



Deriving the physical phenomena of TOUGMA's solution

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Abstract

In this paper, we derive the physical phenomena that implies the second TOUGMA's metric. Due to astrophysical applications, the interest of studying the TOUGMA metric right now is to implement using a metric concrete the new physical concepts that implies this metric

Key-Words: Ricci Tensor, metric of TOUGMA, quantum relativity.

I. INTRODUCTION

Relativistic astrophysics occupies a growing part in contemporary astronomy. raine, particularly in view of the large amount of data generated are either cosmologi- ques, or involve compact objects (black holes, neutron stars). In the both cases, the theoretical basis of their study was general relativity. But TOUGMA have been given recently et unified equation of quantum langrangian and gravity, nommed Quantum Relativity theory, published in 2021 by TOUGMA and all ¹.

$$[(\alpha-3)R_{uv} - \frac{1}{2}g_{uv}R](1 + \frac{2kL_m}{R}) - 2k(\alpha-3)g_{uv}L_m = T_{uv} \quad (1)$$

It has resolved and one of solutions is given by:

$$ds^2 = -[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2 c^2 dt^2 + \frac{dr^2}{[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2} + r^2(d\theta^2 + r^2 \sin^2 \theta d\phi^2) + r^{\alpha-4} d\Omega_{\alpha-4} \quad (2)$$

The gravitational field of bodies with spherical symmetry is obviously of importance capital in astrophysics. To arrive immediately at interesting applications of astrophysical interest, we are going to study the physical concepts of this TOUGMA's metric.

II. METHODS

A solution of TOUGMA's equation that can be defined by the existence of a coordinate system $(x^u) = (ct, r, \theta, \phi, \Omega_{\alpha-4})$, called TOUGMA coordinates, such that the components g_{uv}

of the metric tensor g are written there

$$ds^2 = -[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2 c^2 dt^2 + \frac{dr^2}{[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2} + r^2(d\theta^2 + r^2 \sin^2 \theta d\phi^2) + r^{\alpha-4} d\Omega_{\alpha-4} \quad (3)$$

The first observation that we can made in view of (3) is that the space-time (E,g) is static and spherically symmetric. The metric components are clearly independent of t and, $\vec{\partial}_t \cdot \vec{\partial}_t = -[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2 c^2 < 0$, and then that ∂_t is time-like; , we conclude that spacetime is stationary. As for the spherical symmetry, it is immediate because the components g_{uv} given by (3). Moreover, the space-time described by the TOUGMA metric is asymptotical ^{2 3 4} we have in effect if $r=0$

$$[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2 = [\tan(-\frac{kL_m}{4(\alpha-2)})]^2 \quad (4)$$

that we are going to study the physicals phenomena of this function in the next section. as limites we have:

$$\lim_{r \rightarrow -\infty} [\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2 = +\infty \quad (5)$$

and

$$\lim_{r \rightarrow 1} [\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2 = +\infty \quad (6)$$

A. Finding Radial Light Geodesics

Let us place ourselves in the frame of the TOUGMA coordinates $(x^u) = (ct, r, \theta, \phi, \Omega_{n-4})$. A light geodesic is a geodesic of zero length: we must therefore have along this one

$$ds^2 = g_{uv} dx^u dx^v = 0 \quad (7)$$

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On the other hand, if we assume the radial geodesic, then $d\theta = 0$ and $d\phi = 0 = d\Omega_{n-4}$ along it.

It happens:

$$- [\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2 c^2 dt^2 + \frac{dr^2}{[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2} = 0 \quad (8)$$

$$- [\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2 c^2 dt^2 = \frac{dr^2}{[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2} \quad (9)$$

$$[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2 c^2 dt^2 = \frac{dr^2}{[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2} \quad (10)$$

$$[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])] c dt = \pm \frac{dr}{[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]} \quad (11)$$

$$c dt = \pm \frac{dr}{[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2} \quad (12)$$

$$ct = \pm \int_{r_0}^r \frac{dr}{[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2} \quad (13)$$

$$ct = \pm \left[\frac{4(\alpha-2)(r-1) \cos^2(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])}{kL_m \tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])} \right]_{r_0}^r \quad (14)$$

$$ct = \pm \left[\frac{4(\alpha-2)(r-1) \cos^2(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])}{kL_m \tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])} - \frac{4(\alpha-2)(r_0-1) \cos^2(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r_0)])}{kL_m \tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r_0)])} \right] \quad (15)$$

