities that help shape their future educational endeavors.

Mathematics Proficiency

According to Watson (2023), found out that College calculus is essential for STEM students' academic and professional goals. Proficiency in subjects from earlier mathematics courses, such as algebra and precalculus, is one of the most important variables considered when evaluating a student's potential for success in calculus. They investigate the correlation between students' precalculus proficiency and their performance in introductory calculus, considering their classroom environment. They provide results that show the outcomes of implementing the Modeling Practices in Calculus (MPC) paradigm, which is an innovative and interactive method of learning. They also employ a randomized-controlled trial research methodology where students are randomly allocated to either MPC or standard, lecture-based calculus sessions. The administration of the Precalculus Concept Assessment inventory aimed to assess the level of precalculus ability among students. They revealed that students who were exposed to the MPC model demonstrated a higher likelihood of achieving success in their calculus courses, even if they initially had little ability in precalculus. Furthermore, students who were registered in the MPC sessions experienced substantial improvement in their precalculus skills from the start to the end of the semester. In addition, we saw that this approach helps students belonging to specific demographics in terms of enhancing their proficiency, which they may not receive in conventional classrooms.

According to Maharaj, et.al (2015), they stated that it is always a worry for teachers to comprehend how students interpret mathematics. A group of professors at a university in South Africa conducted a study on the written responses of first-year students. They investigated the mental conflicts that emerged while these students formulated solutions to assigned assignments. The mental tensions, as depicted by their reactions, were classified. They focus on the area of basic algebra. Upon identifying the prevailing conflicts, the instructors devised tasks that are accessible through websites. The tests were designed to address cognitive conflicts to eliminate them from students' cognition. The website tasks were created following the scaffolding principle. The researchers examined the different conflicts within the context of Kilpatrick's five dimensions of mathematical proficiency. The researcher examines the design of activities and interactive partnerships in tutorial sessions with students. They also discovered that the website content appeared to be effective in resolving typical mathematical difficulties experienced by certain undergraduate mathematics students. Furthermore, it was discovered that the usage of non-mathematical language could potentially lead to cognitive conflicts among students.

Cerbito (2020), evaluated the opinions and skill levels in Mathematics of Senior High School Students from various academic disciplines. They used The Attitudes Toward Mathematics Inventory (ATMI) to measure and determine the students' attitudes in terms of value, enjoyment, motivation, and self-confidence. Furthermore, they demonstrated a notable correlation between individuals' attitudes toward mathematics and their level of ability in the subject. Also, they found out that teacher educators should be mindful of the attitudes of Senior

High School students across many strands and strive to enhance them to have a favorable impact on students' mathematical proficiency.

Conic Sections

Conic sections are a fundamental topic in precalculus, encompassing the study of ellipses, parabolas, hyperbolas, and circles. Thompson (2017), indicates that many students struggle with the geometric interpretations and algebraic manipulations involved in conic sections. Furthermore, Thompson's study revealed that only 40% of students demonstrated proficiency in identifying and graphing various conic sections. The difficulties often stem from a lack of spatial reasoning skills and insufficient practice with real-world applications.

Series and Mathematical Induction

Series and mathematical induction are critical for understanding sequences and proving mathematical statements. According to Lee and Engelbrecht (2018), showed that students often find these concepts challenging due to their abstract nature. Only about 35% of students in their study could correctly apply mathematical induction to prove statements. Furthermore, the study highlighted that students who excelled in logical reasoning and had prior exposure to proof-based mathematics were more likely to achieve proficiency in this area.

Circular Functions

Circular functions, including sine, cosine, and tangent, are vital for understanding periodic phenomena. Martinez and Perez (2019), stated that students generally have a moderate level of proficiency with circular functions, with approximately 50% demonstrating a solid understanding. However, the main challenges identified were related to transitioning from right-angle trigonometry to the unit circle approach. Also, visualization tools and interactive learning resources were found to significantly enhance students' grasp of circular functions.

Trigonometric Identities

Trigonometric identities are essential for simplifying expressions and solving equations. Brown and Adams (2016), indicated that students often struggle with memorizing and applying these identities. They showed that only 30% of students were proficient in using trigonometric identities to simplify complex expressions. Furthermore, the study suggested that a more conceptual approach, rather than rote memorization, could improve students' proficiency in this area.

Trigonometric Functions

Understanding trigonometric functions is important for solving various mathematical problems. According to Johnson and Raymond (2017), students exhibit varying levels of proficiency in this area with approximately 45% of students demonstrating a good understanding of trigonometric functions, including their graphs and properties. Furthermore, the study highlighted the importance of linking trigonometric functions to real-world applications to enhance comprehension and retention.

Significant Relationships between the Profile of the Respondents and their Level of Proficiency in Precalculus

General Proficiency Levels

Moore (2016) has found out that levels of competency may differ significantly amongst students with a good proportion struggling with simple concepts such as functions, trigonometry, and limits. Furthermore, less than 45% of the sampled students mastered precalculus skills suggesting the substantial gaps must require attention to guarantee learners' preparedness for calculus and other advanced science courses in terms of skill development.

Influencing Factors

1. **Mathematical Background**

   Prior knowledge of foundation mathematical ideas is necessary for achieving success in precalculus. According to Hoffer (2017), precalculus performance is typically higher for students who have strong algebraic and geometric foundations. He also conducted longitudinal research on high school students and discovered that those who performed well in algebra and geometry classes had better levels of competency in precalculus and emphasized the significance of building upon mathematical knowledge over time.

2. **Instructional Methods**

The choice of instructional methods has a significant impact on students' proficiency of precalculus. According to a study conducted by Johnson and Kuennen (2018), the implementation of active learning strategies, such as collaborative problem-solving and the utilization of technology, has been found to improve students' comprehension and retention of precalculus topics. Furthermore, students in classrooms that used these strategies demonstrated a 20% increase in proficiency compared to students in typical lecture-based settings.

3. Student Attitudes and Motivation

The competence levels of students are significantly influenced by their attitudes and motivation. According to Tapia and Marsh (2016), they found that students who have a more positive attitude toward mathematics and a higher level of intrinsic motivation are more likely to become proficient in precalculus. Further, a strong correlation between students' self-efficacy in mathematics and their performance in precalculus, suggesting that fostering a positive mathematical mindset is essential.

4. Gender Differences

Gender differences in mathematical proficiency have been the subject of extensive research. According to Hyde and Mertz (2019), it suggests that there are no innate discrepancies in mathematical aptitude between genders. However, societal and educational influences can contribute to variances in performance. Further, they discovered that when given the same chances and support, female students achieved the same level of performance as their male peers in precalculus.

5. **Socioeconomic Status**

Socioeconomic status (SES) is another critical factor influencing proficiency in precalculus. According to Reardon (2016), there are differences in mathematical ability related to socioeconomic status (SES), with kids from higher SES families typically performing better since they have access to additional resources like advanced coursework and tutoring. Moreover, they recommend implementing focused interventions to assist students from low socioeconomic backgrounds in bridging the proficiency gap.

Level of Difficulty in Precalculus

According to research by Thompson (2017), students generally perform well on tasks involving the identification and basic properties of these shapes, as these are often introduced in earlier grades. Moreover, students tend to understand basic arithmetic and geometric series, especially when problems are straightforward and involve step-by-step progression. As cited by Lee and Engelbrecht (2018), it indicates that students are comfortable with calculating the sums of simple series and recognizing patterns. Furthermore, many students excel in understanding the unit circle and the basic definitions of sine, cosine, and tangent. Similarly, Martinez and Perez (2019) found that interactive tools and visual aids significantly enhance students' grasp of circular functions, enabling about 50% of students to accurately calculate and interpret these functions in various contexts. Also, students typically perform well with fundamental trigonometric identities such as the Pythagorean identities, which are often introduced with visual aids and straightforward proofs. Additionally, Brown and Adams (2016) found that about 40% of students could correctly use these basic identities in simple problems. Lastly, students generally show a good understanding of basic trigonometric functions and their applications in solving the right triangles. Ultimately, Johnson and Raymond (2017) noted that around 45% of students were proficient in using trigonometric functions to solve real-world problems and interpret their graphs.

