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Determination Approach of the Sudan National Gravimetric Geoid Model

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Abstract
The Geoid Model is mainly used to determine the orthometric height in surveying, mapping, geospatial data infrastructure, and in the industry for public and private sectors to generate and transmit the vertical corrections for the RTK measurements for planning, land reclamation, development, and construction, oil prospection, etc. Such wide use of the data puts special requirements on the data quality and reliability of the geoid model results. To ensure the quality and high reliability of the geoid model, extensive measurements, testing and validation of the geoid model should be implemented.
The development of the Sudan National Gravimetric Geoid Model (SNGGM); requires absolute and relative gravity measurements, GNSS grid observations for existing and newly established benchmarks, and Geodetic Control Points (GCPs). As well as the acquisition and analysis of existing local and global gravity data that is needed for geoid model computations and fitting processes incorporating gravity, leveling, GNSS, and DTM data. The final validation of the computed Geoid Model will be done using the ground truth data, and independent ground control points will be used for the quality check of the derived Geoid model.

Keywords: - Ground Control Point (GCP), Digital Terrain Model (DTM), Global Navigation Satellite System (GNSS), International Terrestrial Reference Frame (ITRF2008), Sequential Multipole Analysis (SMA).

1. Introduction

The geoid is well known in geodesy, as an equipotential surface of the earth's gravity field which approximates the figure of the earth. The main function of the geoid is to be used as an adopted reference surface for leveling and heightening purposes. The determination of precise geoid model has been a significant research subject in geodesy and of interest in today's use of satellite positioning, and many other applications, including but not limited to geomatics, engineering, geospatial, and scientific applications, such as [7, 9, 10]: **1.** Reference surface for leveling. **2.** Vertical datum for Orthometric heights; **3.** Civil engineering applications in building developments and construction, road and rail constructions, tunneling, routes alignment, design and construction of drainage and Sewer systems, water supply pipelines, oil pipelines, electricity power grids, 4**.** A regional and local vertical datum, continental and recent crustal vertical movements, and deformations monitoring. **5.** Studies of the ocean and Earth's interior and exterior. **6.** Studies, alignment, and locations for deposits of gas and oil, and **7.** Marine navigation, marine mapping, and hydrographic surveying.

With a full understanding of the geoid model requirements, Sudan has to create its new enhanced gravimetric geoid model, to replace most of the conventional leveling use and applications in geospatial, engineering, and mapping. The geoid model is considered to be a cost-effective means of orthometric height determination. Additionally, users can use and benefit from information obtained from emerging satellite positioning, satellite gravity missions, and precise global geoid models. This will increase knowledge about the mechanism and how the earth works in the areas of the earth's gravity field and its functionally related geoid heights, height anomalies, gravity disturbances, gravity anomalies, and their variations.

The development of the National Gravimetric Geoid Model requires a. Absolute gravity measurements. b. Relative Gravity and GNSS grid observations for existing and newly established benchmarks and Geodetic Control Points (GCPs). d. Acquisition and analysis of existing gravity data including airborne gravity data; marine gravity data; coastal gravity data; terrestrial gravity data of surrounding areas (absolute and relative), satellite gravity data; satellite altimetry; DTM; Earth Global Models; and e. Computation of fitted Geoid using the adopted processes and software incorporating gravity, leveling, GNSS, and DTM data. The resulting geoid model of Sudan is to be accurate up to 1-3cm level.

2. Methodology of Geoid Modelling

2.1 Definition of the basic terms required for geoid computation

(a) **Gravitational and gravity potential**: - From Newton's law of gravitation, we define the potential, V as [3, 12,18, 19]:

$$V = G\frac{m}{l^2} \quad (1)$$

Where G is the gravitational constant, m is the earth's mass, and l is the length. The gravity potential of the Earth. W at any external point P may be expressed by the simple sum of the gravitational potential V and the centrifugal potential, ϕ by:

$$W = V + \Phi \quad (2)$$

Where;

$$\Phi = ½\, \omega^2\, l \quad (3)$$

Where ω is the angular velocity equals 7.2 92115 x 10^{-5} rad/s.

(b) **Gravitational potential**: - Gravitational potential V of the Earth at the point P may be expressed by the Newtonian integral using the volume density of the body;

$$V = G\iiint_v \frac{\rho}{l}\, dv \quad (4)$$

dv is an element of volume v; dm is an element of mass; l is the length between the mass element dv and the attracted point P with unit mass.

(c) **Centrifugal potential**: - Centrifugal potential, W of the Earth at the point P, can be calculated based on the angular velocity of the Earth's rotation and the radial length.

$$W = W(x,y,z) = V + \Phi = G\iiint_v \frac{\rho}{l}\, dv + \frac{1}{2}\omega^2\left(x^2+y^2\right) \quad (5)$$

(d) **Gravity**: - Gravity vector g is the gradient of the gravity potential.

(e) **Disturbing potential**: - As it is well known, the application of the above definitions of gravity functions to the determination of the Earth's gravity potential requires corresponding reductions from the Earth's surface to the geoid. To do so, the density of masses above the geoid must be known or derived for the computation of gravity.

(f) **Height anomaly** is distance or connected with the conception of normal heights and closely corresponding to the geoid undulation, which is connected with the conception and use of orthometric heights, in such a way that the ellipsoidal height can be represented at every point of the Earth's surface.

(g) **Remove-restore technique**: - The great influence of the terrestrial masses outside the geoid (in the land areas) causes the necessity of applying the so-called "remove-restore technique "for computation of the (quasi)geoid. The effect of these masses should be removed from observations and it must be restored in geoid heights [3,12].

(h) **Sequential Multipole Analysis (SMA) method**: Each multipole is a special point object placed at the point i inside the Earth. It is characterized by its degree and by geocentric spherical coordinates that are geocentric distance, latitude, and longitude, respectively. Location of radial multipole (i) and external point (P). By [12], at any external point P with geocentric spherical coordinates, the disturbing potential may be described by infinite sum, and the functions can be computed using the recursive formula [4, 12], which is based on Legendre polynomials.

(i) **Approximation of disturbing potential by potentials of radial multipoles**: According to [5] "The set of the potentials of radial multipoles of zero degree (without zero-degree solid spherical harmonics) and the set of the potentials of eccentric dipoles are the non-orthogonal base systems in the space. On the whole, every set of the potentials, if n>1, is the linear independent and complete base system on any subset of without all linear combinations of solid spherical harmonics from zero up to n-1 degree." This assertion holds a possibility of approximation of the disturbing potential by potentials of non-central radial multipoles. With special consideration to the disturbing potential. The solution for the disturbing potential T should be obtained by the general variational principle.

2.2 Geoid Modelling Activities

Determination of the precise geoid model will include new datasets collection and assessment of the available geoid models in Sudan [3, 15, 16] and orthometric height data, gravity measurements, gravity reduction, and geoid modelling. This work can be divided into four main activities.

2.2.1 Activity.1 Gravity observations

Gravity measurements should be made across the entire Sudan area and its surroundings (if possible), observations will be made on a grid at 1km spacing in the buildup areas and 2.5 or 3 km in other areas, but precise adherence to the grid pattern is not essential [16]. Gravity observations should be tied to the absolute gravity stations and maybe to international gravity networks with the accuracy compatible with the new gravimetric geoid model determination. The purpose of the determination of the national precise geoid model; includes increasing the density of ground control detail, with particular attention to obtaining accurate orthometric heights. This activity can be divided into the collection of the existing data and gravity observation:

(a) Collation of Control Data

The following sub-activities will take place:

- Collection of the existing height data;
- Field-check and GNSS measurement of existing control points;
- Establishment of new ground control points and benchmarks.
- Preparation of the survey database;
- Extension of the database with GNSS height information;
- GNSS Static observations;
- Baseline calculation, and survey database completion.

Additional data collection, processing, and verification will also be needed, such as: -

i. Global Geopotential Model: - Concerning the remove-restore technique for the geoid computation, the newly computed global geopotential model should be used for removing the long wavelength part of the gravity field. The solution represents a high-resolution global gravity model constructed from the GOCE/GRACE/LAGEOS satellite mission and altimetry/gravimetry surface gravity data [3]. Comparing XCM2019e to EGM08 global geoids one can observe small differences (up to 12cm) due to different data sets coming from GOCE satellite and perhaps altimetry-gravimetry marine/land gravity. Also, gravity data from the global model can be used as an extension for the area to avoid the border effect during the geoid modeling process. For such extensions of the existing data set, gravity values from satellite-only solution global models may be used.

ii. Gravity anomalies from other satellite missions: - In addition to inverted altimetry data, the global set of ocean-wide 1'×1' derived gravity anomalies, can be taken for processing as well. As mentioned above this data set can be also used as an extension to cover neighboring countries' areas. Such a strategy finally gives us much more stable results and higher accuracy of the computed geoid values close to the selected area borders [18, 19].

iii. DEM and corrections: - Due to the topography signal over some areas in Sudan such as the Red Sea mountains in the east, Nuba mountains in the south, and Jebel Marra mountains in the west, a terrain correction should be applied for the gravity data. One of the impacts of local terrain corrections is to remove the correlation of free-air anomalies with heights and avoid the aliasing that might appear when gravity stations are systematically observed at different levels than the average topographic level, as gravity points tend to be located in valleys of the mountains. Such aliasing errors can be very large and critical for the geoid computation, but we can avoid this by applying terrain corrections. For this purpose, the global digital elevation model may be used, or any local DEM. Before the corrections, the DEM data set should be resampled to the same grid size as gravity data. Using the Gauss-Kolmogorov-Wiener interpolation algorithm with 9 nodes [9], topography data may be converted to the grid. By applying the topographic correction all onshore free-air anomaly data will be reduced. In addition to terrain correction, computation of the topographic-isostatic corrections in the frame of the Airy-Heiskanen model [17] can be done for the isostatic geoid computation for geophysical studies and interpretation of the tectonic of the region.

iv. Other gravity data sets: - In addition to the existing worldwide gravity data, all possible existing local gravity data sets should be checked and included for the geoid modeling.

(b) Gravity Observations

This task is comprised of the precise measurement of the gravity all over the Sudan area. The measurements will be executed with gravimeters. The gravity measurement locations and the ellipsoid heights are done with differential GNSS. The gravity measurements will be tied to absolute gravity points. The accuracy of the gravimeter instruments used should be 0.05 mGal or better. The data will be adjusted by rigorous least squares adjustment software. The gravity points will be measured with advanced recent high-quality dual-frequency differential GNSS instruments and the positions will be post-processed to ensure the quality.