Due to the \pm , we obtain two families of radial geodesics, which can be classified as following⁵⁶:

- the outgoing geodesics, for which $dr/dt > 0$; their equations are

$$ct = \frac{4(\alpha-2)(r-1) \cos^2(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])}{kL_m \tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])} - \frac{4(\alpha-2)(r_0-1) \cos^2(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r_0)])}{kL_m \tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r_0)])} \quad (16)$$

- incoming geodesics, for which $dr/dt < 0$; their equations are

$$ct = \frac{4(\alpha-2)(r_0-1) \cos^2(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r_0)])}{kL_m \tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r_0)])} - \frac{4(\alpha-2)(r-1) \cos^2(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])}{kL_m \tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])} \quad (17)$$

the physicals phenomena are going to be studied in the next section

B. Orbits of material bodies

Let us now examine the mass bodies trajectories(orbts) of $m \ll M$ around of the central body of the TOUGMA metric. As we saw in § 2., these trajectories must be time-like geodesics.If the subsequent trajectory deviates towards one of the two hemispheres separated by this equator, this would represent a break in the spherical symmetry. Thus the particle must remain in the plane and for a specific $\Omega_{\alpha-4}$

$$\theta = \frac{\pi}{2} \quad (18)$$

and

$$\Omega_{\alpha-4} = cste \quad (19)$$

From Kelling vectors we have:

$$\varepsilon = -\frac{c}{m} \vec{\chi}_0 \vec{p} = -c^2 \vec{\chi}_0 \vec{v} \quad (20)$$

$$l = \frac{1}{m} \vec{\chi}_z \vec{p} = c \vec{\chi}_z \vec{v} \quad (21)$$

that implies:

$$\varepsilon = c^2 \tan^2(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)]) \frac{dt}{d\tau} \quad (22)$$

$$l = r^2 \sin^2 \theta \frac{d\phi}{d\tau} \quad (23)$$

the 5-components of the pentavector \vec{v} are

$$v^0 = \tan^{-2}(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)]) \frac{\varepsilon}{c^2} \quad (24)$$

$$v^\theta = 0 \quad (25)$$

$$v^\phi = \frac{l}{cr^2} \quad (26)$$

$$v^{\Omega_{\alpha-4}} = 0 \quad (27)$$

by $\vec{v} \cdot \vec{v} = -1$ the renormalization equation, it happens:

$$g_{00}(v^0)^2 + g_{rr}(v^r)^2 + g_{\theta\theta}(v^\theta)^2 + g_{\phi\phi}(v^\phi)^2 + g_{\Omega\alpha-4}(v^{\Omega\alpha-4})^2 = -1 \quad (28)$$

$$-\tan^{-2}\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)\frac{\varepsilon^2}{c^2} + \tan^{-2}\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)(v^r)^2 + \frac{l^2}{c^2 r^2} = -1 \quad (29)$$

$$-\frac{\varepsilon^2}{c^2} + (v^r)^2 = -\left(\frac{l^2}{c^2 r^2} + 1\right) \tan^2\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right) \quad (30)$$

$$\frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 = -\frac{1}{2}\left[\left(\frac{l^2}{c^2 r^2} + 1\right) \tan^2\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right) - \frac{\varepsilon^2}{c^2}\right] \quad (31)$$

$$V_{eff}(r) = -\frac{1}{2}\left[\left(\frac{l^2}{c^2 r^2} + 1\right) \tan^2\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right) - \frac{\varepsilon^2}{c^2}\right] \quad (32)$$

the physicals phenomena of this are going to be studied in the next section

III. RESULTS

Now, we are going to give the physicals phenomena of last section equation. The firsts is

$$\left[\tan\left(-\frac{kL_m}{4(\alpha-2)}\right)\right]^2 \quad (33)$$

Ploted it with L_m and α variables we have We can see in the first figure that there is no universe forms with $\alpha \in [1, 3]$, also we can see the best possibility of universes formation according quantum Lagrangian L_m . The next figure shown the probability of universe formation according L_m and $\alpha \in [1, 100]$