However, students often struggle with more complex aspects of conic sections, such as the equations and deeper properties of ellipses and hyperbolas. As cited by Thompson (2017), he found that only 40% of students demonstrated proficiency in these areas. The difficulties are attributed to challenges in visualizing the shapes and translating geometric properties into algebraic equations. This indicates a need for more integrated teaching approaches that connect algebraic and geometric perspectives. Furthermore, students often find mathematical induction challenging due to its abstract nature and the logical rigor it requires. According to Lee and Engelbrecht (2018), they reported that only about 35% of students could correctly apply mathematical induction to prove statements. Students frequently struggle with the initial step of the induction process and maintain logical consistency throughout the proof.

Despite this, students often face difficulties when transitioning from right-angle trigonometry to the unit circle approach. As cited by Martinez and Perez (2019), they noted that students struggle with the abstract nature of radians and the periodic properties of circular functions. They also highlight the need for continuous reinforcement and the use of diverse teaching strategies to solidify understanding. In addition, the same study revealed significant challenges with more complex identities, such as sum and difference identities, product-to-sum, and multiple-angle identities. Students often resort to rote memorization without a deep understanding of the underlying principles, leading to difficulties in application. Brown and Adams (2016) suggest that more emphasis on conceptual learning and derivation of identities could help mitigate these issues. Lastly, challenges arise with inverse trigonometric functions and their properties. Johnson and Raymond (2017) reported that students often find it difficult to understand the restricted domains and ranges of these functions, which complicates their ability to solve equations involving inverse trigonometric functions and indicates a need for more targeted practice and conceptual explanations in this area.

Lesson Plan Exemplar–Based Design

Structured Lesson Plans

According to Johnson and Rampersad (2017), structured lesson plans with clear objectives, detailed instructions, and formative assessments are essential for effective precalculus teaching. However, teachers demonstrated that using detailed lesson plans observed significant improvements in student performance. Such plans ensure comprehensive coverage of content and provide opportunities for continuous assessment, feedback and enabling timely interventions to address learning gaps.

Active Learning Strategies

According to Freeman et al. (2014), stated that active learning strategies integrated into lesson plans, such as group work, peer teaching, and interactive problem-solving, have been shown to enhance student engagement and understanding. Additionally, active learning significantly increases the students' performance in STEM subjects, including precalculus, and these strategies will promote students' participation and foster deeper comprehension of mathematical concepts.

Differentiated Instruction

According to Tomlinson (2017), stated that differentiated instruction involves tailoring lesson plans to meet the diverse needs of students. He also emphasized the importance of adapting instructional materials and activities to accommodate different learning styles, abilities, and interests. By providing varied approaches and resources, differentiated instruction helps all students achieve proficiency, regardless of their starting point.

Impact on Proficiency in Precalculus

Enhanced Conceptual Understanding

According to Siegle and McCoach (2019), the use of diverse instructional materials and lesson plan exemplar-based enhances students' conceptual understanding in precalculus topics. Also, he found out that students taught using a variety of instructional materials, including visual aids and interactive tools, showed significant improvement in understanding complex concepts. These resources make abstract ideas more accessible and relatable.

Improved Problem-Solving Skills

According to Hiebert and Grouws (2007), stated that effective lesson plans that include problem-solving activities and real-world applications help students develop critical thinking skills. Furthermore, they noted that students are regularly exposed to challenging problems and encouraged to explore multiple solutions and improve their problem-solving abilities and overall mathematical proficiency.

Increased Student Engagement

According to Kogan and Laursen (2014), stated that engaging lesson plans that incorporate interactive and collaborative activities increase student motivation and interest in precalculus. Additionally, they found out that students in classrooms with high levels of active learning and engagement were more likely to participate, perform better in precalculus, and increased engagement leads to better retention and understanding of mathematical concepts.

CHAPTER II

METHODOLOGY

This chapter presents the research design, population and locale, research instruments, data gathering procedure, statistical treatment of data, data categorization, and ethical considerations that were used during the conduct of the study.

Research Design

This study will use a descriptive research design employing correlational and developmental approaches. Johnson and Christensen (2020) utilized a descriptive research design to systematically analyze demographic and educational data, yielding valuable insights into trends and patterns in student performance. By employing descriptive statistical methods, they were able to provide a detailed portrayal of various student populations, highlighting key factors that influence academic achievement and identifying significant disparities among different demographic groups. Similarly, Albrecht and Karabenick (2018) conducted a descriptive study to investigate the demographic characteristics and academic outcomes of students across diverse educational settings. The descriptive approach allowed them to map out the relationships between these variables and student achievement, offering a detailed picture of how demographic factors intersect with educational outcomes.

Additionally, the study will use correlational methods to determine the significant relationships of variables. According to Khan et al. (2020), they employed a correlational design to investigate the relationship between students' socioeconomic status and academic achievement, finding significant associations that provide insights into educational disparities. Similarly, Zhang and Zhang (2021) used correlational research to examine the link between students' engagement in digital learning activities and their academic performance, highlighting important trends that inform educational practices and policies.

Meanwhile, developmental research has been defined as the systematic study of designing, developing, and evaluating instructional programs, processes, and products that must meet criteria of internal consistency and effectiveness (Richey, 1994) as cited by Bongolan (2018). Furthermore, according to Richey & Klein (2014), development research is a type of inquiry unique to the instructional design and technology field dedicated to the creation of new knowledge and the validation of existing practice. Thus, this developmental research design was considered in the formulation of a lesson plan exemplar-based design for the subject Precalculus.

Population and Locale of the Study

The study's respondents are mainly composed of Grade 11 Science, Technology, Engineering, and Mathematics (STEM) students in Candon National High School with Precalculus subjects. They are the ones that fit the study's criteria since STEM students studied the said subject. The researchers chose Grade 11 STEM students since they are the most appropriate to achieve the study's goal, which is to determine the proficiency of Grade 11 STEM students in Pre-Calculus.

Since the total number of Grade 11 STEM Students in Candon National High School is less than 250, there is no need for sampling anymore. For this study, the researchers used total population sampling. Total population sampling is a type of purposive sampling technique that involves examining the entire population with a particular set of characteristics, attributes/traits, experience, knowledge, and skills.

The table shows the total population of students from Grade 11 STEM 1 to 4 of Candon National High School.

Table 1. Total Number of Students in each Grade 11 STEM Section

Grade and Section	Population
11 STEM 1	44
11 STEM 2	49
11 STEM 3	47
11 STEM 4	46
Total	**186**

Research Instrument

To further develop this study, the researcher used research made a two-part survey questionnaire. Part I used to elicit the demographic profile of the respondents such as age, sex, type of residence, educational attainment of parents, and occupation of parents.

Part II was a test questionnaire used to determine the level of proficiency of Grade 11 STEM students in Precalculus. It is composed of a 50-item teacher-constructed multiple-choice test covering the five main topics for the first semester which are: Conic Sections, Mathematical Induction, Circular Functions, Trigonometric Identities, and Inverse Trigonometric Functions to describe their proficiency level in Precalculus.

Validity and Reliability

To ensure the test's validity, the researcher had it evaluated by five experts, a Master Teacher II in teaching Precalculus subject, a Mathematics Coordinator teaching Precalculus and Basic Calculus in Private School, a Head Teacher VI in Mathematics Department, a Doctor in Mathematics expert in teaching Differential and Integral Calculus, and a Research Teacher/Mathematics Teacher to examine the effectiveness of the questions for collecting data. These validators were selected from Candon City, Ilocos Sur, Vigan City, Ilocos Sur, and San Fernando City, La Union. After validation, the researcher solved the validity index and came up with a computed grand mean of 4.64 interpreted as "Very High Validity".