2.2.2 Activity.2: Gravity reductions

Gravity anomalies shall be computed at all observed gravity points. Methods of gravity reductions will be conducted using the following major formulae in the computation. The following equation [2, 17, 18], summarizes the corrections (reductions) to the observed gravity:

$$g_o = g_{stn} + C_{tide} - C_{drift} - g_{base\text{-}out} + g_{0(base)} \qquad (6)$$

Where: g_o=observed gravity in milligals, g_{stn}=raw gravity reading at each station, C_{tide}=tidal effect (in milligals), $g_{0(base)}$ = absolute value of gravity at the base.

$$C_{drift} = (g_{base\text{-}out} - g_{base\text{-}in})/(t_{base\text{-}out} - t\, g_{base\text{-}in}) \times t_{stn} \quad (7)$$

Where, $g_{base\text{-}out}$, $g_{base\text{-}in}$ are base readings at start and end of loops $tbase_{-out}$, $t_{base\text{-}in}$ are base times at start and end of loops

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right) \qquad (8)$$

Here, the Latitude Correction should be performed to correct observed gravity for the effects of the differential centrifugal acceleration due to the reduction in angular velocity at the surface of the earth with latitude (this acceleration could be at its maximum at the equator where the observed gravity would be at its minimum). This correction is solely a function of latitude (ϕ) obtained from survey coordinates. From this, expanding until second order, we obtain

$$\gamma_h = \gamma\left[1 - \frac{2}{a}\left(1 + f + m - 2f\sin^2\varphi\right)h + \frac{3}{a^2}h^2\right] \qquad (9)$$

Where a is the semi-major axis of the reference ellipsoid, f is the flattening, γ_h denotes the normal gravity for a point at latitude φ, in height h above the ellipsoid and γ is the normal gravity on the ellipsoid itself, and h is the ellipsoidal height.

Free Air correction would be performed to correct for the fact that gravity decreases (as the square of the distance) from the elevation datum [3, 4]. This correction is a function of station elevation above the datum and is approximated by:

$$C_{fa} = 0.3085958\ H \qquad (10)$$

Where, C_{fa}= free air correction and H= orthometric height

Bouguer gravity is calculated using the following equation:

$$G_b = G_o - G_t + C_{fa} - C_b + C_t \qquad (11)$$

Where: G_o=observed gravity; G_t= theoretical gravity calculated from the station's latitude; C_{fa}= free air correction; C_b= Bouguer correction; C_t= terrain correction; G=universal gravity constant; ρ=density. Bouguer correction would correct for the gravitational attraction of the masses between the station and elevation datum (geoid). This correction is dependent on elevation and the mass density approximated by: -

$$C_b = 2\pi G \rho h = 0.041896325 \rho\ H \text{ mgal.} \qquad (12)$$

This indicated that the following reductions and geodetic functionals shall be applied to the gravity input data:

a) Free-air gravity anomalies and terrestrial gravity anomalies.
b) Gravity disturbances.
c) Absolute gravimetry data processing.
d) (Quasi) geoid heights and indirect effect of geoid modeling.

The coordinates of data points and coordinates of result points (grid nodes) are to be expressed in geodetic latitude and geodetic longitude; using one of the following heights: Orthometric height; Normal height, or Ellipsoidal height. All the data shall be processed in an adopted global geodetic reference system (for example, in GRS80). Despite of this requirement; geodetic coordinates of data and result points can be referred to as different ellipsoids such as the WGS84 ellipsoid. Results of the (quasi) geoid computation can be presented in the two systems [17], i.e. Tide-free system (all tidal effects are excluded) and the Zero-tide system (direct tidal effects are excluded whereas indirect effects have remained).

2.2.3 Activity.3 Geoid determination

The geoid should be computed, based on the agreed method, using heterogeneous data composed of gravity anomalies, satellite and global models' data, and DEM, and the method of geoid computation made with the required accuracy standards. One of the well-known geoid computation models that should be used, such as: -

(a) **Modified Stokes' formula:** The Stokes' formula or Stokes' integral shall be used for geoid model computation. It is considered to be the most important formula in physical geodesy because it is used to determine the geoid from gravity data [3]. The geoid determination problem is expressed as a boundary value problem in the potential theory based on Stokes' integral. Hence, the gravitational disturbing potential, T can be computed, and from which the geoid undulation can be derived. The disturbing potential (T) is the difference between the actual gravity potential on the geoid surface W and the normal potential value U on the reference ellipsoid surface. Here, Bruns' formula (one famous formula in physical geodesy), which relates the geoid undulation to the disturbing potential can be expressed as [2,17, 19]:

$$N = \frac{T}{\gamma} \quad (13)$$

where γ stands for the normal gravity (or standard gravity) on the surface of the reference ellipsoid. By substitution, the disturbing potential, Stokes' formula, and the surface integral in Stokes' formula can be applied and N can be derived as:

$$N = \frac{R}{4\pi\gamma} \iint_{\sigma} S(\psi)\Delta g \, d\sigma \quad (14)$$

where R is the mean Earth radius, ψ is the geocentric angle, Δg is gravity anomaly, dσ is an infinitesimal surface element of the unit sphere and S is the Stokes function, which can be expressed as a series of Legendre polynomials over the sphere [15].

The surface integral has to be truncated to the gravity anomaly area and then an estimator of the geoid height can be obtained, which is usually called the truncation error of Stokes' formula. In such a formula, the remote zone (the area outside the gravity area) can be considered. Molodensky stated that the truncation error of the remote zone can be reduced when Stokes' formula combines the terrestrial gravity anomalies and long-wavelength, as a contribution of the Global Gravitational Model (GGM). Within the satellite's era, it becomes possible to generate geoid models in a global sense, by combining information from the GGM with Stokes' integration over local gravity data [3]. In Geoid modeling, two components should be considered: the long-wavelength component provided by GGM (using spherical harmonics) and the short-wavelength component from local gravity observations. By using local gravity data, Stokes' formula will be truncated to the inner zone. This causes truncation errors due to the lack of gravity data in remote zones. These errors could be ignored or reduced by modifying Stokes' Kernel. The modification parameters are varied, depending on the quality of local gravity data, the radius of integration, and the characteristics of the GGM.

(b) **Fast Fourier Transformation (FFT) method:** This method can also be used to compute the gravimetric geoid model. Stokes formula can be given in the approximate form of the planar kernel [18], [19], in which the residual geoid height and the gravity residuals are calculated from the terrestrial gravity data. At the same time, the gravity anomaly is computed from the geopotential model and the terrain correction by FFT. The final geoid solution, N, is obtained based on the remove-compute-restore technique using the FFT method. It should be noted that this method requires one regularly distributed grid of gravity points.

2.2.4 Activity.4: Geoid model fitting

The gravimetric geoid should be computed across the entire Sudan area using the obtained data: - observed gravity anomalies, high-accuracy global model of the geoid, existing and Open-access DEMs, and newly determined and existing Orthometric/ Ellipsoidal heights. Fit the gravimetric geoid with special consideration to the indirect effects of gravity reductions, including: -.

(a) The Ellipsoidal Correction: Geoid determination by Stokes' formula holds only on the spherical boundary, the mean Earth sphere with radius R. Since the geoid is assumed to be the boundary surface for the gravity anomaly, the ellipsoid is a better approximation for it.

(b) The Downward Continuation Correction: The analytical continuation of the surface gravity anomaly to the geoid is necessary for the application of Stokes' formula for geoid estimation. The necessity of this is when the topographic effect is reduced; the observed surface gravity anomalies must be downward continued (DWC) to the geoid. DWC has been done in different methods, but the most common method is the inversion of Poison's integral [5, 18], which reduces the surface gravity anomaly for direct topographic effect and then continues the reduced gravity anomaly downward to the sea level.

(c) Molodensky gravity anomalies: As it is supposed that all masses inside the geoid are known, it is very difficult to fulfill this assumption in practice. For this reason, Molodensky introduced his theory to overcome this problem in Stokes' theory. Molodensky used Earth's surface and the telluroid to describe the anomalous gravity field, similar to the geoid, and reference ellipsoid in Stokes's theory. The gravity anomaly of Molodensky can be defined as the difference between the actual gravity on the Earth's surface and the normal gravity on the telluroid. After removing gross errors, gravity data will be ready to be regularly gridded using interpolation.

(d) DEM and corrections: Due to the topography of the area (especially over the mountainous areas in Sudan) a terrain correction should be applied for all our gravity data. One of the impacts of local terrain corrections is to remove the correlation of free-air anomalies with heights and avoid the aliasing that might appear when gravity stations are systematically observed at different levels than the average topographic level, as gravity points tend to be located in valleys of the mountains. Such aliasing errors can be very large and critical for geoid computation [3], but this can be avoided by applying terrain corrections. For the purpose, of the terrain correction, a Digital Terrain Model Data Set should be provided and used. As necessary the available global digital elevation models, DTM of required accuracy and resolution shall also be used.

(e) The Combined Topographic Correction: The combined topographic effect is the sum of direct and indirect topographical effect on the geoid; it can be added directly to the approximate derived geoid height value, considering the mean topographic mass density and H is the orthometric height. These errors can also, be reduced significantly by using a special interpolation technique.

(f) Fit GNSS/Levelling Points: the last step is to ensure consistency with the levelling and GPNSS systems by fitting the gravimetric geoid to the available points with information obtained by:

$$N^{GPS} = h - H \qquad (15)$$

Here h is the ellipsoidal height in the WGS84 system, and H is orthometric height in the national vertical datum. The function models the final geoid

$$\varepsilon = N^{GPS} - N^{gravimetric} \qquad (16)$$

The difference is modelled by a 4-parameter trend function (corresponding to a Helmert transformation) and least squares collocation, using an empirical second-order Markov model. The parameters of the collocation process (the correlation length and the signal noise) will be determined from the fit to the gravimetric geoid to the GNSS/levelling values. The requested correlation length in the region of 50 km indicates the need of a smooth correction surface.

2.2.5 HEIGHT DISCREPANCY ANALYSIS

The obtained heights were defined through the geoid and the correction surface and will be compared to the archive heights for areas where mapping exists. Examination and evaluation of these height discrepancies will be conducted, and major outliers will be flagged. Graphical representations of misfits of different points should be used extensively. Errors should be flagged as outliers depending on the assumed class of points. It is expected that some lower accuracy height points could be used along the Sudan international boundaries to constrain possible geoid errors.

