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Dynamical System on Three – Dimensional Almost f-Cosymplectic Manifolds

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Abstract

The study addressed the Dynamical System on three-Almost f-Cosymplectic Manifolds and its applications in Mechanics using differential manifold techniques. It has been found that triple (M^3,φ.ξ,η,g) is Hamiltonian mechanical system on 3- Almost f-Cosymplectic Manifolds (M^3,φ.ξ,η,g).

Key words; Almost f-Cosymplectic Manifolds, A differential Manifold,Dynamical SystemEquations.

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1 Introduction

The geometric study of dynamical systems is an important topicin contemporary mathematics due to its applications in many branches, such, Mechanics and Theoretical Physics. If M is a differentiable manifold thatcorresponds to the configuration space, a dynamical system can be locallygiven by a system of ordinary differential equations. Mathematicians in [1],[2],[3] and [4] were introduced some important works dealt withconcerned issuesand its applications in many Mathematical fields, these motivated the researcher to deep insight in to dynamical system on three Almost f- Cosymplectic Manifold and its applications.

In this paper, we studiedDynamical System on Three-Almost f-Cosymplectic Manifold and its implementations in Mathematical Physics fields.

The work has been organized as follow, in section1, we considered historical background as paperbase,section 2 dealt with the study preliminary idea,section 3 devoted to study Three - dimensional Almost f-Cosymplectic Manifolds, section 4 devoted to studyLagrangian Dynamical System Equations on Three- Almost f-Cosymplectic Manifolds and section 5 is devoted to studyHumiliation Dynamical System Equations Three-Almost f-Cosymplectic Manifolds.

2.Preliminary

In this preliminary chapter, we recalled basic definitions, results and formulas that we usedthemin the subsequent chapters of thepaper.

DefinitionAlmost f-Cosymplectic Manifolds2.1

Let M^n be a $(2n + 1)-$dimensional differentiable manifold equipped with atriple (ϕ, ξ, η), where ϕ is a type of (1, 1) tensor field, ξ is a vector field, η isa 1-form on M such that

$$\eta(\xi) = 1 \quad , \quad \phi^2 = -1 + \eta \otimes \xi \qquad (1)$$

which implies

$$\phi\xi = 0, \quad \eta \circ \phi = 0 \quad , rank\,(\phi) = n - 1 \qquad (2)$$

where $\emptyset$ is a(1-1) tenser field η is a 1- form and the Riemannian metric$g$ .It well known that

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \quad , g(X, \xi) = \eta(X) \qquad (3)$$

The fundamental 2- form of metric manifolds is defined by

$$g(\phi X, Y) = -g(X, \phi Y) \qquad (4)$$

for any vector fields X, Y on M.

i) An almost contact metric manifold(M,ϕ,ξ,η,g)such that $d\omega = -f\eta\wedge\omega\ \ and\ \ d\eta = 0\ \ for\ smooth\ function\ f\ satisfying\ df\wedge\eta = 0$
If the almost f–cosymplectic structure on Mnormal

Proposition 2.2

Anf–cosymplectic manifoldis cosymplectic manifoldif f vanishesalongξ**.**

Lemma 2.3

For a three – dimensional f –cosymplectic manifoldM^3 , we have

$$QY = \left(-3\tilde{f} - \frac{R}{2}\right)\eta(Y)\xi + \left(\tilde{f} - \frac{R}{2}\right)Y \qquad (6)$$

Where R is the scalar curvature of M

Lemma 2.4 Iff, g and k $-$form berespectively then

(i) $f\wedge g = -g\wedge f$

(ii) $(f\wedge g)(x) = f(x)g - g(x)f$

(iii) $\left(dx^i\wedge dx^j\right)\left(\frac{\partial}{\partial x^k}\right) = \frac{\partial x^i}{\partial x^k}dx^j - \frac{\partial x^j}{\partial x^k}dx^i$