GSJ: Volume 13, Issue 4, April 2025
ISSN 2320-9186

Table 2. Table of Validity

Statistical Range	Descriptive Equivalent Rating
4.21 – 5.00	Very High Validity (VHV)
3.41 – 4.20	High Validity (HV)
2.61 – 3.40	Moderate Validity (MV)
1.81 – 2.60	Low Validity (LV)
1.00 – 1.80	Very Low Validity (VLV)

Table 2.1. Summary of Validation

Indicators	V1	V2	V3	V4	V5
Format					
The data gathering tool is nearly prepared.	4	5	5	5	5
The data gathering tool is complete in terms of its necessary parts.	5	5	4	5	5
The tool gives clear instructions/directions.	5	5	5	5	5
Consistency/Appropriateness					
The questions/items agree with the overall objectives of the study.	5	5	4	5	5
The research tool is constructed based on various authoritative sources.	4	4	4	5	5
Purpose					
The questions have a coherent objective. The questions, once taken together, bring out a holistic assessment of the context.	4	5	4	5	4
The questions are so structured that they can aid later in the overall interpretation of the data.	4	5	5	5	4
Sentence Structure					
The questions are free from any errors in syntax and grammar.	4	4	4	4	5
The words used are appropriate for the respondent's level of understanding.	5	5	5	5	4
Mean	**4.44**	**4.78**	**4.44**	**4.89**	**4.67**
Grand Mean	**4.64 (Very High Validity)**				

Additionally, to ensure the tests' reliability, pilot testing was carried out with 30 Grade 11 students selected randomly from Saint Joseph Institute, Inc.

as participants. The reliability coefficient, calculated using Cornbach's alpha, yielded a score of *0.726*, indicating a very high level of reliability. This means that the validated research instrument can elicit dependable and consistent results.

Table 3. Table of Reliability

Statistical Range	Descriptive Equivalent Rating
1	Perfect Reliability
0.71 – 0.99	Very High Reliability
0.51 – 0.70	High Reliability
0.21 – 0.50	Low Reliability
0.01 – 0.20	Negligible Reliability
0.00	No Reliability

Statistical Treatment of Data

The Statistical Package for Social Science (SPSS) was utilized in the statistical analysis and treatment of data gathered. Specifically,

Frequency count (*f*) and **Percentage (%)** were used to describe the profile of the respondents.

Arithmetic Mean was utilized to describe the level of proficiency of the students in Precalculus and the level of acceptability of the lesson plan exemplar-based design.

In addition, the following formula was used to compute the difficulty index of the test items.

$$D = \frac{students\ with\ correct\ answer}{total\ number\ of\ students}$$

Pearson *r* Correlation Analysis was used to determine the significant relationship between the profile and level of proficiency of the students.

Data Categorization

To determine the level of proficiency of the students, the following Likert scale was used in data categorizations for every subtopic:

Table 4. Conic Sections, and Trigonometric Identities

Statistical Range	Descriptive Equivalent Rating
12.01 – 15.00	Very High (VH)
9.01 – 12.00	High (H)
6.01 – 9.00	Moderate (M)
3.01 – 6.00	Low (L)
0.00 – 3.00	Very Low (VL)

Table 4.1. Mathematical Induction, and Inverse Trigonometric Functions

Statistical Range	Descriptive Equivalent Rating
4.01 – 5.00	Very High (VH)
3.01 – 4.00	High (H)
2.01 – 3.00	Moderate (M)
1.01 – 2.00	Low (L)
0.00 – 1.00	Very Low (VL)

Table 4.2. Circular Functions

Statistical Range	Descriptive Equivalent Rating
8.01 – 10.00	Very High (VH)
6.01 – 8.00	High (H)
4.01 – 6.00	Moderate (M)
2.01 – 4.00	Low (L)
0.00 – 2.00	Very Low (VL)

To determine the level of difficulty on the various topics, the 1-point Likert scale was used:

Table 5. Table of Difficulty Level

Statistical Range	Descriptive Equivalent Rating
0.81 – 1.00	Very Easy (VE)
0.61 – 0.80	Easy (E)
0.41 – 0.60	Moderate Difficult (MD)
0.21 – 0.40	Difficult (D)
0.00 – 0.20	Very Difficult (VD)

To validate the lesson plan exemplar–based design, the 5-point Likert scale was used:

Table 6. Table of Validity

Statistical Range	Descriptive Equivalent Rating
4.21 – 5.00	Very High Validity (VHV)
3.41 – 4.20	High Validity (HV)
2.61 – 3.40	Moderate Validity (MV)
1.81 – 2.60	Low Validity (LV)
1.00 – 1.80	Very Low Validity (VLV)

Ethical Considerations

To substantiate and guarantee ethical conduct in the process of this research, the researchers will certainly observe the following:

The researcher will ensure that participants are fully informed about the purpose, procedures, potential risks, and benefits of the research before they agree to participate.

The researchers will be after the respondents' responses, and it was an assurance that they had never been emotionally and physically harmed just to be participants in this study.

The researchers will observe the confidentiality of the respondents' responses on the demographic profile.

Accurate and proper document sourcing or referencing of materials used in the study will be done to avoid copyright infringement.

A communication letter will be presented to the validators concerned with verifying the questionnaire/survey that will be raised to observe transparency, and reliability and to avoid misinterpretation in the data collection process.

The research instrument will be subjected to validity and reliability tests. Their suggestions will be incorporated for the coherence of the questions. A list of summaries and corresponding actions of the researchers will be appended.

CHAPTER III

RESULTS AND DISCUSSION

This chapter includes the presentation, interpretation and analysis of significant findings of the current study. This also contains the conclusions and recommendations of the study.

FINDINGS

Profile of the Respondents

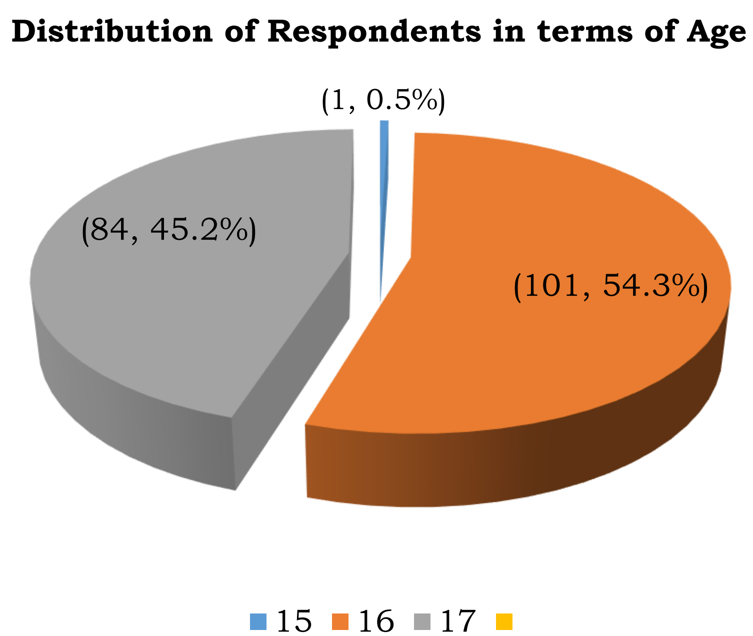

Figure 1. Age of the Respondents

The pie chart shows that most of the students ages 16 or 54.3%. Only one respondent, 0.5%, is 15 years old, The age distribution shown in the pie chart, where most of students are 16 and 17 years old, is consistent with the expected age range for senior high school students in the Philippines. Under the K to 12 Basic Education Program, students typically enter Grade 11 at age 16 and complete Grade 12 at age 17 (Department of Education, 2023). The minimal representation of 15-year-olds supports the idea that early school entry is rare, as the formal school starting age in the country is generally six years old for Grade 1. Additionally, the presence of 17-year-olds in notable numbers may be attributed to factors such as late school entry, temporary dropouts, or grade repetition—common issues identified in public education systems (Philippine Statistics Authority [PSA], 2022.