FIG. 1: alpha and Lm variables with α lower

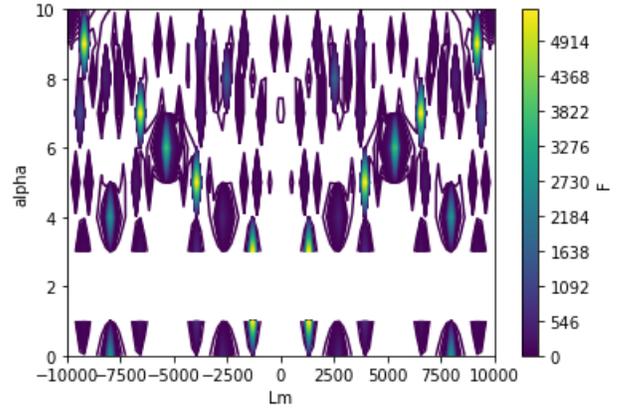
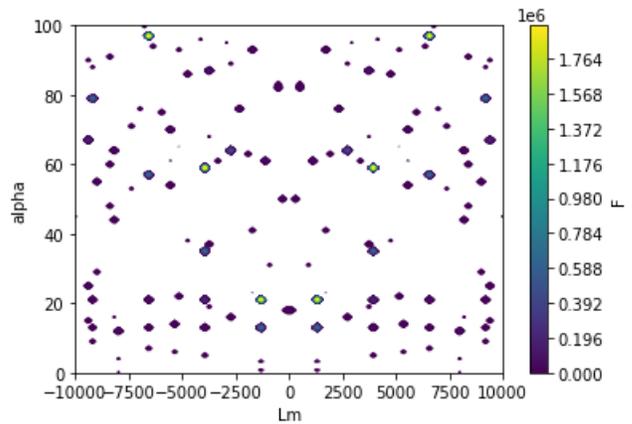


FIG. 2: alpha and Lm variables with α upper

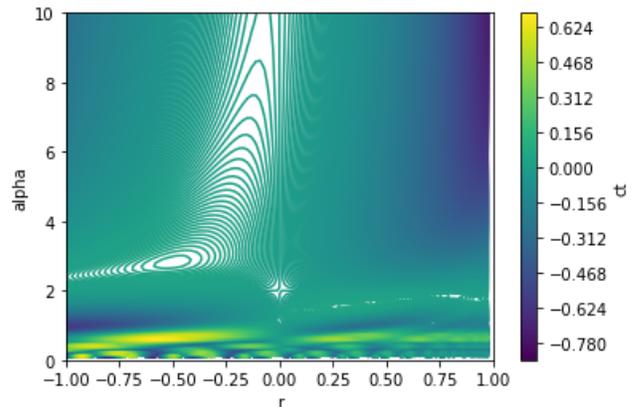


A. Radial Light Geodesics

We are going to represent the outgoing geodesic

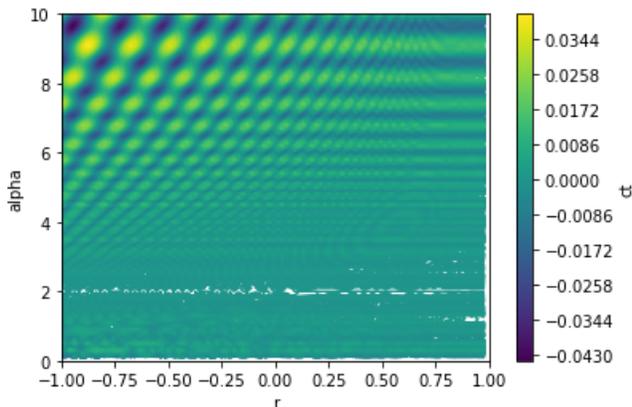
- for lagrangian fields lower, we represent $ct(r, \alpha)$