3. Absolut and Relative Gravity Measurements

3.1 General

The number and location of absolute gravimetric stations have to be selected, designed, constructed, and then observed. Relative gravity observations will be conducted together with the absolute gravity observations. Usually, the existing absolute gravity station sites in a country should be checked and evaluated. The following absolute gravity station specifications are to be considered:

- The number of absolute gravity stations, their distribution, and required territory area coverage.
- The absolute gravity stations should be designed before construction & observation, following international standards and specifications (such as NGS standards).
- preserving stability and the long existence of gravity points.
- field observation of the network using most modern techniques, adopting the National Geodetic Network.
- planning to observe several grid points for the entire area under consideration with the grid spacing adopted (such as a grid of 1x1 km spacing where highly accurate geoid is required, 2.5 X2.5 km grid spacing for built-up and developed areas, and with the grid spacing 3X3 km for rest of territory and 5X5 Km spacing for desert areas).

- The gravimetric observations will be carried out using most modern relative digital gravimeters (such as CG-5 and similar) that ensure the high accuracy of the field observations (better than 0.01 mGal).
- the gravity measurements should also be made on the geodetic control points and leveling benchmarks for the refinement of the computed geoid model.
- The methodology of the gravimetric surveys and data processing must ensure a high precision of the Geoid Model computation, so that, the resulting Geoid Model accuracy in all areas should be in the range of 1-3 cm.

For absolute and relative gravity observations, the following should be conducted: -

(a) Monumentation: The process of monumentation should start with a geological survey for the identification of suitable sites to construct the gravity and geodetic control points. The absolute gravity stations have to meet modelling-specific demands.

1. Stations must be accessible by a vehicle within an appropriate distance if the gravimeter is operated from a car.
2. The pillar must be made from a single block of concrete.
3. The diameter of the top of the pillar must be at least 45 cm and flat, and the mass concrete pad should be 0.5 m thick.
4. Stations should be located as far as possible from any urban/settlement or any source that might generate seismic noise.

(b) GNSS Observation: To carry out GNSS observation for the newly constructed Geodetic control points along with the existing Geodetic First order control points.

(c) Ground Gravity Survey: To acquire accurate GNSS-positioned ground gravity measurements; field checking by gravity tie-in data of every absolute base network gravity station; use only verified absolute base network gravity stations to provide unbiased control points for the survey.

(d) Geoid Computation: Use all available gravity data (the new ground gravity data, the existing gravity data and gravity from other available sources) for the geoid computation.

(e) Quality Assurance Plan: the quality approach should be used for describing the mechanisms implemented within the framework. So, in this respect, the quality management means quality assurance procedures and control of processes and products in order to reach the expected quality. The quality assurance requirements concern: assigning responsibility; meeting the deadlines set, the intervention process (organization, means and methods); production monitoring and associated quality controls.

3.2 Gravity Measurement Procedures

Detailed schedule for absolute gravity survey and vertical gravity gradient determination at each station should be presented, such as: -

- Absolute gravity survey (methodology for single station), in which, two independent setups of the absolute gravimeter can be used, together with the instrument setup for about 15 to 20 minutes; each setup consists of 8 measurement sets; and each measurement set consists of 120 single drops (gravity determinations) and the drops performed at the rate of 1hz, 1-minute break between sets, total single setup time is 24 minutes, total single station occupation is estimated at ~2 hours.

- Vertical gravity gradient determination: - Two independent vertical gravity gradient determinations are to be used; readings taken on two or multiple heights; instrument setup takes 15 to 20 minutes; total single gravity gradient determination is ~1-hour, total single station occupation is estimated to ~2 hours
- The On-site vertical gravity gradient verification: if < 10 µgal/m, result approved, if > 10 µgal/m, consecutive vertical gravity gradient determination should be performed.

In practice, absolute and vertical gravity gradient measurements are to be performed, at the same station. The absolute gravity measurement is to be done first, followed by the vertical gravity gradient. Total measurement time will be 4-5 hours, knowing that, gravimetric surveys require good weather conditions (i.e. no rain, light wind, moderate heat), so that, weather suitability is to be included at the planning stage.

3.3 Absolute Gravity Instruments

Two types of gravimeter instruments are to be used for absolute and relative gravity measurements: -.

(a) Absolute gravimeter: In general, the accuracy of 10 µGal will be assured by a careful measurement methodology assuming the performance of two independent setups which will be evaluated in terms of their consistency to account for gross errors. During reprocessing, special care should be taken to include up-to-date calibration parameters for the gravimeter and up-to-date correction models in the reprocessing process.

As the gravity value will be provided at the benchmark level, the accuracy of the vertical gravity gradient will contribute to the accuracy of the determined absolute gravity value. Therefore, the vertical gravity gradient determination has to undergo a strict requirement of consistency of 10 µGal/m between two independent determinations. The accuracy of the vertical gravity gradient determination has a direct impact on the results from the gravimeter.

The following considerations are to be used for absolute gravity measurements:

- Accuracy (Uncertainty): 0.010 mGal or better from extensive experience and usages,
- The uncertainty of the gravimeter can be as small as 0.007 mGal.
- The gravimeter operates by using a free-fall method. The object is dropped inside a vacuum chamber and its position is monitored very accurately using a laser interferometer in time controlled by a precise frequency standard.

In principle, the free-fall trajectory of a dropped object is referenced to a very stable active spring system called a “Super spring”. The super spring provides seismic isolation for the reference optic to improve the noise performance of the gravimeter. The optical fringes generated in the interferometer provide a very accurate distance measurement system that can be traced to absolute wavelength standards. The very accurate and precise timing of the occurrence of these optical fringes is done using an atomic rubidium clock that is also referenced to absolute standards.

Figure 1. A10-020 absolute gravimeter setup on a bedrock station in Sweden

b. Instrumentation calibrations: All instruments to be used are to be calibrated to assure the standard of their performance for all gravimetric surveys. The Calibrations of the absolute gravimeter are to be performed for the internal components of the gravimeter. The internal calibrations include comparing absolute gravimeter standards for the laser, rubidium clock and barometer with corresponding national metrological standards, while the external verifications will include additional gravimeter stability measurements against the gravity reference.

3.4 Absolute gravity Stations

The gravity observation and processing performed can be summarized as follows:

- Design of the Gravimetric Network, gravimetric Control, Gravity Datum and Network field reconnaissance, GNSS instruments and methods of observation.
- Observation of the Gravity Network, Instruments for absolute gravity measurement, operations of Absolute gravimeters, Monumentation of the stations for gravity measurements and Data processing and computation of Absolute Gravity stations, gravity reductions and corrections.

- Observation of the 1x1 km and 2.5 x 2.5 km grid network, Planning of the observations, methodology, Instruments and equipment for Relative gravity measurements
- Gravimetric data processing and computation of geoid, General approach and methodology, Modified Stokes' formula, The Ellipsoidal Correction, The Downward Continuation Correction, Molodensky gravity anomalies, DEM and corrections, the Combined Topographic Correction
- Additional data collection, processing and verification: Global Geopotential Model, Gravity anomalies, gravimetric satellite missions
- Software for the Geoid Model Computation

3.4.1 Design of the Gravity Control

The principle of the design of the Absolute Gravity Control (AGC) is to create a single order gravity control consisting of stations of homogenous distribution across the territory area. As all stations will obtain absolute gravity values no adjustment of the gravity control will be needed. Furthermore, co-located GNSS and gravity stations will serve as an integrated geodetic network. Overall, homogenously distributed stations will be used. The distances between the stations have to considered, and enough for the efficient use of gravity control, especially for the performance of relative gravity measurements.

3.4.2 Gravity Datum

The gravity datum is to be realized by means of absolute gravity surveys conducted with absolute gravimeters of metrological consistency verified during comparison campaigns of absolute gravimeters. The main principles of the definition of a gravity reference level should be taken in to account to resolve variations between the fields of absolute gravimetry and metrology.

The gravity reference level of the gravity control should be defined by the absolute gravimeter, which will be carefully controlled during the observation. As; The solution should make the designed gravity control an up to date with the leading world standards in the field of gravimetry.

3.4.3 Network field reconnaissance

The network field reconnaissance should be done based on the results of initial design of the network. All the station's location selected at the design stage should be inspected, check the site conditions compliance with the requirements. If the site found unsuitable for its purpose, a new location should be selected.

At the field reconnaissance it will be checked if all the stations are suitable for the gravimetric measurements. This will include the field inspection of all the sites for the gravimetric measurement of the fundamental and basic networks including the designed Control Points (checked at the stage of the monumentation of the Geodetic Network), precise levelling benchmarks, GCPs stations. The characteristic of these locations will have to satisfy both GNSS sites suitable location requirements and Gravimetric sites location parameters.

3.5 Observation of the Gravity Network

3.5.1 Observation methodology

All absolute gravity surveys will be performed with the selected absolute gravimeter. Measurement methodology for the absolute gravity determinations at each station with the absolute gravimeter consists of 2 independent setups and their corresponding single drops. On site both setups will be verified in terms of their consistency and elimination of gross errors. If the difference between two setups will be larger than 0.010 mGal another setup will be performed. For each site a specific uncertainty of the determined gravity value will be evaluated. The data processing and quality control measures will be done.

3.5.2 Determination of the vertical gradient

At each absolute gravity station vertical gravity gradient must be determined in order to provide data required for the reduction of absolute gravity measured with the gravimeter at the height of about 80 cm above the benchmark – to the benchmark level (or any needed level for relative gravity measurements). The vertical gravity gradient will be calculated from gravity survey with precise relative gravimeters on at least three levels above the benchmark, with the use of a special stand. Two independent vertical gravity gradient determinations will be performed. On site both determinations will be verified in terms of their consistency and elimination of gross errors. If the difference of the reduction of the gravity value (from gravimeter measurement level to benchmark level) will be larger than 0.010 mGal another measurement will be performed.

Standard observation log for each gravimeter setup will include station information, monumentation state description, weather parameters, file name of the measurements, instrumental parameters, field obtained absolute gravity value. Due to the sensitivity of gravimeters to high temperatures the measurements are to consider the season and time of observation.

3.5.3 Principe of operation of Absolute gravimeters

Absolute gravimeters operate by using a free-fall method. An object is dropped inside a vacuum chamber and its position is monitored very accurately using a laser interferometer. The free-fall trajectory of the dropped object is referenced to a very stable active spring system called a “Super spring”. The Super spring provides seismic-isolation for the reference optic to improve the noise performance of Absolute gravimeters. The optical fringes generated in the interferometer provide a very accurate distance measurement system that can be traced to absolute wavelength standards. Very accurate and precise timing of the occurrence of these optical fringes is done using an atomic rubidium clock that is also referenced to absolute standards.