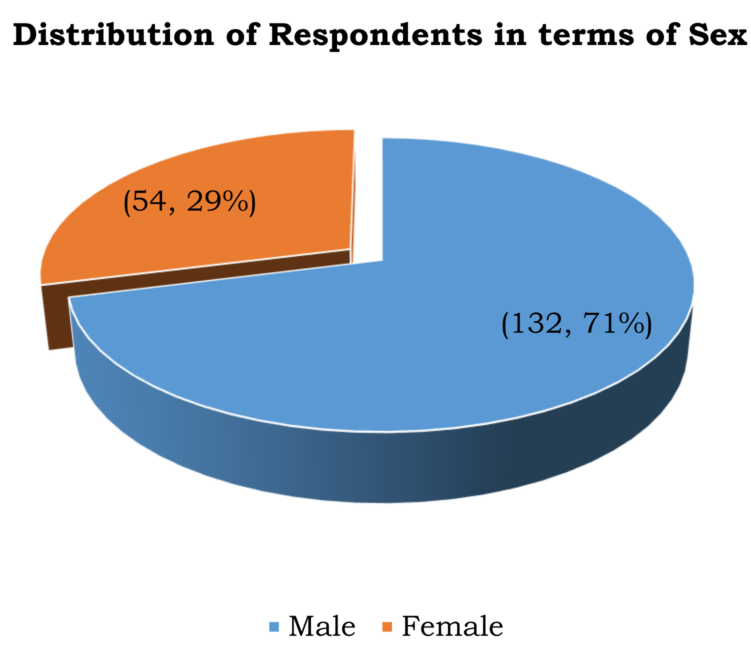

Figure 2. Sex of the Respondents

The pie chart shows that 132 or 71% are female, while only 54 or 29% are male. With more than twice as many females as males.

The gender distribution presented in the pie chart, where 71% of respondents are female and only 29% are male, reflects a broader trend observed in Philippine education. According to the Philippine Statistics Authority (PSA), females consistently demonstrate higher completion rates across educational levels, with 70.1% of females aged six and over having completed at least elementary education, compared to only 65.1% of males (PSA, 2022). This trend continues into secondary and tertiary education, where more females pursue academic tracks such as humanities, education, and healthcare, fields that typically attract a higher proportion of female enrollees (PSA, 2021).

On the other hand, male students often gravitate toward vocational and technical education programs, which may not have been represented in the surveyed population. This divergence in academic track preference contributes to the observed gender disparity. Moreover, societal and economic factors such as early employment and lack of personal interest further affect male educational participation. In fact, PSA (2023) reported that among out-of-school youth, 25.9% of males were not attending school due to employment and 17.9% due to lack of personal interest, both significantly higher than female counterparts. These statistics support the finding that male underrepresentation in certain academic settings may be influenced by a combination of educational preferences and socio-economic constraints.

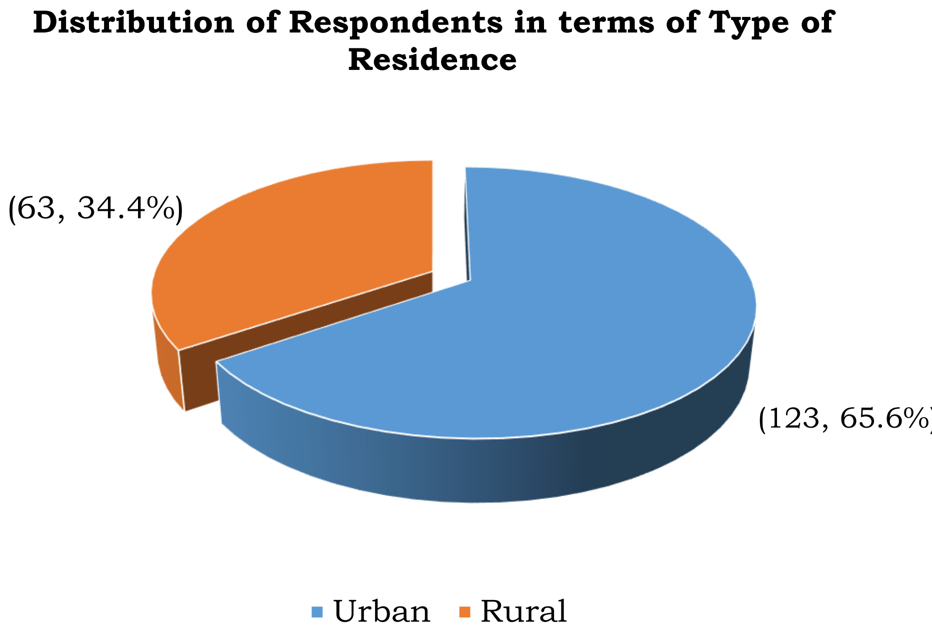

Figure 3. Type of Residence of Respondents

The pie chart shows that 123 respondents or 65.6% live in rural areas, while only 63 or 34.4% reside in urban areas, reflects the broader national demographic trend in the Philippines. As of early 2023, approximately 51.8% of the Philippine population lived in rural regions, while 48.2% resided in urban centers (DataReportal, 2023). This rural majority suggests that educational institutions located in or near rural areas naturally attract more students from these communities.

This indicates that the access to quality education in rural areas is often hindered by limited funding and resources. Studies have shown that rural schools frequently face challenges such as inadequate facilities and a scarcity of qualified teachers, which can impact educational outcomes (Children of the Mekong, 2023). Conversely, urban areas typically offer a wider array of educational institutions and programs, providing students with more choices that align with diverse academic interests and career aspirations. This disparity may contribute to the higher proportion of rural students attending nearby institutions that cater to their socioeconomic backgrounds and available opportunities.

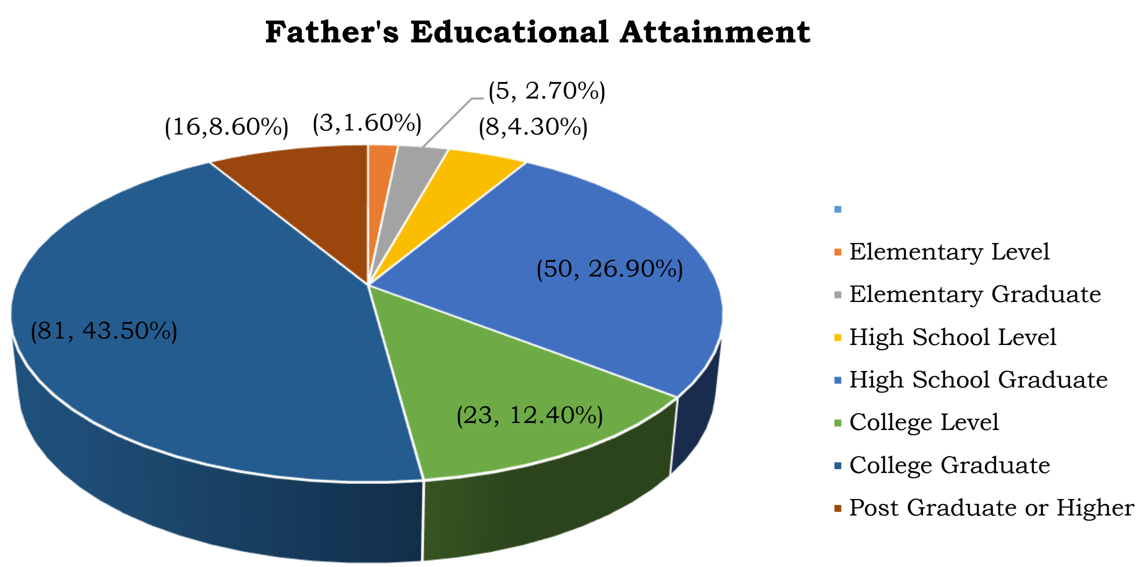

Figure 4.1. Father's Educational Attainment

The pie chart shows the educational attainment of fathers, with data distributed across several educational levels. A significant portion of the respondents' fathers, 43.5% (81 individuals), are college graduates, and none of the respondents' fathers reported having no formal education.