FIG. 3



- for lagrangian fields upper, we represent $ct(r, \alpha)$

FIG. 4



- for $\alpha = cste$, we represent $ct(r, L_m)$

FIG. 5

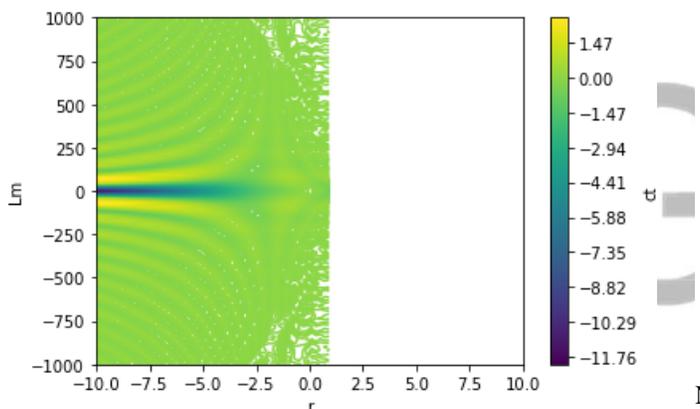
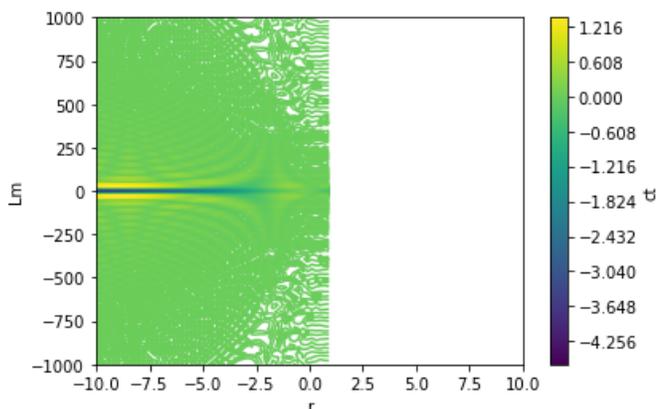
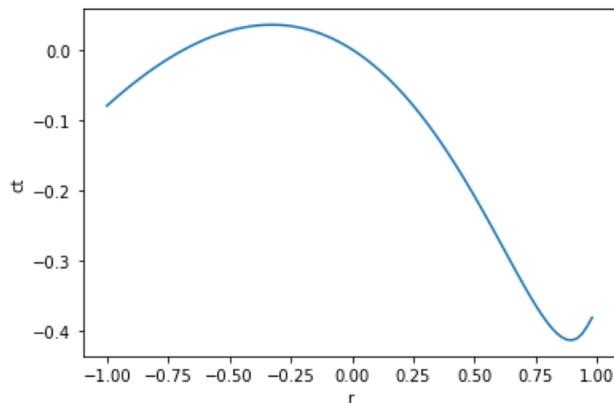


FIG. 6



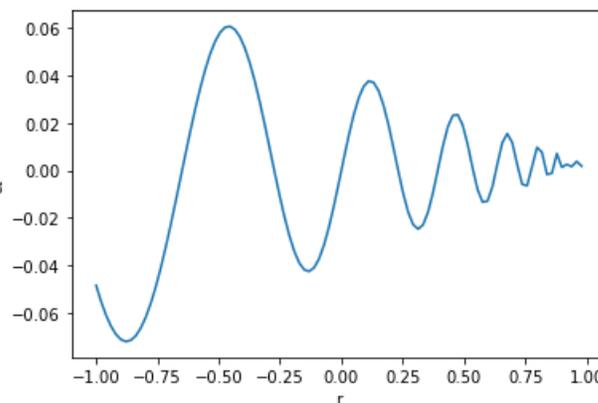
- for $\alpha = cste$ and L_m lower, we represent $ct(r)$