The measurement is directly tied to international standards, and this is what makes them absolute gravimeters. By basing the measurement on these standards, the system is inherently calibrated and will neither drift nor tare over time.

3.5.4 Data processing and computation of Absolute Gravity stations

Data processing will be performed with the provided Software during the survey. Data will then be re-processed with the use of up-to-date calibration parameters of laser

wavelengths and rubidium oscillator frequency. All absolute gravity measurements will be corrected with the most up to date correction models (solid Earth tides, ocean tidal loading), Earth Orientation Parameters (polar motion correction). In addition, the Influence of gravity variations due to global hydrology will be taken into account.
The offset of the gravimeter determined during International Comparisons of Absolute Gravimeters will be taken into consideration to establish the gravity reference level. The Gravity Observation main deliverables will be as follows:

(1) Technical Report on the establishment of the absolute gravity network;
(2) Data Sets in the agreed formats and structures with the results if the gravity measurements, preliminary processed and checked;
(3) Network computation data and results in agreed format;
(4) Site Logs and descriptions of each gravity station with necessary information;
(5) List of coordinates and heights of the stations;
(6) Layout of the location of the stations of Absolute Gravity Network;

3.5.5. Data processing and quality check

After each working day, data analysis will be performed. Gravity data will be dumped to notebook every day, and checked for anomalous drift or tilt. Processed gravity data should be available at the end of each day. On site processing of gravity data will contain:
- calculation of gravimeters drift and conversion of readings to milli gals,
- Earth tied corrections.
- Drift corrections and meter height correction.

The data processing will be carried out upon the completion of the field blocks designed at the planning stage to ensure smooth data processing and discovering of any possible errors. Special attention will be paid for this procedure as it is an important part of the data preparation for the geoid computation. Necessary correction to the raw data will be applied including instrumental, tidal, drift and any correction to the observed data.
All field observation and the results will be a subject of the rigorous quality control according to the established Quality Assurance and Quality Control (QA/QC) procedures. The QA/QC procedures will be applied to the gravity measurements as well as similar standard data quality control procedures will be applied to the GNSS observations of the gravity points. If any issues of the data quality or non-compliance with the procedures will be discovered during the data quality check additional measures will be implemented to avoid similar issues in the course of further observations and proper instructions should be provided to the field crews on the resolution of the issues.
The quality check and control will be organized as continuous process and all the observation blocks will be checked up on completion to ensure that any issues or errors will be discovered at earliest possible stage and necessary corrective measures will be applied for further observations process.
The Gravimetric data processing and computation of geoid should include:
- Gravity-meter drift rate evaluation will be done.
- Site calibration will be done using absolute gravity Locations
- Tying stations to stations by following the sequence of difference loop.
- Corrections to the relative measurements by post processing.

- Difference loop sequence adjustment.
- Network adjustment of the relative gravity measurement at each station.
- applying a mathematical model to determine absolute gravity values
- Lease square fit analysis for the data sets to test the accuracy for each station

3.5. RELATIVE GRAVITY MEASUREMENTS

The primary surveying technique to be used for gravity survey requirements is relative gravimetry. Gravity value at each station will be determined with relative gravimeters an example is shown in figure.2. An amount of approximately 10% of all stations will be resurveyed for accuracy assessment and all measurements will be done with reference to absolute gravity network.

To assure efficient execution of the gravity surveys with the gravimeters, they need to be placed on stable ground to a station marked in the field (i.e. at least with a wooden pole) to determine the exact position of the station.

Figur.2 LaCoste & Romberg relative gravimeter measurement setup with carrying case.

2.3 Calibration of the relative gravimeter

Relative gravimeter requires scale factor determination which is needed to convert the gravimeter readings to gravity values. Relative gravimeters are calibrated on the gravimetric calibration baselines. The calibration procedure is performed as follows: Relative survey is carried out with the scheme [15] between two stations of known gravity difference, on which absolute gravity measurements were performed. Gravity difference measured by the relative gravimeter is divided by the gravity difference calculated from absolute gravity surveys. This procedure is carried out on a few spans of the gravimetric calibration baseline to provide independent scale factor determinations.

4.2 Relative gravity survey methodology

The methodology used should maintained and verified as follows: -

- o multiple independent recording periods for the relative gravimeter 1-3 minutes long each to verify on site consistency;
- o on site mean gravity reading to reading value agreement verification: if < 10 µGal, result approved, if > 10 µGal, consecutive records are performed;
- o instrument occupation per station takes up to 10-15 minutes, once gravimeter is set up, including position determination;
- o single gravity campaign will begin and finish on the absolute gravity station with repeating at least one of the surveyed stations along the way (for gravimeter drift control) if the duration of the campaign exceeds 3-4 hours;
- o Single gravity run will cover as many stations as possible but will be planned not to last longer than 4-5 hours for gravimeter drift control;
- o Up to 2 gravimeters are planned to perform measurements at the same time,

4. Geodetic Control Network

4.1 Geodetic Network design

The geodetic network design will be developed to achieve the accuracy requirements for both gravity stations and leveling benchmarks. The network and observation methodology will be designed with sufficient redundancy to detect or isolate possible blunders and ensure high reliability of the observation results.

The network design will be based on the use of the geodetic reference stations for the GCP observations. The reference station data will be analyzed for consistency, availability, and quality before using according to the procedures adopted for data computation.

A Geodetic Network of minimum constraints consisting of the required Geodetic Control Points shall be designed, with the following properties:

- This Network shall form the basis of GNSS observations as well as Gravity observations.
- Sides of triangles shall be the baselines for observations.
- Triangles have been selected with the shortest side lengths.
- Oblique triangles with small acute angles to be avoided
- GNSS instruments shall occupy the vertices of a single triangle or adjacent triangles to have small baselines.
- Absolut Gravity Network shall be part of this Network.

5.2 Network Monumentation

A typical benchmark/geodetic control disks should be made as specified (for example of aluminum, brass, or bronze; 9 centimeters in diameter, and have a domed surface to support the foot of a levelling rod, and a center point for plumbing survey equipment). Soil penetration test (SPT) shall be carried out at each location of the BM/GCP site before construction to know the properties of underlying strata for drilling/digging of holes to a specific depth.

Monument Design: - A concrete monument shall be prepared/constructed as per the agreed design specification and parameters. Local ground conditions, such as hard soil types with subsurface rock, may prohibit desired monument depth; whereas, softer, sandy soil types may require slightly deeper monuments to assure stability. Typical, monuments for the geodetic control network are designed to satisfy the requirement of the GNSS observation and Gravimetric observations of the basic network. To avoid setting concrete monuments in areas affected by sliding or other potential movement, such as in slopes and all earth-fill situations should consider the following: -

1. Station Designation: Station designation and setting year on the top surface of the disk shall be stamped before setting.
2. Drilling / Digging the Hole: The hole shall be dug/ drilled as per the requirements, and then backfilled with concrete mix. A smooth surface near the top of the monument is less susceptible to damage by frost or other forces than unfinished tops.

4.3 Geodetic Network Observation

Differential GNSS carrier phase surveying is used to obtain the highest precision from GNSS and has direct application to civil works, topographic, and engineering survey activities. There are six different GNSS differential surveying techniques [18] in use today: a. Static. b. Pseudo-kinematic. c. Stop and go kinematic. d. Kinematic. e. Rapid static. f. On-the-fly (OTF)/Real-time kinematic (RTK).

For the geodetic control points observations, the Static GNSS survey method and technique will be used. The static survey method will be used as the most common method for the establishment of the geodetic control networks. The standard observation procedures and techniques will be used for the geodetic network observations as per requirements and adopted standards. The observation methodology, procedures, and observation schemas will be based on the requirements that will be updated upon the completion of detailed design and reconnaissance of the network depending on its final configuration.

The observation of the horizontal and vertical (ellipsoidal heights) positioning will be done simultaneously. However, to achieve the best possible accuracy of the ellipsoidal heights the observation of the ellipsoidal heights will be done according to a special procedure. The following are some general GNSS field survey procedures [8, 11, 18] that should be performed at each station, observation, and/or session on a GNSS survey.

Most modern dual-frequency geodetic GNSS receivers with proper Geodetic antennas should be used for the network observations. The receivers will be able to track the latest available satellite signals (including the L2C, L5, etc.) as well as at least the GPS and GLONASS satellites. The receiver's specifications will ensure tracking of a minimum of 72 channels. All the equipment will be properly checked and verified in the process of the observations according to the standard survey procedures. A minimum of four receivers will be used in the sessions of the observations, however, more receivers can be used to ensure the homogeneity of the observations and increase the efficiency of the time use.

GNSS receivers shall be set up according to manufacturer's specifications before beginning any observations. Most receivers will lock on to satellites within 1-2 min of powering up. All tri-braches with optical plummets will be used for the antenna centering over the control points. The optical centering devices will be calibrated and verified before the use as well as the standard procedures for testing and adjustment of the plummets will be regularly carried out during the observation process. The accuracy of the centering will be better than 1 mm. The field crew will regularly check the centering of the antenna.

Height of instrument (HI) refers to the correct measurement of the height of the GNSS antenna above the reference point of the monument over which it has been placed. HI measurements will be made both at the beginning and at the end of each observation session. The HI will be made from the monument to the Antenna Reference Point (ARP) marked on the antenna. The methods of antenna height measurement will ensure high accuracy and reliability of the results and exclusion of possible errors and blunders.

All measurements of the HI will be made in meters. Height of instrument will be determined to the nearest millimeter. It should be noted whether the HI is vertical or diagonal. All the measurement of the height of instrument will be recorded to the station's logs and the standard sketch of the measurements will be provided demonstrating how the height was measured.

In general, the standard static observation technique will be used. During the sessions of observation optimal satellite geometry will be used with proper PDOP (max 5) and VDOP values. The minimal number of reference stations will be not less than three. Minimum of 5% of the repeated baseline observations will be also ensured. The maximum data sampling interval will be no more than 5 seconds (the 1 sec interval will be also considered). The satellite mask angle above the horizon will be in general 10 degrees.

Station occupation time is dependent on baseline length, number of satellites observed, and the GNSS equipment used.