This data suggests that most students come from households where the father has attained at least a high school education, with a substantial portion completing college. This educational background may contribute positively to the academic and motivational environment in which students are raised. Recent studies have further explored the relationship between parental education levels and students' academic performance. Additionally, a study conducted by Almira, Wibowo, and Zahra (2024) at Sekolah Indonesia Davao in the Philippines examined the influence of parental education level, socio-economic status, and learning motivation on high school students' academic performance. The findings indicated that these factors did not significantly impact academic performance, suggesting that other variables might better explain the variability in students' academic outcomes.

On the other hand, a study by Escol and Alcopra (2024) in the Manolo Fortich IV District, Division of Bukidnon, Philippines, found a significant relationship between parental involvement and learners' academic performance. The study revealed that parents who were highly involved in their children's education had children who performed very satisfactorily during the Second Quarter of the School Year 2023-2024. This underscores the importance of parental engagement in fostering academic success.

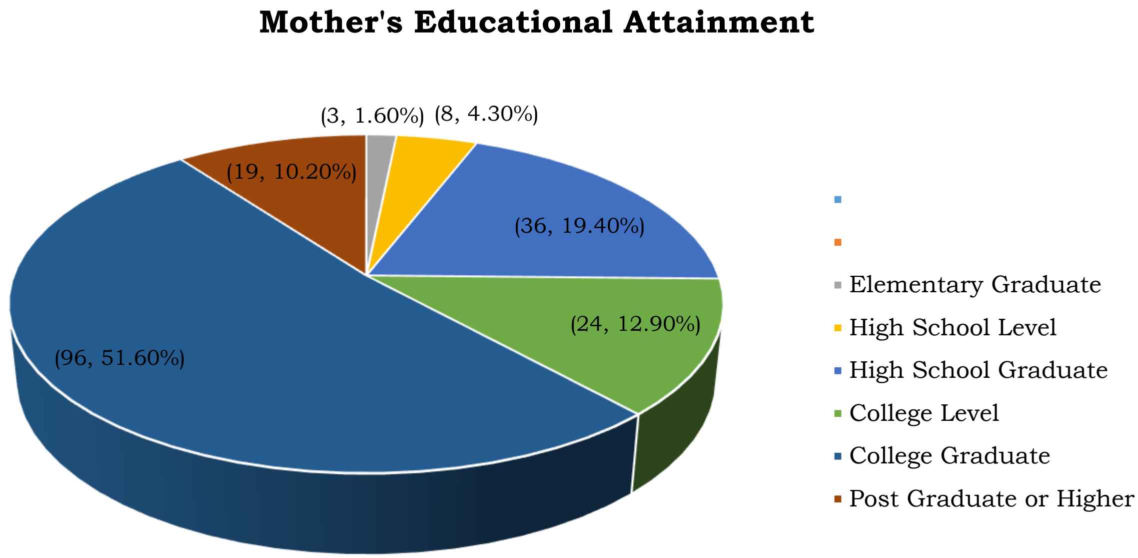

Figure 4.2. Mother's Educational Attainment

The pie chart shows the educational attainment levels of mothers based on survey responses. Most of the respondent's mothers, accounting for 96 or 51.60%, are college graduates. and 0 respondent's mothers with no formal education and elementary level.

The data suggests that a significant portion of the student population benefits from having mothers with relatively high educational backgrounds. Maternal education is widely acknowledged to have a direct influence on children's academic success, fostering positive attitudes toward learning and increased parental involvement (OECD, 2021). The fact that more than half of the mothers are college graduates may positively correlate with students' access to educational resources, guidance at home, and encouragement to pursue higher academic goals.

Recent studies continue to affirm the significant impact of maternal education on children's academic achievement, highlighting the mechanisms through which this influence operates. A study by Weis et al. (2023) demonstrated that higher maternal education correlates with elevated self-transcendence values, such as altruism and social responsibility, in mothers. These values are associated with less restrictive parenting practices, fostering better behavior regulation in children, which in turn enhances their school performance. This suggests that maternal education contributes to academic success not only through cognitive stimulation but also by shaping parenting approaches that support children's behavioral development.

Furthermore, research by Esteban-Cornejo et al. (2021) found that cardiorespiratory fitness partially mediates the relationship between maternal education and academic achievement, particularly among boys. Children of more educated mothers tend to have higher fitness levels, which are linked to better academic outcomes. This indicates that maternal education may influence children's academic success through both direct educational expectations and indirect factors like promoting healthier lifestyles that enhance physical fitness and cognitive function.

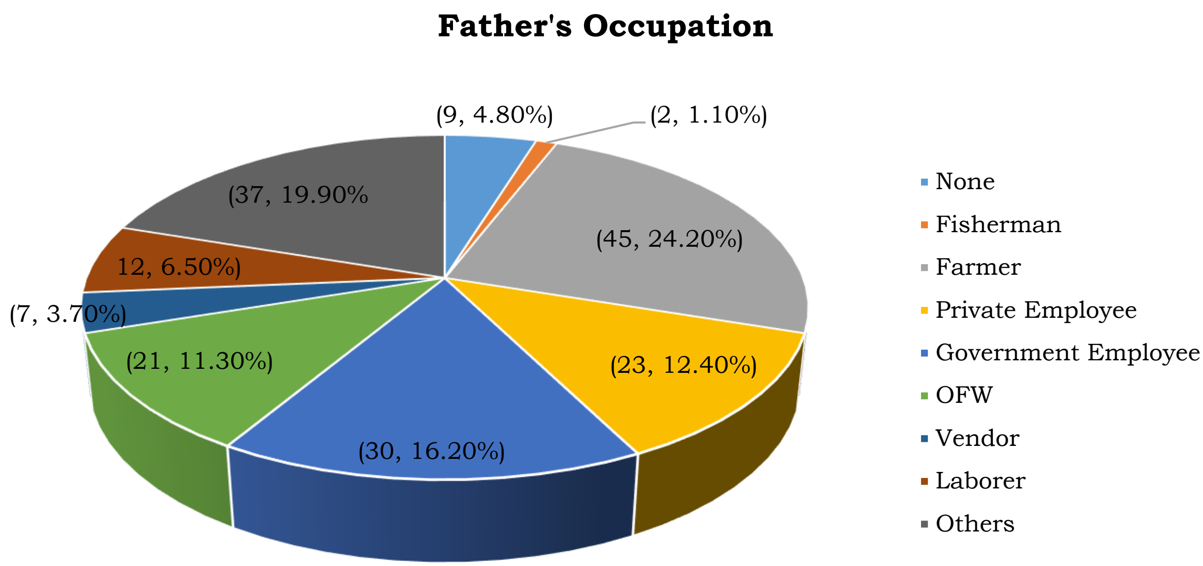

Figure 5.1. Father's Occupation

The pie chart shows the distribution of various occupations held by fathers. The most common occupation is "Farmer," with 45 (24.20%), and the least being those with "Fisherman" (2 or 1.10%).

The data reveals that a significant number of fathers engage in blue-collar and manual labor jobs, such as laborers, vendors, and farmers. This may indicate a socioeconomic profile leaning towards lower to middle-income brackets. The presence of private and government employees suggests that a portion of the population enjoys more stable employment, possibly with better benefits and job security. The minimal number of unemployed fathers is encouraging, indicating high economic participation among male guardians. Occupational roles, particularly manual labor and informal work can impact parental involvement in education, as irregular working hours or financial stress may hinder. Recent studies support the view that a father's occupation significantly influences children's educational outcomes. According to Malapit and Quisumbing (2021) on gender and labor in low-income households in the Philippines highlights that fathers in informal or unstable jobs often contribute less to educational support due to time and resource constraints. Furthermore, a study by De Guzman and Uy (2023) indicates that children from households where fathers are engaged in formal employment tend to have better school attendance and academic performance due to greater financial stability and routine.