FIG. 7



- for $\alpha = cste$ and L_m grather, we represent $ct(r)$

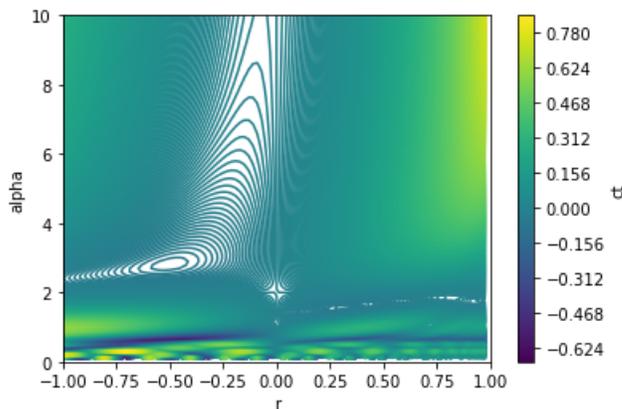
FIG. 8



Now, we represent the incoming geodesic

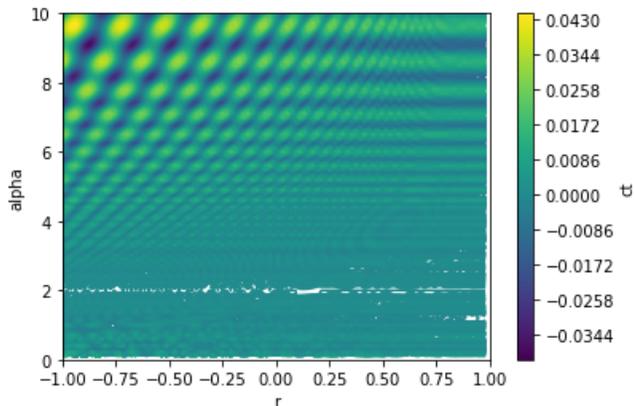
- for lagrangian fields lower, we represent $ct(r, \alpha)$

FIG. 9



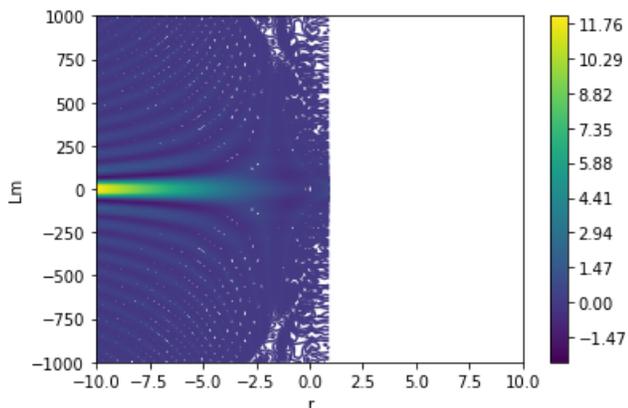
- for lagrangian fields upper, we represent $ct(r, \alpha)$

FIG. 10



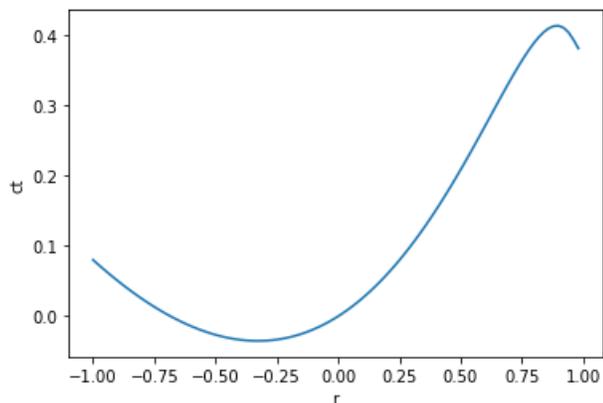
• for $\alpha = cste$, we represent $ct(r, L_m)$

FIG. 11



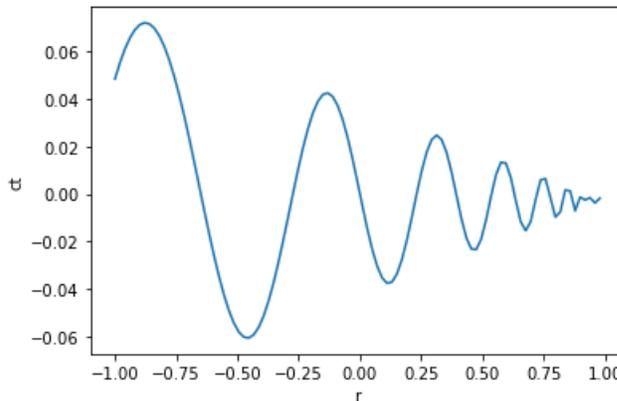
• for $\alpha = cste$ and L_m lower, we represent $ct(r)$