4.4 Processing Software

Minimum one post-processing software's shall be used for GNSS data processing and network adjustment. The software will be used for processing and producing relative position coordinates, the corresponding statistics, and least squares network adjustment results (Table 1). The general principle in data pre-processing for session solutions can be summarized as follows:

a. Orbit Computation

(i) IGS final orbit referred to as IGS00 will be used in all computations with IGS final Earth Rotation Parameter (ERP). Two programs associated with the orbit computation are PRETAB and ORBGEN.

(ii) PRETAB programs compute a table of satellite positions in the inertial frame system from the available orbit information, i.e. from the precise ephemerides that is referred to the IGS00 frame. The programs also produce the satellite clock file that may be used to compute satellite clock corrections for each observation epoch.

(iii) ORBGEN programs in general will produce standard orbit with one or more standard arcs, each of which characterized by a start and end time. Ocean tides correction OT-SCRC model is introduced at this stage with development planetary ephemeris.

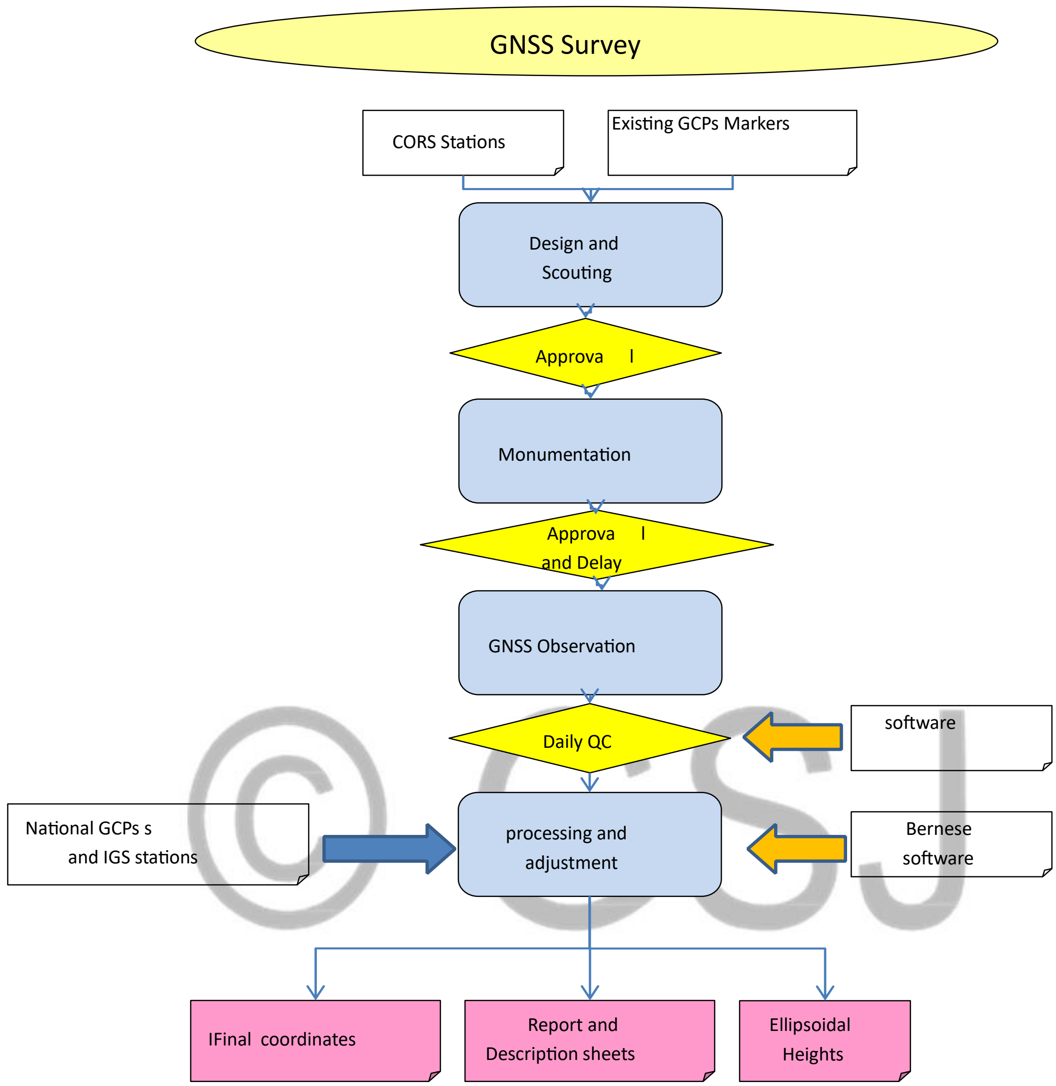

Figure 3: GNSS processes

b. Data Conversion

(i) Data will be converted from Rinex to Bernese format is using the RXOBV3 programs. The output of the data conversion is four (4) files, namely: the code zero observation and header (*.CZO and *.CZH) and phase zero observation and header (*.PZO and *.PZH).

(ii) The interval of observation data will be set accordingly.

c. Processing Part:

The general principle in processing stage can be summarized as follows:

a. Code Single Point Positioning (CODSPP): - The main task of CODSPP program is to compute the receiver coordinates and clock correction with high accuracy. The most important output of this program is to write the receiver clock correction in the code and phase zero difference for further computation.

b. Forming Single Difference Baseline (SNGDIF): - The program SNGDIF creates the single-differences (between two receivers) and stores them in files. The programs create both code and phase single differences but for further computation only phase single differences will be used

Table 1: Specifications for GNSS Processing and Network Adjustment

GNSS Processing & Network Adjustment Specifications	
1. Fixed integer solution required for all baselines	Yes
2. Ephemeris	Precise
3. Loop closure analyses, maximum number of baselines per loop	6
4. Maximum loop length	150 km
5. Maximum misclosure per loop in terms of loop length	10 ppm
6. Maximum misclosure per loop in any one component (XYZ) not to exceed	5 cm
7. Repeat baseline length not to exceed	50 km
8. Repeat baseline difference in any one component (XYZ) not to exceed	10 ppm
9. Maximum length misclosure allowed for a baseline in a properly-weighted least squares network adjustment	10 ppm
10. Maximum allowable residual in any one component (XYZ) in a properly-weighted least squares network adjustment	4 mm

c. Phase Data Screening (MAUPRP): - The phase data screening is using Manual Automatic Pre-Processing program (MAUPRP) and the main task is to screen the cycle-slips using L3 linear combination triple difference. The program will try to fix the cycle-slip and marked unpaired observation (L1 without L2 and vice-versa), small pieces of data, gaps and observations at low elevation and setup the ambiguities. The input for the program is the single difference baseline files, a priori stations coordinate with 1m accuracy and standard orbit. The programs have to be run twice for each baseline in order to totally fix all the slips as well as to mark the problems data (small pieces etc.).

d. Parameter Estimation (I) – Residuals: - Bernese has the capability of the used of ocean tide correction, estimated troposphere and mapping function. L3 linear combination will be used for the computation to check the post-fit residuals.
The mapping function of the troposphere will use Dry-Niell and the cut-off angle will be set at 15 degrees using elevation-dependent weighting.

e. Parameter Estimation (II) – Ambiguity Resolution> - Ambiguity is the unknown integer number of the reconstructed carrier phase contained in an unbroken set of measurements. The receiver counts the radio waves (from the satellite as they pass to the antenna) to a high degree of accuracy. However, it has no information on the number of waves to the satellite at the time it started counting. This unknown number of wavelengths between satellites and antennae is the ambiguity. Also known as Integer ambiguity or integer bias. Quasi Ionosphere Free (QIF) ambiguity resolution strategy and Melbourne-Wubbena wide-lane technique will be also used in the computation.

5. Precise leveling Network

The leveling Benchmarks should be designed as a wall-mounted leveling benchmark in urban areas in an old building (5 years) and as a ground-leveling benchmark in sand soil. To generate a Hybrid Geoid Modelling, geoid enhancement, and Geoid testing, the use of a Precise Leveling Network is required. The leveling network preferably [13] be at least first-order class II, in which all measurements will start from the adopted vertical datum point. The design should show all new and existing points' numbers, station locations, instruments, observations and network adjustment, methodology, and the software.

6.1 Observation and Adjustment Specifications

The first-order precise leveling will be carried out for the newly constructed benchmarks connecting the previous loops' Nodal points, which have been specifically given to use. the leveling connection has to be made to all established Geodetic Control Points. The leveling lines/loops are to be designed to take care of the route close to the GCP/Benchmarks stations.

Taking the above specifications, the methodology and guidelines shall be as follows: -

- Levelling Network design including construction plans for and final design of permanent and intermediate benchmarks.
- The leveling network shall be linked to the existing leveling network, and an analysis to be made if an error exists in any of the loops to match the order of leveling.
- The final methodology for leveling, network adjustment, and orthometric height computation to meet the accuracy requirements.
- Proposed methodology will be based on NGS and best international practice and present properly documented evidence that the leveling results will meet the required accuracy.
- The methodology shall envisage the use of modern digital first-order leveling instruments with technical characteristics to meet the leveling accuracy.
- The leveling shall be done according to high precision differential leveling technique, in two directions (fore and back-run) in each loop or section.
- The final network layout and location of benchmarks should be made before the fieldwork of benchmarks documentation and leveling starts.
- The benchmarks must be located in places that ensure the durability and vertical stability of the marks over a long period. Ground-based monuments must be designed