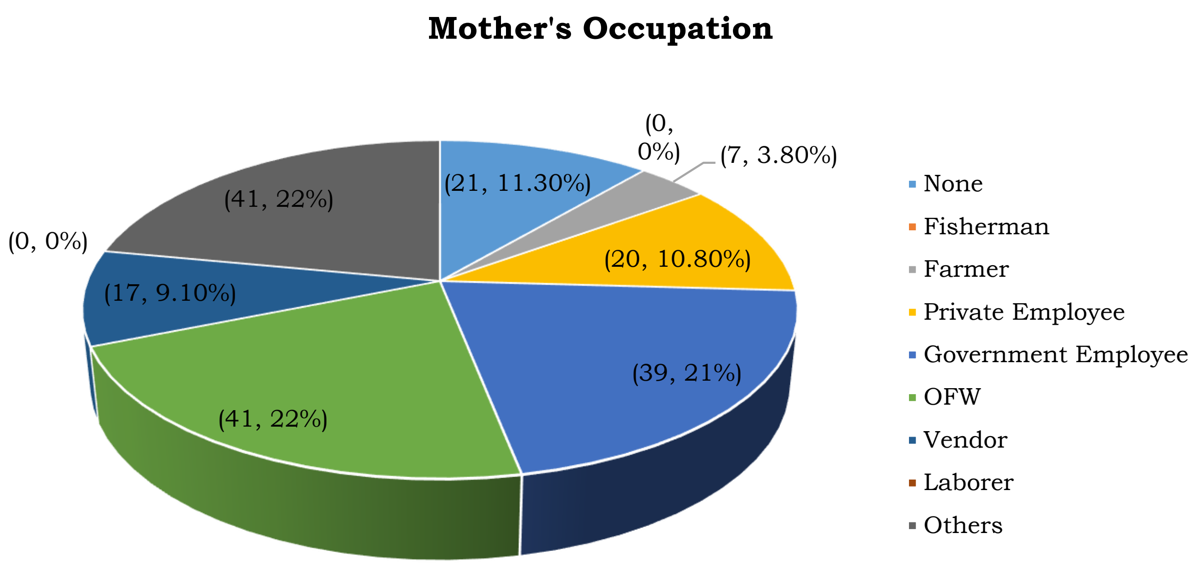

Figure 5.2. Mother's Occupation

The pie shows the employment distributions among mothers. The largest segments are tied between "OFW" and "Others," each with 41 respondents (22%) and no mothers are recorded under "Fisherman" and "Laborer".

The data reveals most of the mothers are engaged in working outside the country and in informal roles, with the "OFW" and other categories. Meanwhile, the strong representation of mothers as private and government employees suggests increasing female participation in formal employment. These employment trends may affect their capacity to support their children's education—while employed mothers may bring in financial stability, irregular or demanding work hours can reduce the time spent on academic guidance at home.

According to Serrano and Magno (2022), children whose mothers are engaged in formal employment tend to perform better academically, primarily due to improved financial resources and structured environments. However, the same study noted that when maternal work involves long hours or informal sectors, it can limit direct involvement in educational supervision. Similarly, research by Reyes and Bautista (2021) found that while maternal employment contributes to household resilience, work-related stress and time poverty can hinder active educational support. These findings align with the occupational distribution in the chart and underscore the importance of balancing work and parental involvement.

Proficiency Level in Precalculus of the Respondents

The succeeding table shows the proficiency level in Precalculus of the Grade 11 STEM students in Candon National High School (CNHS).

Table 2. Level of Proficiency in Precalculus along the following various topics.

Statistical Range		*f*	%
a. Conic Sections			
12.01 – 15.00		9	4.8
9.01 – 12.00		40	21.5
6.01 – 9.00		77	41.4
3.01 – 6.00		58	31.2
0.00 – 3.00		2	1.1
Mean	**7.44**		
Descriptive Rating	**M**		
b. Mathematical Induction			
4.01 – 5.00		5	2.7
3.01 – 4.00		21	11.3
2.01 – 3.00		61	32.8
1.01 – 2.00		57	30.7
0.00 – 1.00		42	22.6
Mean	**1.91**		
Descriptive Rating	**L**		
c. Circular Functions			
8.01 – 10.00		3	1.6
6.01 – 8.00		28	15.1
4.01 – 6.00		53	28.5
2.01 – 4.00		50	26.9
0.00 – 2.00		52	28.0
Mean	**3.71**		
Descriptive Rating	**L**		
d. Trigonometric Identities			
12.01 – 15.00		5	2.7
9.01 – 12.00		36	19.4
6.01 – 9.00		39	21.0
3.01 – 6.00		88	47.3
0.00 – 3.00		18	9.7
Mean	**6.25**		
Descriptive Rating	**M**		
e. Inverse Trigonometric Functions			
4.01 – 5.00		22	11.8
3.01 – 4.00		33	17.7
2.01 – 3.00		37	19.9
1.01 – 2.00		55	29.6
0.00 – 1.00		39	21.0
Mean	**2.20**		
Descriptive Rating	**M**		

**Legend*

6.01 – 9.00	*4.01 – 6.00*	*2.01 – 3.00*	*Moderate (M)*
3.01 – 6.00	*2.01 – 4.00*	*1.01 – 2.00*	*Low (L)*

Conic Sections

The data show a moderate level of understanding, with a mean score of 7.44. Particularly, 41.4% of students scored within the 6.01–9.00 range, while a minimal 1.1% fell within the 0.00–3.00 range, suggesting that most learners possess at least a foundational grasp of the topic. The relatively higher performance in Conic Sections compared to other Precalculus topics may be attributed to its visual nature, allowing students to comprehend concepts more effectively through graphs and diagrams. This is supported by Arcavi (2019) that geometric visualization plays a crucial role in mathematics education, aiding students in understanding abstract concepts and enhancing problem-solving skills. Furthermore, According to Jones & Smith (2020), highlighted that there are so many applications in real-life situations of Conic Sections, such as satellite orbits and architectural designs, making the subject more relatable and easier to conceptualize for learners.

Although this general proficiency, certain students encounter challenges, particularly in distinguishing between equations of different conic shapes and applying transformations. As cited by Duval (2018) stated that students often struggle with articulating different representations linked to the concept of functions, which may impact their ability to solve problems involving Conic Sections. However, Hollebrands et al. (2021) addresses these challenges, by incorporating active engagement in applets or geometry software and hands-on activities that can be beneficial for learners in reinforcing conceptual understanding in Conic Sections. Additionally, the use of different geometry software has been shown to positively impact students' performance in Conic Sections, providing interactive and visual learning experiences.

Mathematical Induction

The data shows a low level with a mean score of 1.91, the lowest among the Precalculus topics. Most of the students 32.8% scored within the 2.01 – 3.00 range, while a considerable percentage fell within the 0.00 – 2.00 range, suggesting that many struggle with the fundamental concepts of mathematical proof. As cited by Almeida (2020) stated that Mathematical Induction requires a strong understanding of logical reasoning and proof structure, which many students find difficult to grasp. It has been shown that students often struggle with the abstraction of mathematical proof, as it requires shifting from computational thinking to formal reasoning. The difficulty in recognizing how the base case, inductive step, and assumption work together may contribute to students' poor performance in Mathematical Induction.

As cited by Norton et al. (2022), it stated that proof by mathematical induction remains a challenge for students transitioning from procedural problem-solving to theoretical justification. It also highlights that students often struggle with the abstract nature of induction, particularly in understanding the inductive step and the principles of logical reasoning. Additionally, difficulties also arise in generalizing statements—comprehending how proving a base case and an inductive step extends to all natural numbers (Jiang et al., 2023).