FIG. 12



• for $\alpha = cste$ and L_m grather, we represent $ct(r)$

FIG. 13



B. Orbits of material bodies Geodesics

The potential equation is given by:

$$V_{eff}(r) = -\frac{1}{2} \left[\left(\frac{l^2}{c^2 r^2} + 1 \right) \tan^2 \left(-\frac{kL_m}{4(\alpha-2)} [1 - \ln(1-r)] \right) - \frac{\epsilon^2}{c^2} \right] \quad (34)$$

$$\frac{dV_{eff}(r)}{dr} = -\frac{1}{2} \left[\left(\frac{l^2}{c^2 r^2} + 1 \right) \frac{kL_m}{4(\alpha-2)(1-r) \cos^2 \left(-\frac{kL_m}{4(\alpha-2)} [1 - \ln(1-r)] \right)} - \frac{l^2}{c^2 r^3} \tan^2 \left(-\frac{kL_m}{4(\alpha-2)} [1 - \ln(1-r)] \right) \right] \quad (35)$$

and the extremuns of V_{eff} are given by ⁷:

$$0 = -\frac{1}{2} \left[\left(\frac{l^2}{c^2 r^2} + 1 \right) \frac{kL_m}{4(\alpha-2)(1-r) \cos^2 \left(-\frac{kL_m}{4(\alpha-2)} [1 - \ln(1-r)] \right)} - \frac{l^2}{c^2 r^3} \tan^2 \left(-\frac{kL_m}{4(\alpha-2)} [1 - \ln(1-r)] \right) \right] \quad (36)$$

$$\left(\frac{l^2}{c^2 r^2} + 1 \right) \frac{kL_m}{4(\alpha-2)(1-r) \cos^2 \left(-\frac{kL_m}{4(\alpha-2)} [1 - \ln(1-r)] \right)} = \frac{l^2}{c^2 r^3} \tan^2 \left(-\frac{kL_m}{4(\alpha-2)} [1 - \ln(1-r)] \right) \quad (37)$$

$$\left(\frac{l^2}{c^2 r^2} + 1 \right) \frac{kL_m c^2 r^3}{4l^2(\alpha-2)(1-r)} = \sin^2 \left(-\frac{kL_m}{4(\alpha-2)} [1 - \ln(1-r)] \right) \quad (38)$$

$$\arcsin \left(\sqrt{\left(\frac{l^2}{c^2 r^2} + 1 \right) \frac{kL_m c^2 r^3}{4l^2(\alpha-2)(1-r)}} \right) = -\frac{kL_m}{4(\alpha-2)} [1 - \ln(1-r)] \quad (39)$$

$$\frac{4(\alpha - 2)}{kL_m} \arcsin\left(\sqrt{\left(\frac{l^2}{c^2 r^2} + 1\right) \frac{kL_m c^2 r^3}{4l^2(\alpha - 2)(1 - r)}}\right) + 1 = \pm \ln(1 - r) \quad (40)$$

$$(1 - r) = \exp\left(\pm \left[\frac{4(\alpha - 2)}{kL_m} \arcsin\left(\sqrt{\left(\frac{l^2}{c^2 r^2} + 1\right) \frac{kL_m c^2 r^3}{4l^2(\alpha - 2)(1 - r)}}\right) + 1\right]\right) \quad (41)$$

$$r = 1 - \exp\left(\pm \left[\frac{4(\alpha - 2)}{kL_m} \arcsin\left(\sqrt{\left(\frac{l^2}{c^2 r^2} + 1\right) \frac{kL_m c^2 r^3}{4l^2(\alpha - 2)(1 - r)}}\right) + 1\right]\right)$$

REFERENCES

¹J. L. W. TOUGMA and al, "The quantum relativity in extra-dimensions," *International Journal of Science and Research (IJSR)*, 2021.

²K. Schwarzschild, "Idotsber das gravitationsfeld eines massenpunktes nach der einsteinschtheorie. sitzungsberichte der deutschen akademie der wissenschaftenzu berlin, klasse furmathematik, physik," *und Technik*, p. 424.

³S. Carroll, "Spacetime and geometry: An introduction to general relativity," *Addison Wesley(Pearson Education)*, San Fransisco, 2004.

⁴S. W. Hawking, "Black holes in general relativity," *Commun. Math. Phys.* 25, (1972), 152166., .

⁵P. M. C, T. M, and W. H. F, ".,," *Ann. Phys.* 327 1078, 2012.

⁶L. G. i. e. P. P. Valeria Ferrari, "General relativity and its applications : black holes, compact stars and gravitational waves," *Boca Rat on,CRC, hors coll*, 2020.

⁷T. H. et Francisco S. N. Lobo, "Extensions of f (r) gravity :curvature-matter couplings and hybrid metric-palatini theory," *Cambridge monographs on (42) mat hemat ical physics*, 2018.