(iv) $\left(\frac{\partial x^j}{\partial x^i}\right) = \delta^i_j = \begin{cases} 1 & ,i = j \\ 0 & ,i \neq j \end{cases}$

3.Three – Dimensional Almost f-Cosymplectic Manifolds

Definition3.1

Letthree-dimensional manifold $M = f(x,y,z) \in R^3, z \neq 0$;where (x,y,z) are the standard coordinates in R^3: The vector fields

$$e_1 = e^{z^2}\frac{\partial}{\partial x} = e^{\theta(z)}\frac{\partial}{\partial x} \quad , \quad e_2 = e^{z^2}\frac{\partial}{\partial y} = e^{\theta(z)}\frac{\partial}{\partial y} \quad , e_3 = \frac{\partial}{\partial z} \qquad (6)$$

Where $e^{\theta(z)}$ is a smooth function on$M$
are$\{e_1,e_2,e_3\}$ linearly independent at each point of M: Let g be the Riemannian metricdefined by

$$g(e_1,e_3) = g(e_2,e_3) = g(e_1,e_2) = 0$$
$$g(e_1,e_1) = g(e_2,e_2) = g(e_3,e_3) = 1$$

and given by the tensor product

$$g = \frac{1}{e^{z^2}}(dx\otimes dx + dy\otimes dx) + dz\otimes dz \qquad (7)$$

Proposition 3.2

Let η be the 1-form defined by $\eta(X) = g(X,e_3)$for any$Z \in \aleph(M)$.
,Letϕ bethe (1,1) tensorfield defined by

$$\phi(e_1) = e_2, \qquad \phi(e_2) = -e_1 and \quad , \quad \phi(e_3) = 0 \qquad (8)$$

Thenusing the linearity of ϕ and g we have

$\eta(e_3) = 1$; $\phi^2(X) = -X + \eta(X)e_3$;
$g(\phi Z,\phi W) = g(X;Y) - \eta(X)\eta(Y)$;
for any $Z,W \in \phi(M)$: Thus for $e_3 = \xi$, (ϕ,ξ,η,g) defines an almost contactmetric structure on M: Now, by direct computations we obtain

$$[e_1,e_2] = 0 \quad ; [e_1,e_3] = -2Ze_1 \quad , \qquad [e_2,e_3] = -2Ze_1 \qquad (9)$$

Consequently, the Nijenhuis torsion of ϕ is zero, i.e., M is normal.

$\omega = \frac{1}{e^{\theta(z)}} dx \Lambda \mathrm{dy} = \frac{1}{e^{z^2}} dx \Lambda \mathrm{dy}$ (10)

$d\omega = 2\grave{\theta}(z)\, \omega \Lambda \eta$

Therefore M is an f-cosymplectic manifold with $f(x, y, z) = \grave{\theta}(z)$.

Proposition 3.3

the following expressions are given

$\phi\left(e^{z^2} \frac{\partial}{\partial x}\right) = e^{z^2} \frac{\partial}{\partial y}$, $\phi\left(e^{z^2} \frac{\partial}{\partial y}\right) = -e^{z^2} \frac{\partial}{\partial x}$, $\phi\left(\frac{\partial}{\partial z}\right) = 0$ (11)

The dual form ϕ^* of the aboveϕ is as follows

$\phi^*\left(e^{z^2} dx\right) = e^{z^2} dy$, $\phi^*\left(e^{z^2} dy\right) = -e^{z^2} dx$, $\phi^*(dz) = 0$ (12)

Proposition 3.4

The vector fields

$e_1 = \frac{\partial}{\partial x}$, $e_2 = \frac{\partial}{\partial y} and$, $e_3 = \frac{\partial}{\partial z}$

If $\phi$is defined a complex manifold$\mathcal{M}$then$\phi^2 = \phi \circ \phi = -1 \ \ or \ \ 0$

Proof:

$\phi^2(e_1) = \phi(\phi(e_1)) = \phi(e_2) = \phi(e_2) = -e_1 = -1$

$\phi^2(e_2) = \phi(\phi(e_2)) = \phi(-e_1) = -\phi(e_1) = -e_2 = -1$

$\phi^2(e_3) = \phi(\phi(e_3)) = \phi(0) = \phi(0) = 0$

As can ϕ^2 is -1 (complex) or 0

4.Lagrangian Dynamical System Equationson Three- Almost f-Cosymplectic Manifolds

In this section we introduce the concept of Lagrangian Dynamical Systems. We start by the following definition.

Definition 4.1

A Lagrangian function vector field X on$\mathcal{M}$ is a smooth function $\mathrm{L} : \mathrm{T}\mathcal{M} \rightarrow \mathrm{R}$ such that

$\mathrm{i}_X \phi_L = \mathrm{dE}_L$ (13)

Definition 4.2

Let ξ be the vector field by

$$\xi = \mathrm{x}\frac{\partial}{\partial \mathrm{x}} + \mathrm{y}\frac{\partial}{\partial \mathrm{y}} + \mathrm{z}\frac{\partial}{\partial \mathrm{z}} \quad , \quad \mathrm{x} = \dot{\mathrm{x}} \ , \quad \mathrm{y} = \dot{\mathrm{y}} \ , \quad \mathrm{z} = \dot{\mathrm{z}} \qquad (14)$$

By Liouville vector field

space form $(M^3, \phi. \xi, \eta, g)$, we call the vector fielddetermined by $\mathrm{V} = \phi\xi$ and calculated by

$$\phi\xi = \phi\left(\mathrm{X}\frac{\partial}{\partial \mathrm{x}} + \mathrm{Y}\frac{\partial}{\partial \mathrm{y}} + \mathrm{Z}\frac{\partial}{\partial \mathrm{z}}\right) = X\phi\left(\frac{\partial}{\partial \mathrm{x}}\right) + \mathrm{Y}\phi\left(\frac{\partial}{\partial \mathrm{x}}\right) + Z\phi\left(\frac{\partial}{\partial \mathrm{x}}\right)$$

$$\phi\xi = \mathrm{x}\frac{\partial}{\partial \mathrm{y}} - \mathrm{Y}\frac{\partial}{\partial \mathrm{x}} + \mathrm{Z}(0) = Xe^{z^2}\frac{\partial}{\partial \mathrm{y}} - \mathrm{Y}e^{z^2}\frac{\partial}{\partial \mathrm{x}} + \mathrm{Z}(0)$$

$\phi\xi = Xe^{z^2}\frac{\partial}{\partial \mathrm{y}} - \mathrm{Y}e^{z^2}\frac{\partial}{\partial \mathrm{x}}$ (15)

Definition 4.3

1- Denote T by the **kinetic energy** and P by the **potential energy** of mechanics system on 3- Almost f-CosymplecticManifolds. Then we write by

$\mathrm{L} = \mathrm{T} - \mathrm{P}$

2- Lagrangian function and by

$$\mathrm{E_L} = \mathrm{V}(\mathrm{L}) - \mathrm{L} \qquad (16)$$

is called the interior product with ϕ, or sometimes the insertion operator, or contraction by ϕ.

Definition 4.4

The exterior vertical derivation d_ϕ is defined by

$\mathrm{d}_\phi = [\mathrm{i}_\phi, \mathrm{d}] = \mathrm{i}_\phi \mathrm{d} - \mathrm{d}\mathrm{i}_\phi \quad (17)$

whered is the usual exterior derivation. For almost product structure ϕ determined bythe closed 3- Almost f-Cosymplectic Manifoldsform is the closed 2-form given by