Circular Functions

The data shows a low level of understanding, with a mean score of 3.71. Most of the students 28.5% scored within the 4.01–6.00 range, while a significant number fell into lower score brackets, indicating that many learners encounter difficulties with key concepts in this topic. According to Orhani (2024), identified that students often struggle with the graphical representation of trigonometric functions and the conceptual link between angles and their corresponding coordinates on the unit circle. These difficulties are compounded when introducing concepts such as negative angles, radian measures, and the periodic properties of trigonometric functions, which require both algebraic manipulation and spatial reasoning. Similarly, Suparman et al. (2021) observed that students exhibit errors in solving trigonometric problems due to misunderstandings in these areas. These challenges may contribute to lower performance in trigonometry compared to more visual or algebraic topics like Conic Sections.

Trigonometric Identities

The data shows a moderate level of understanding, with a mean score of 6.25. Most of the students 47.3% scored within the 3.01–6.00 range, indicating that while many learners possess a basic grasp of the topic, they encounter challenges in effectively applying and solving trigonometric identities. According to Rohimah & Prabawanto (2020), it stated that mastery of these identities necessitates strong algebraic manipulation skills and pattern recognition, which can be discouraging for students who rely mostly on memorization rather than conceptual comprehension. It has been identified that students often face difficulties in solving trigonometric equations and identities due to challenges in interpreting problem forms, factoring trigonometric quadratic equations, and utilizing fundamental trigonometric formulas. This lack of conceptual understanding impedes their ability to recognize transformation strategies and select appropriate identities to simplify expressions.

Inverse Trigonometric Functions

The data shows a mean score of 2.20, one of the lowest among the Precalculus topics. Most of the students 29.6% scored within the 1.01 – 2.00 range, indicating that a significant percentage of learners struggles with the fundamental concepts of inverse trigonometry. This difficulty may stem from the fact that inverse trigonometric functions require an understanding of both function restrictions and domain/range transformations, concepts that are often abstract and difficult to visualize. According to Sierpinska (2018), students frequently confuse inverse trigonometric functions with reciprocal trigonometric functions, leading to conceptual errors. Additionally, working with inverse functions algebraically, especially in solving equations and graphing transformations, requires strong prerequisite knowledge in function operations, which many students find challenging.

Level of Difficulty on the Various Topics Covered in Precalculus

The level of difficulty on the various topics covered in precalculus is presented in Table 2.

Table 3. Level of Difficulty on the Various Topics Covered in Precalculus

Topics	Difficulty Index	Descriptive Equivalent Rating
A. Conic Sections		
a. Circles	0.72	E
b. Parabola	0.53	MD
c. Ellipse	0.46	MD
d. Hyperbola	0.45	MD
B. Mathematical Induction		
a. Introduction to Mathematical Induction	0.70	E
b. Proving Summation Identities	0.30	D
C. Circular Functions		
a. Angles Measure	0.59	MD
b. Circular Functions on Real Numbers	0.41	MD
c. Reference Angle	0.27	D
D. Trigonometric Identities		
a. The Fundamental Trigonometric Identities	0.34	D
b. The cofunction Identities and the Sine Sum and Difference Identities	0.47	MD
c. The Cosine Difference and Sum Identities	0.52	MD
d. The Tangent Sum and Difference Identities	0.54	MD
E. Inverse Trigonometric Function		
a. Inverse Sine Function	0.74	E
b. Inverse Cosine Function	0.48	MD
a. Inverse Tangent Function and the Remaining Inverse Trigonometric Functions	0.48	MD

Legend: *0.61 – 0.80 - Easy (E)*
0.41 – 0.60 - Moderate Difficult (MD)
0.21 – 0.40 - Difficult (D)

Conic Sections

The data shows the level of difficulty of the following subtopics in Conic Section; The Circle with the highest difficulty index of 0.72 as interpreted as

Easy, while the Hyperbola as the lowest difficulty index of 0.45 interpreted as Moderate Difficulty.

The variation in student mastery suggests that more familiar and visually intuitive shapes like Circles and Parabolas are easier for students to understand. The abstract properties and asymptotic behavior of Hyperbola and Ellipse make it more conceptually difficult, contributing to its lower index of difficulty. Students likely benefit from concrete examples and visual representations for Circles and Parabolas, while Hyperbolas and Ellipse demand a stronger grasp of algebraic manipulation.

A study by Andrade and Coloma (2021) highlights that students perform better in geometry-based algebra topics when visual aids and real-life applications are incorporated, which aligns with the ease observed in Circle-related questions. On the other hand, more abstract forms like Hyperbolas require instructional scaffolding and repeated exposure to avoid cognitive overload (Delgado-Reyes et al., 2022).

Mathematical Induction

The data shows that the subtopic "Introduction to Mathematical Induction" with a difficulty index of 0.70 interpreted as Easy and "Proving Summation of Series" at 0.30 interpreted as Difficult.

The drastic difference indicates that while students can grasp the basic concept of induction, applying it to prove summation formulas introduces significant complexity. The step-by-step logical structure involved in proofs requires abstract reasoning, which many students find challenging without enough practice.

According to Li and Schoenfeld (2021), the application of mathematical induction demands high levels of symbolic thinking, and most students struggle when transitioning from concept to application. Their study suggests incorporating scaffolded examples and interactive proving examples and problems help the students improve their understanding in procedural topics like this.

Circular Functions

The data shows the difficulty index of the following subtopics such as Angle Measure with a difficulty index of 0.59 interpreted as Moderate Difficult, the highest among the three subtopics and Reference Angle, the lowest with a difficulty index of 0.27 interpreted as Difficult.

The relatively low scores suggest that the circular function topics are conceptually difficult, especially Reference Angles, which interpreted as the lowest score in this category. These topics require spatial reasoning, understanding of angle direction, and radian measure—concepts unfamiliar to many students transitioning from basic trigonometry.

According to Delgado-Reyes et al. (2022) found that integrating dynamic geometry software helped students visualize angles and circular motion, leading to improved performance in topics like arc length and reference angles. This supports the idea that abstract spatial concepts require visual and interactive strategies.

Trigonometric Identities

The data shows the index of difficulty of the following subtopics such as The Tangent Sum and Difference Identities with a difficulty index of 0.54 interpreted as Moderate Difficult, the highest among the four subtopics and Trigonometric Fundamental Identities with difficulty index of 0.34 interpreted as Difficult.

While students generally manage to work with most identities, the low score on Fundamental Identities suggests they struggle with simplification and symbolic manipulation. It is likely that students can memorize identities but find difficulty applying them in problem-solving contexts, especially when multiple steps are involved.

Berg and Heller (2021) argue that while memorization of trigonometric identities is common, deep understanding requires meaningful practice and contextual problem-solving. They emphasize the need for targeted instruction to bridge the gap between knowledge and application, particularly in foundational identities.

Inverse Trigonometric Functions

The data show that only one topic "Inverse Sine Function" is listed with a difficulty index of 0.74 interpreted as Easy, the highest among all topics, while the other two subtopics are listed with a difficulty index of 0.48 interpreted as Moderate.

This suggests students find inverse functions more manageable, possibly due to prior exposure and simpler domains/ranges involved in the Sine function. The concept is also more concrete and tied to real-world applications, aiding comprehension and recall.

A study by Abad & Salazar (2023) indicates that topics involving direct function relationships (like Inverse Sine) are easier for students when presented through graphing calculators and real-life scenarios. Their research shows increased accuracy and retention in inverse trigonometric applications compared to complex identity manipulations.

Relationship between the Profile of the Respondents and Proficiency level of the Students in Precalculus

The relationship between the profile of the respondents and proficiency level of the students in precalculus is presented in Table 5.