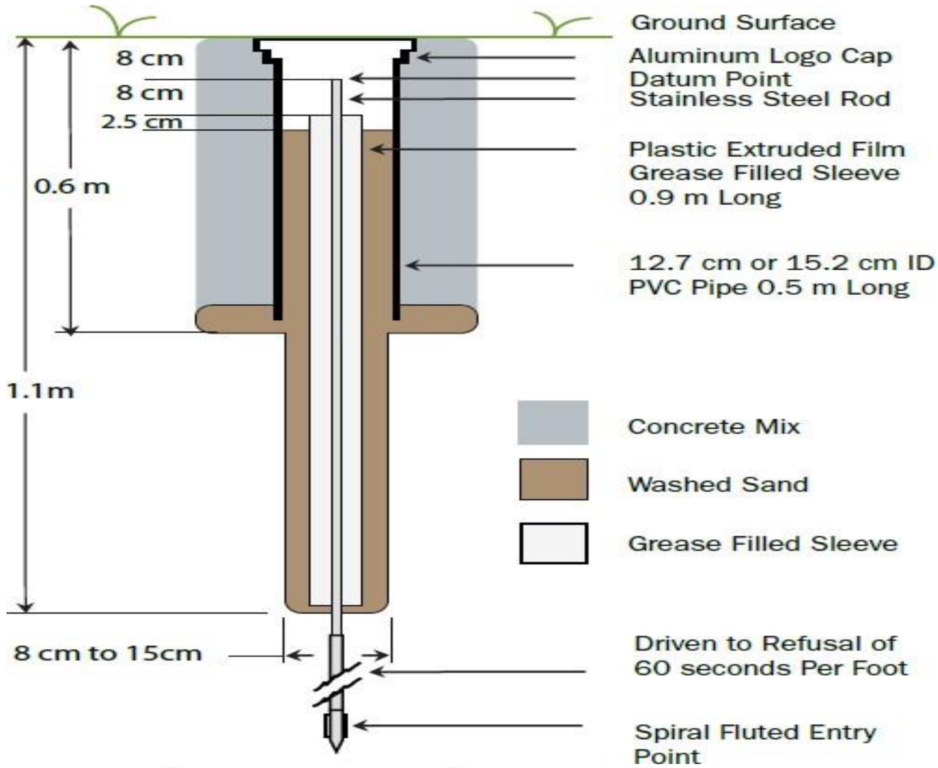

Figure 4: Standard Benchmark Monument - as NGS 3-D Rod Mark

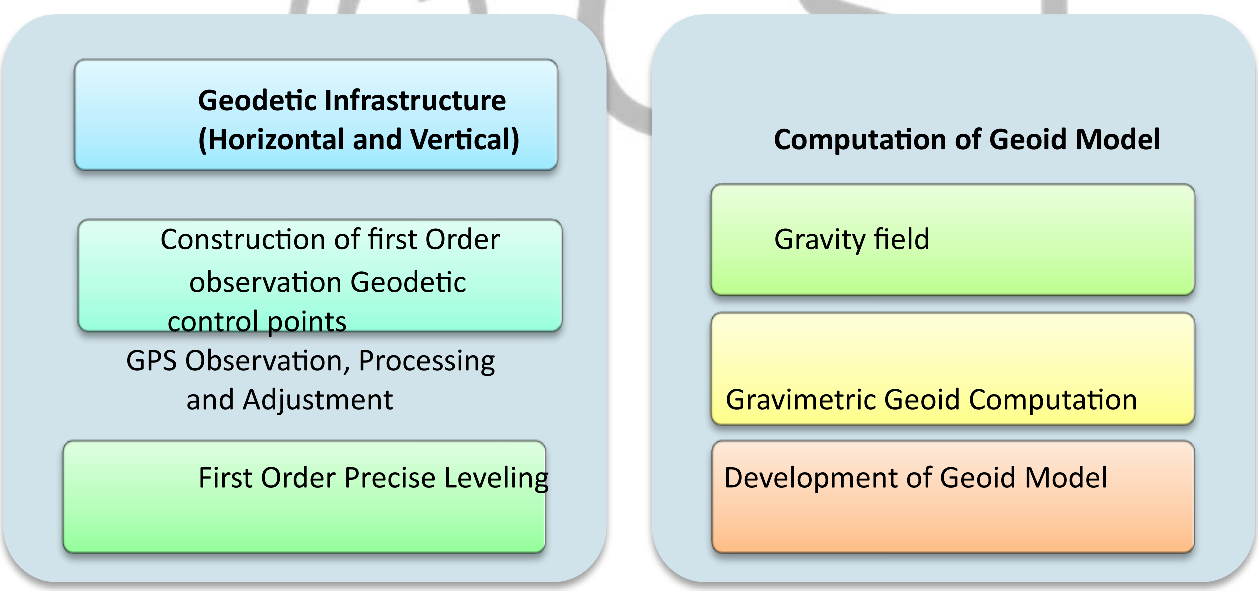

Figure 5: Precise Levelling Development of Geoid Model

- and constructed. The leveling benchmarks shall be designed and have physical characteristics and quality that are commensurate with the order of the leveling survey.
- Benchmarks shall be of a stable, permanent nature; e.g., galvanized steel pipe; steel rod driven into a firm soil base; or cast in place or prefabricated concrete.

• The benchmarks (brass made) can also be implemented in solid, stabilized objects, such as bridges, basements of other permanent structures, or rock outcrops must have engraved (on the mark or mark reference plate) - mark identification number, inscription "Levelling mark", year of establishment and the Client name. The design of benchmarks will be presented for approval.

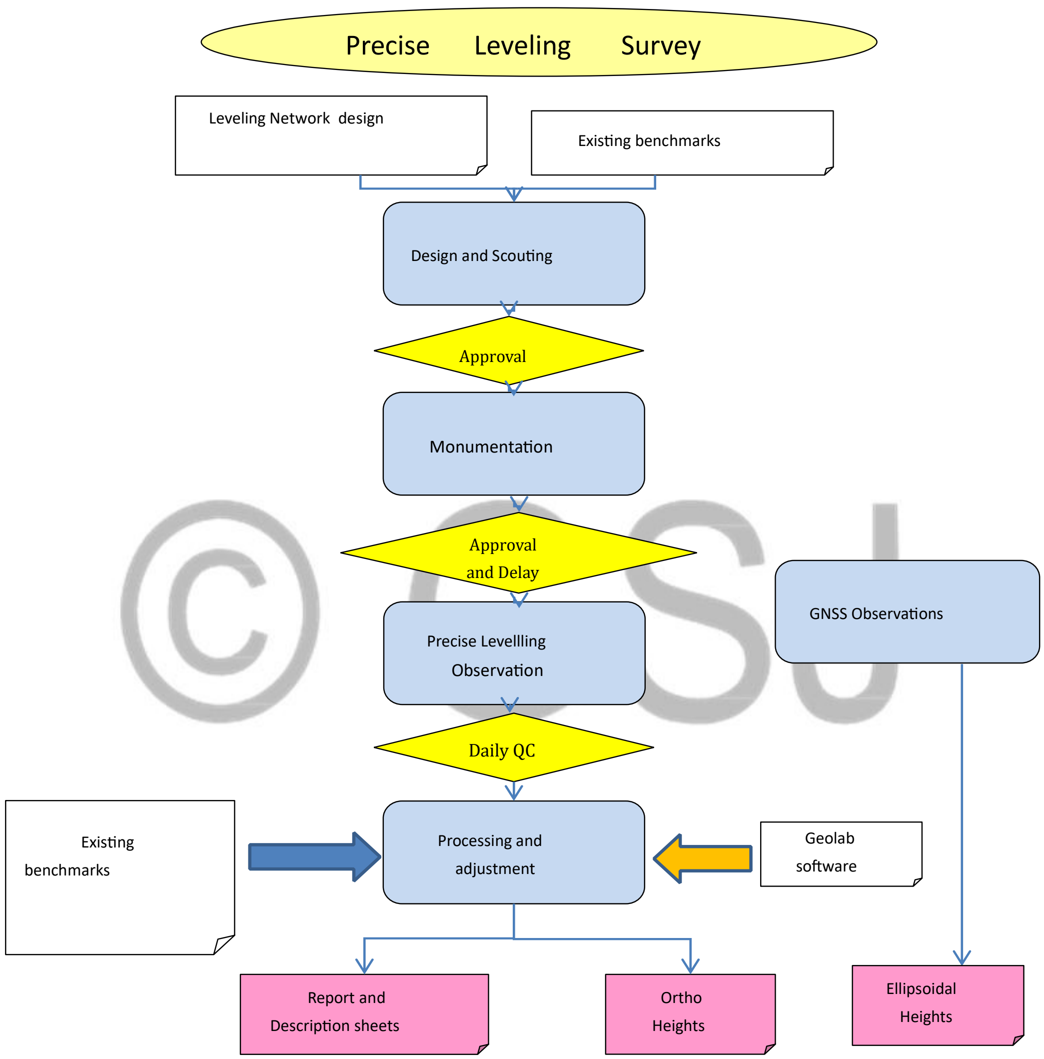

Figure 6: Precise leveling process

• The end of the leveling daily observation shall be on the permanent benchmarks from which the observation should continue the next day. Otherwise, the end and the beginning of the observation should be done on the three temporary (intermediate) marks established with a spacing of about 60 - 80 m at the places that will guarantee the stability of the marks. The discrepancy of the elevation is to be observed on these points the next day for the continuation of leveling shall not be more than 1 mm, otherwise, the section should be re-observed starting from the permanent mark.

• The misclosure should be calculated for each section between permanent points and each loop and it should not exceed the required accuracy of leveling. The control of the closure error on each section shall be done after completion, the fore and back-run leveling for the section. If the disclosure is more than required, the leveling in this section shall be repeated in one direction, and for final computation shall be included the leveling results meeting the requirements and obtained from the leveling in opposite directions. All three observations may be included for final computation if the disclosure of initial observations is not more than ±4mm√K, where K is the length of the section in km and the repeated leveling results differ not more than ±4mm√K from each of initial levelling results.

6. Modelling for Geoid heights

6.1 Roles and responsibilities for computing Geoid heights

The detailed information for each of the following main points has to be provided:

- Evaluating the different geopotential gravity models derived from satellite missions using the available ground-based gravity field and GNSS/Levelling datasets.
- Deciding on which processing strategy and analysis method would be used to compute the geoid height model
- Computing the geoid model after getting the final evaluation of datasets and comparisons of processing techniques.
- Assessing the accuracy of the geoid model using geometric geoid heights from the GNSS/Levelling dataset to reach the adopted accuracy (say 1 or 2 cm).
- Design and provide a web application (geodetic calculator) for height conversion.
- data evaluation process through global gravity models, this evaluation process plays an important role here in validating the measured terrestrial datasets (GNSS/leveling data and gravity anomalies).
- to compute geoid heights models by different processing strategies, e.g. calculating the residual gravity anomalies by removing the long and very short wavelength components from terrestrial free-air gravity anomalies, and estimating the model parameters for various areas. Moreover, the GNSS/leveling data can be also incorporated into the processing technique to compute the hybrid geoid model (i.e. using different datasets together).
- the quality of the geoid model for an exact accuracy assessment must be given by comparing the developed model with the GNSS/Levelling data. Since the resulting error from the combination of the ellipsoidal heights obtained from GNSS observations with normal heights obtained from spirit Leveling usually within 1 cm, the accuracy of geoid heights obtained from GNSS/Levelling data is estimated to be 1 – 2 cm (the optimistic estimation worldwide).