Table 4. Relationship between the Profile of the Respondents and Proficiency level of the Students in Precalculus

Level of Proficiency in Precalculus	Profile of the Respondents						
	Age	**Sex**	**Type of Residence**	**Father's Educational Attainment**	**Mother's Educational Attainment**	**Father's Occupation**	**Mother's Occupation**
Conic Section	-0.013	-0.037	-0.024	0.002	0.022	-0.021	-0.041
Mathematical Induction	0.116	0.041	-0.010	0.021	-0.062	0.024	0.124

Circular Functions	-0.066	0.063	0.031	0.160*	0.024	-0.188*	0.007
Trigonometric Identities	-0.072	-0.129	-0.023	0.037	-0.047	-0.029	0.054
Inverse Trigonometric Functions	0.015	-0.027	-0.003	0.088	0.036	-0.077	-0.032

**correlation is significant at the 0.05 level*

Table 4 shows the relationship between students' demographic profiles and their proficiency levels in Precalculus. Among the correlations, father's educational attainment with a computed correlation value of ($r = 0.160$, $p < 0.05$) and father's occupation ($r = -0.188$, $p < 0.05$) show significant relationships with students' proficiency in Circular Functions. This means that students whose fathers have higher educational attainment tend to perform better in Circular Functions, potentially due to greater academic support at home. On the other hand, the negative correlation with father's occupation implies that students whose fathers are engaged in demanding jobs might receive less academic guidance, affecting their understanding of Circular Functions. These findings align with the study of (Fan & Williams, 2010), as educated parents often provide stronger academic support and learning resources at home.

The overall weak correlations across other profile variables suggest that age, sex, residence, and parental occupation generally have minimal influence on Precalculus proficiency. However, the significant correlations in Circular Functions reinforce the idea that parental background plays a crucial role in mathematical comprehension. As cited by Wang & Wei (2024), it highlighted that students with academically supportive environments demonstrate stronger problem-solving skills and conceptual understanding in mathematics. Moreover, Jiang et al. (2023), emphasizes that parental involvement when supportive rather than intrusive positively impacts students' mathematical achievement, fostering independence and deeper learning.

Level of Validity of the Lesson Exemplar in Precalculus

Table 5. Results of Validity Test

Indicators	Mean Rating	Descriptive Equivalent Rating
A. Content		
1. Activities are aligned with the objectives of the program.	5.00	VHV
2. Content is accurate.	5.00	VHV
3. Content is appropriate to the target user.	5.00	VHV
4. The content is relevant to the participants	5.00	VHV
B. Organization		
5. Topics and activities are well organized.	4.60	VHV
6. Organization is flexible, permitting variation in sequence.	4.60	VHV
7. Material within the topics is well organized.	4.60	VHV
8. Approach is suitable to the wide range of participants.	4.40	VHV
C. Mechanics		
9. Topics, titles, and strategies are concrete, meaningful, and interesting.	4.60	VHV
10. Activities are well-constructed and meaningful.	4.60	VHV
11. Activities are suited to the participants' interest.	5.00	VHV
Grand Mean	**4.76**	**VHV**

Legend:
4.21 – 5.00 Very High Validity (VHV)

The table shows the results of a validity test conducted on a lesson exemplar in Precalculus. The assessment is based on three major indicators: Content, Organization, and Mechanics. Each item under these indicators received ratings between 4.40 and 5.00, which fall under the "Very High Validity (VHV)" category according to the legend. Specifically, all four content-related criteria scored a perfect 5.00, indicating exceptional alignment, accuracy, appropriateness, and relevance. In terms of organization, scores ranged from 4.40 to 4.60, suggesting very high but slightly variable perceptions of the exemplar's flexibility and structure. The mechanics of the lesson, including how meaningful and engaging the topics and activities are, also scored between 4.60 and 5.00. The overall grand mean is 4.76, which is interpreted as "Very High Validity."

These results suggest that the lesson exemplar in Precalculus is of excellent quality, particularly in terms of its content accuracy, alignment with objectives, and appropriateness for the target learners. The high ratings for organization and mechanics indicate that the structure of the lesson is logical, flexible, and engaging for diverse learners. The slightly lower scores in organizational flexibility (4.40) may hint at areas where the sequence or adaptability of topics could still be enhanced for differentiated instruction.

This finding aligns with the study by Gonzales and Trinidad (2021), who emphasized that content validity and logical organization are pivotal for effective learning materials, particularly in subjects like mathematics that require conceptual clarity. Furthermore, Delos Santos et al. (2023) found that lesson exemplars with highly valid mechanics—clear objectives, engaging strategies, and meaningful tasks—improve student motivation and conceptual understanding in senior high school STEM tracks. These corroborating studies affirm that the validated exemplar is a strong instructional tool likely to support effective teaching and learning in Precalculus.

CONCLUSIONS

Based on the findings, the following statements were concluded:

1. Majority of the respondents are aged 16 to 17, predominantly female, and reside in rural areas. Most of their parents, particularly mothers, have attained a college-level education, and a significant number are engaged in formal or informal employment.
2. The findings reveal a generally low to moderate proficiency level among Grade 11 STEM students in various Precalculus topics. While students demonstrated a moderate understanding of Conic Sections, Trigonometric Identities, and Inverse Trigonometric Functions they struggled significantly with more abstract and conceptually demanding areas such as Mathematical Induction, and Circular Functions.
3. The analysis of difficulty levels across various Precalculus topics reveals that students generally find geometry-based and visual topics such as Circles and Inverse Sine Functions easier to master, as reflected in their high difficulty indices and "Mastered" ratings. In contrast, abstract and symbol-heavy topics like Proving Summation Identities, Reference Angles, and Fundamental Trigonometric Identities pose greater challenges, receiving low difficulty indices and being classified as "Least Mastered."

4. The results found a significant relationship between certain aspects of the respondents' profiles and their proficiency level in Precalculus, specifically in Circular Functions. Particularly, the father's educational attainment showed a positive correlation, suggesting that students whose fathers have higher levels of education tend to perform better in this area, likely due to increased academic support at home. On the other hand, a negative correlation was observed with the father's occupation, indicating that students whose fathers are employed in more demanding jobs may receive less academic assistance.
5. The lesson exemplar in Precalculus addresses the needs of the students in helping them to improve their proficiency level in Precalculus.
6. The researcher made lesson exemplar in Precalculus demonstrates very high validity across all assessed dimensions—content, organization, and mechanics—with a grand mean of 4.76.

RECOMMENDATIONS

Based on the conclusions made, the researcher hereby recommends the following:

1. Schools should provide tailored support for students whose parents may have limited availability due to work commitments, such as offering flexible parent-teacher engagement opportunities or academic mentoring programs. Efforts should also be made to address the unique needs of rural students and promote gender inclusivity across all programs.

2. Teachers should use instructional approaches in Precalculus that emphasize conceptual understanding using visual aids, interactive tools, and real-life applications—especially in complex topics like Mathematical Induction and Inverse Trigonometric Functions. Incorporating technology-based learning, such as graphing software and dynamic geometry applications, can enhance students' spatial and logical reasoning.
3. Teachers should use different teaching strategies incorporating more visual, interactive, and scaffolded learning experiences, particularly for abstract and symbolically dense topics. Tools such as GeoGebra/Desmos, guided discovery activities, and real-life applications can enhance conceptual understanding and retention.
4. Schools should strengthen their parent engagement programs, particularly focusing on awareness and capacity-building among parents with lower educational attainment or demanding occupations. Workshops or resource materials could help equip parents with strategies to support their children academically, regardless of their own educational background or work schedule.
5. Interventions like lesson exemplar and learning activity sheets should be given to students for them to meet the expected level of proficiency or to come up with a higher proficiency level.
6. The lesson exemplar is recommended for classroom use, especially in senior high school STEM tracks. To further enhance its effectiveness, minor improvements in organizational flexibility and feedback from actual

classroom implementation should be considered to better support diverse learning needs.

7. A further study along this line should be conducted to validate and strengthen the results and findings of the study.

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