• the better fitting of the developed geoid model to GNSS/Levelling data. This is essential to minimize the datum shift between the estimated geoid heights and the geometric ones from GNSS/Levelling data. In addition, the computed geoid model should be delivered with a web application for the Geodetic Calculator Progress.

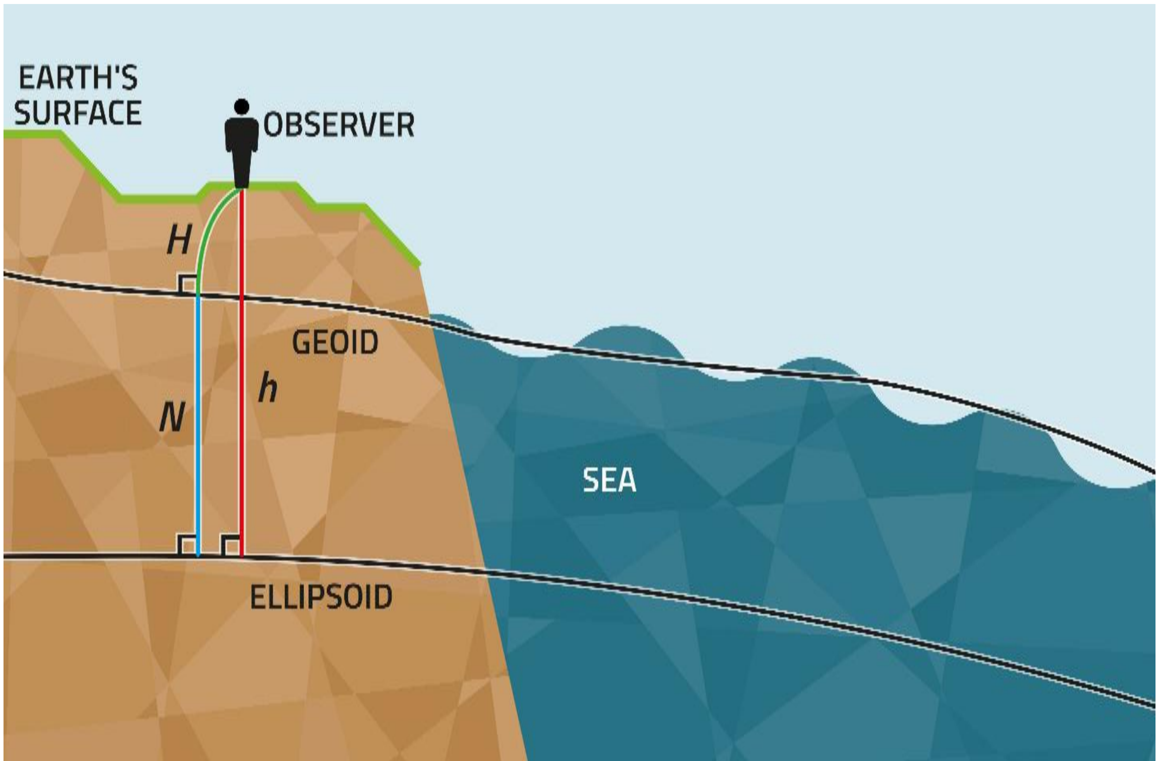

Figure 7: General representation of the geoid surface.

6.2 Geoid Computation

The key to an accurate geoid model is the existence and accessibility of a reliable and robust gravity database that contains:

- Gravity network observations that can be used to adjust the national network about the International Absolute Gravity Base Network (IAGBN) through absolute gravity observations appropriately reduced according to international standards (IERS)
- Terrain gravity observations, in the form of traverses have been observed over the years to provide point gravity values across the country. These gravity values must be referenced to the national gravity network for consistency.
- Gravity observations obtained by airborne gravimetry referenced to the national gravity network.
- Other gravity observations, such as those from shipboard gravimetry.
- Use the latest global geoid model as a reference, such as EGM08, pure satellite, Global Geopotential Models (CGM), such as from GRACE and GOCE missions, satellite combination models (e.g., GOCO03S and other more recent ones) as well as various combination models. In this type of work, various models are to be established to select the most suitable for the region; Analysis of the long wavelength field of EGM08 and then

add the land and airborne components which are bandwidth limited; Use state-of-the-art geoid modeling software for the modeling of the geoid as well as the least-squares collocation approach to derive covariance functions for the gravity field and spatial interpolation and gridding of gravity data. Other software that implements the direct numerical integration of Stokes integral can also be used for testing and comparison purposes. Geoid computation will be carried out in two parts: data preparation and calculation of the geoid undulation; deriving the error pattern for the final geoid model; fitting, the computed geoid to the First order Levelling Network within the entire Sudan area.

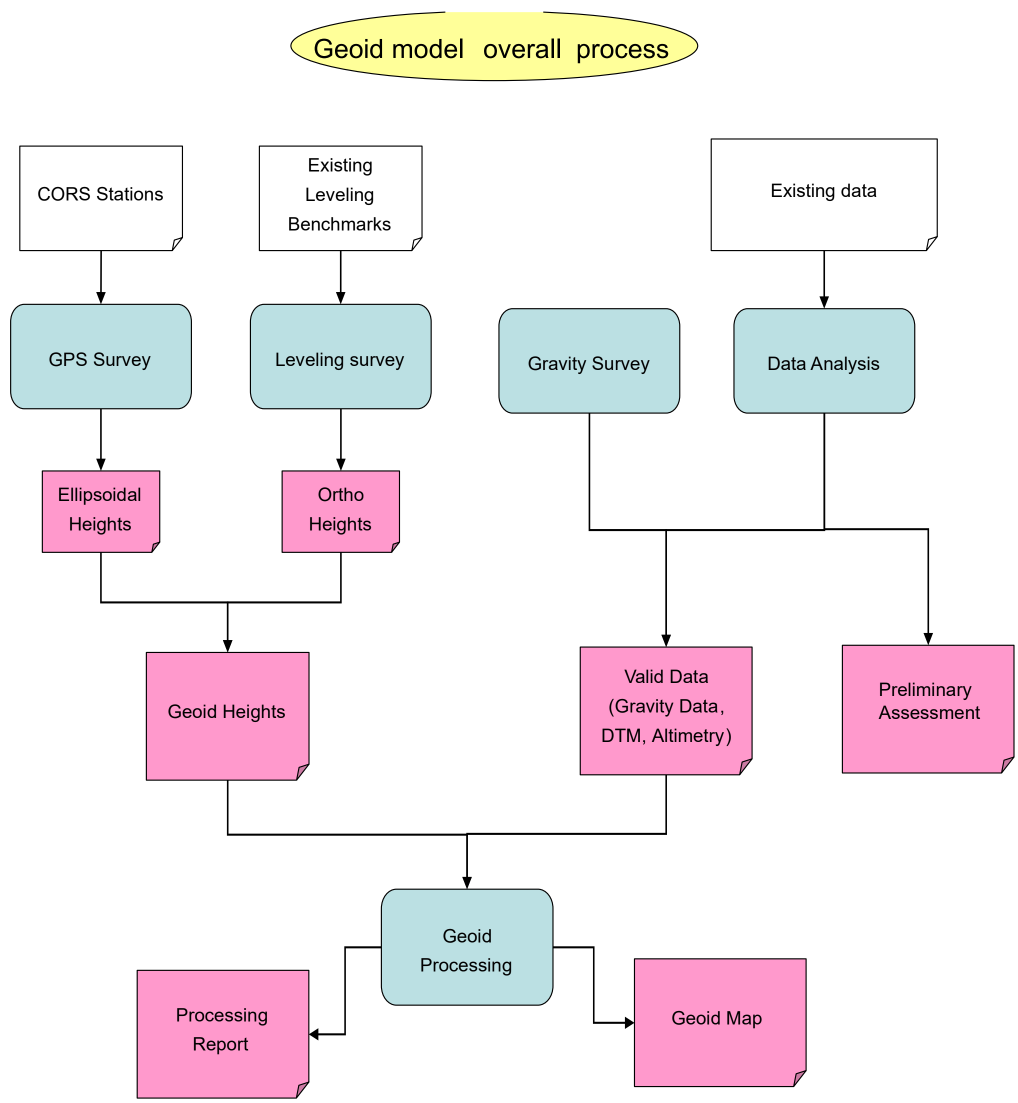

Figure 8: Geoid model overall process

The global scheme for gravity geoid processing is summarized in the following:

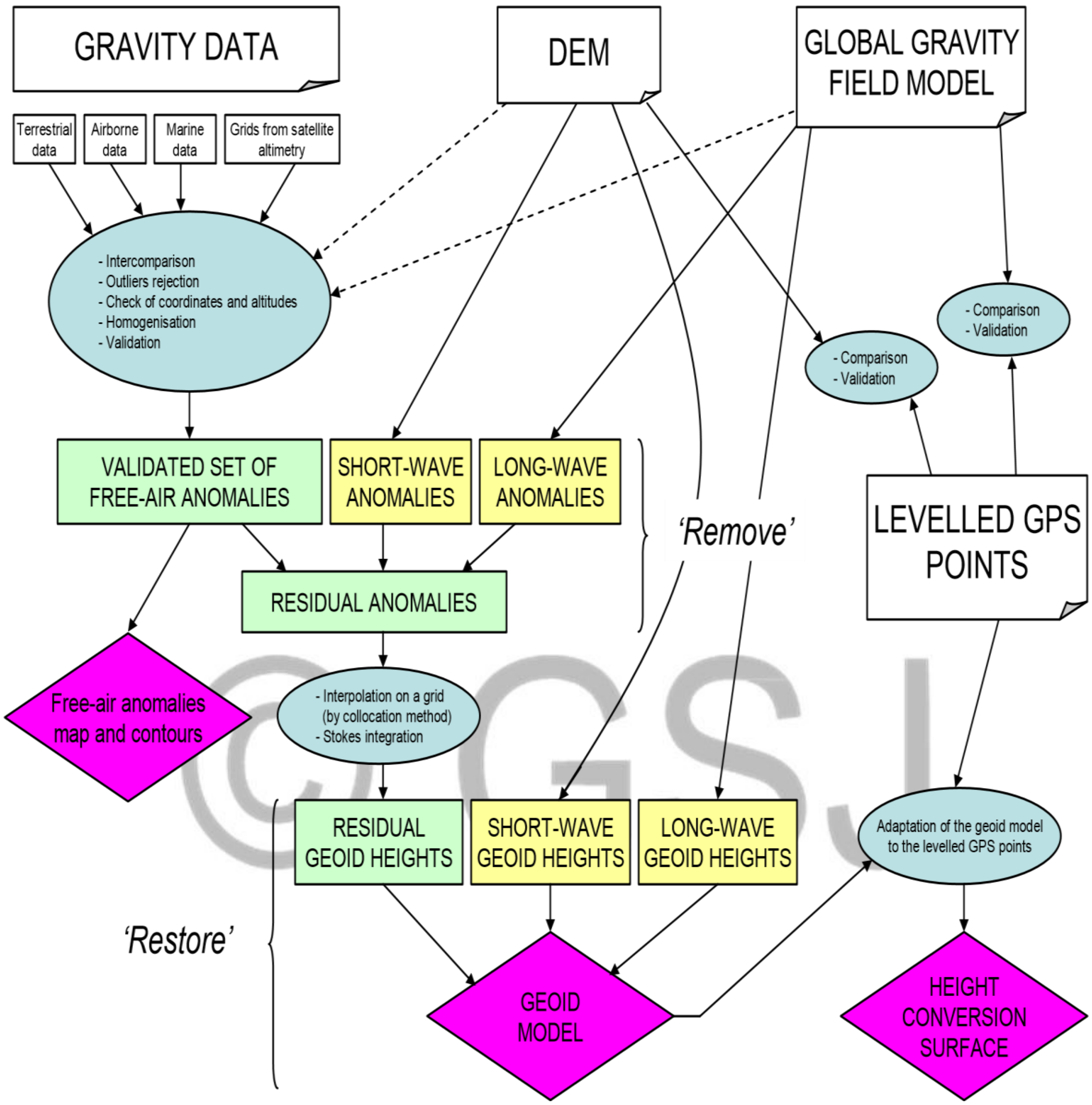

Figure 9: Global Schame for gravimetric Geoid Model Processing

by utilizing sufficient amount of selected collocation points/benchmarks (GNSS-on-benchmarks); Provide the final geoid model on a grid of appropriate resolution defined by and supported by the data density along with an appropriate interpolation algorithm. Compare the final geoid model to publicly available tide gauge reference level. The geoid model will undergo many iterations as more data and more advanced approaches and satellite models become available, particularly with the advent of satellite gravity missions (GRACE, GOCE and follow on missions) that have been revolutionizing the determination of the Earth's gravitational [3].

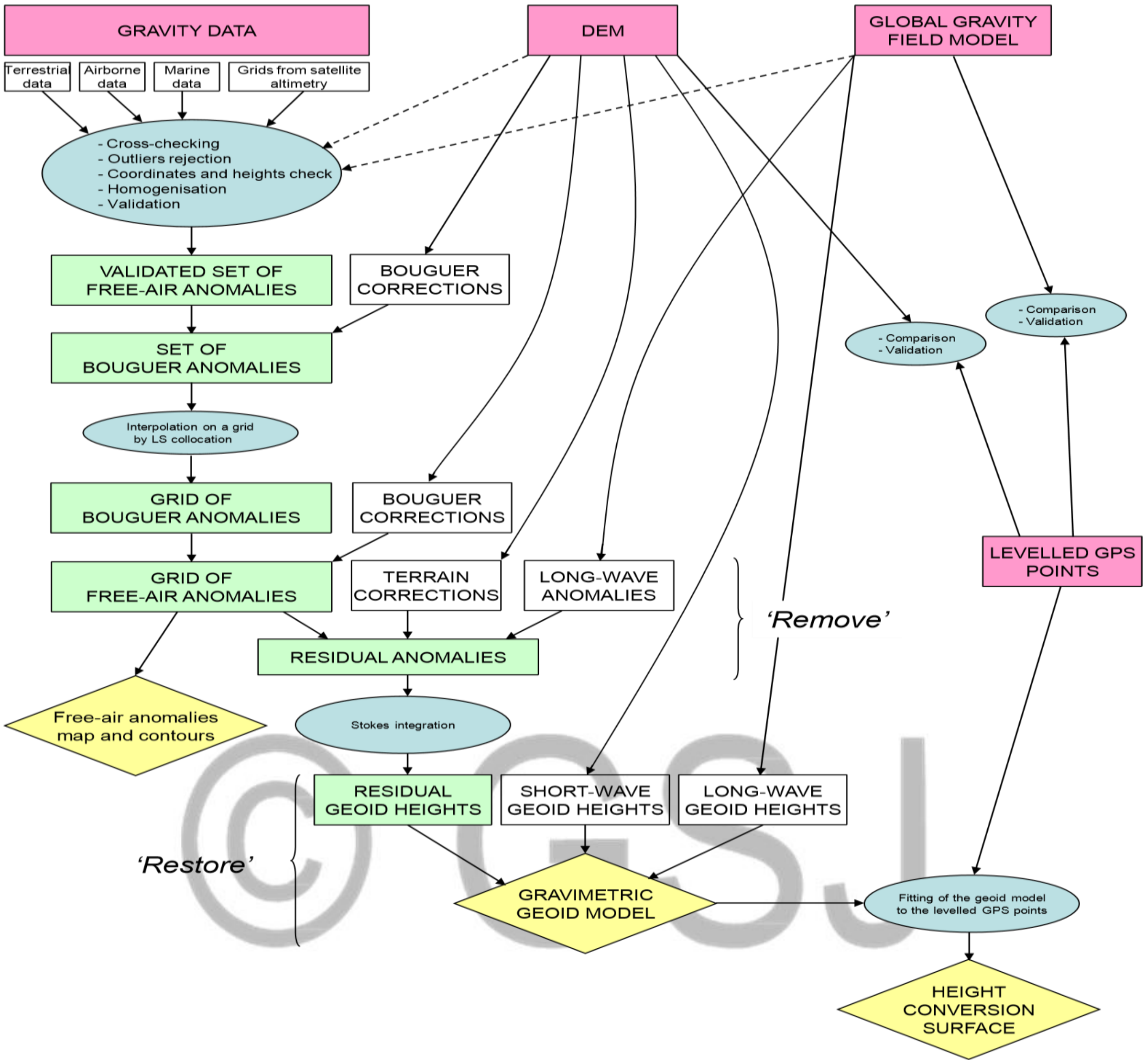

Figure 10: Detailed geoid processing

Tremendous experience with gravity network adjustments and statistical evaluation of the quality of the data is needed, together with the provision of the appropriate software. National gravity network adjustment will be carried out once the database is reasonably reliable and accessible. This is an important step since the national gravity network will serve as a skeleton for referencing all gravity observations and eventually calculating the geoid.

6.3 Geoid collocation and adjustment

Several important geodetic application areas that will benefit from the optimal combination of the heterogeneous height data include (but are not limited to): modernizing regional vertical datums, unifying national/regional vertical datums for a global vertical datum, transforming between different types of height data, and refining and testing existing gravimetric geoid models. The datums vary due to different types of definitions, different methods of realizations, and the fact that they are based on local/regional data. As the move is towards an increased use of space-based data acquisition technologies for coordinate/and height information the ability to correctly combine traditionally obtained measurements with newer measurements becomes an essential tool.

A common approach for defining Sudan vertical datums is to average sea level (known as mean sea level (MSL)) observations over approximately 19 years for one or more fundamental tide gauges at the Red Sea coast. Another common approach is the combined use of co-located ellipsoidal and orthometric heights (or height differences) to compute geoidal height values at the GNSS-levelling benchmarks. This trivial combination leads to GNSS-derived geoid heights, which are invariably different from the values interpolated from a gravimetrically-derived geoid model. For instance, a gravimetrically computed geoid model, obtained from the remove-compute-restore process, will (theoretically) refer to the geocentric reference system implicit in the used geopotential model. This reference system will in turn correspond to the adopted coordinate set for the satellite tracking stations used in the global geopotential solution.

This coordinate set will not necessarily agree with the adopted reference system for the ellipsoidal heights obtained from the GNSS measurements. Furthermore, the local Port Sudan levelling datum to which the orthometric heights refer will not likely correspond to the reference potential value of the geopotential model or the GNSS reference system.

After the final geoid model computation, resulting model will be predicted to the adopted spacing grid using least squares prediction. At the next step, resulting grid will be re-adjusted with the available geometrically computed geoid heights at the points of first order levelling network using the least squares adjustment technique.

Comparisons between different geoid solutions provide insight into the accuracy of the geoid determination techniques. To date, comparisons of gravimetrically- derived geoid model values interpolated at GNSS-on-benchmarks with geometrically computed geoid values provide the best external means of evaluating the geoid model accuracy. In order for this method to provide an indication of the ‘accuracy’ of the gravimetric geoid model, it is important and well known, that the GNSS-levelling data used for testing is not incorporated in the original geoid solution.

Long-wavelength errors present in gravimetrically-derived geoid models may be reduced by constraining the geoid solution to the observed geoid values at GNSS- levelling benchmarks. This is a common procedure implemented in many recent national geoid models through the use of least-squares collocation procedures, and reported to give positive results [17].

The cross-covariance function between measured (GNSS/Levelling) and calculated Geoid heights will be constructed in the frame of classical least squares collocation for the best fitting of the resulting geoid model to the adopted vertical datum, using part of the existing GNSS/Levelling points as a constrain. Then, another part of GNSS/Levelling points will be used for comparison of the fitted geoid [14] to the local vertical datum. In addition to this, other possible techniques and methods for the transformation/shifting of the resulting surface to the Sudan local datum can be used as quality control.

7. Conclusion

The Sudan Precise Geoid Heights Model shall be computed based on gravimetric observations using advanced instruments and techniques, methodologies, and more recent satellite gravity data and global geoid models. So, to compute a new enhanced geoid model by using all heterogenous gravity information provided from both ground-based gravimetric and satellite-based datasets.

The paper described the methodology and the approach which can be adopted for the determination of the Sudan enhanced gravimetric geoid model. Comparisons between different geoid solutions that provide insight into the accuracy of the geoid determination techniques are discussed.

Long-wavelength errors present in gravimetrically-derived geoid models may be reduced by constraining the geoid solution to the observed geoid values at GNSS- levelling benchmarks, to be as a common procedure to be implemented for the national geoid model through the use of least-squares collocation procedures. The cross-covariance function between measured (GNSS/Levelling) and calculated the Geoid heights will be constructed in the frame of classical least squares collocation for the best fitting of the geoid model results, together with the other techniques and methods for the transformation/shifting of the geoid surface to the Sudan local datum.

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