$\phi_\mathrm{L} = -\mathrm{d}\mathrm{d}_\phi \mathrm{L}(18)$

such that

$$\mathrm{d}_\phi = Xe^{z^2}\frac{\partial}{\partial \mathrm{y}} - \mathrm{Y}e^{z^2}\frac{\partial}{\partial \mathrm{x}}$$

$$\mathrm{d}_\phi \mathrm{L} = \left(Xe^{z^2}\frac{\partial}{\partial \mathrm{y}} - \mathrm{Y}e^{z^2}\frac{\partial}{\partial \mathrm{x}}\right)L = Xe^{z^2}\frac{\partial \mathrm{L}}{\partial \mathrm{y}} - \mathrm{Y}e^{z^2}\frac{\partial \mathrm{L}}{\partial \mathrm{x}}$$

Thus we get

$$\phi_\mathrm{L} = -\mathrm{d}(\mathrm{d}_\phi \mathrm{L}) = -\mathrm{d}\left(Xe^{z^2}\frac{\partial \mathrm{L}}{\partial \mathrm{y}} - \mathrm{Y}e^{z^2}\frac{\partial \mathrm{L}}{\partial \mathrm{x}}\right)$$

$$\phi_\mathrm{L} = -\mathrm{X}e^{z^2}\frac{\partial^2 \mathrm{L}}{\partial \mathrm{x}\,\partial \mathrm{y}}\mathrm{dx}\wedge\mathrm{dx} - \mathrm{X}e^{z^2}\frac{\partial^2 \mathrm{L}}{\partial \mathrm{y}\,\partial \mathrm{y}}\mathrm{dy}\wedge\mathrm{dy} + \mathrm{Y}e^{z^2}\frac{\partial^2 \mathrm{L}}{\partial \mathrm{x}\,\partial \mathrm{x}}\mathrm{dx}\wedge\mathrm{dy} + \mathrm{Y}e^{z^2}\frac{\partial^2 \mathrm{L}}{\partial \mathrm{y}\,\partial \mathrm{y}}\mathrm{dy}\wedge\mathrm{dy} + \mathrm{Z}e^{z^2}\frac{\partial^2 \mathrm{L}}{\partial \mathrm{x}\,\partial \mathrm{z}}\mathrm{dx}\wedge\mathrm{dz} + \mathrm{Z}e^{z^2}\frac{\partial^2 \mathrm{L}}{\partial \mathrm{y}\,\partial \mathrm{z}}\mathrm{dy}\wedge\mathrm{dz} \qquad (19)$$

Because of the closed 3- Almost f-Cosymplectic Manifoldsform ϕ_L on 3- Almost f-Cosymplectic Manifoldsspace form $(M^3, \phi.\xi, \eta, g)$ is para-symplectic structure, one may obtain

$$\mathrm{E_L} = \mathrm{X}e^{z^2}\frac{\partial \mathrm{L}}{\partial \mathrm{Y}} - \mathrm{Y}e^{z^2}\frac{\partial \mathrm{L}}{\partial \mathrm{x}} - \mathrm{L} \qquad (20)$$

Considering (0,1) we calculate

$$\mathrm{dE_L} = \mathrm{d}\left(\mathrm{X}e^{z^2}\frac{\partial \mathrm{L}}{\partial \mathrm{Y}} - \mathrm{Y}e^{z^2}\frac{\partial \mathrm{L}}{\partial \mathrm{x}} - \mathrm{L}\right)$$

$$\mathrm{dE_L} = -\mathrm{X}e^{z^2}\frac{\partial^2 \mathrm{L}}{\partial \mathrm{x}\,\partial \mathrm{y}}\mathrm{dx} - \mathrm{y}e^{z^2}\frac{\partial^2 \mathrm{L}}{\partial \mathrm{y}\,\partial \mathrm{y}}\mathrm{dx} + \mathrm{X}e^{z^2}\frac{\partial^2 \mathrm{L}}{\partial \mathrm{x}\,\partial \mathrm{x}}\mathrm{dy} + \mathrm{Y}e^{z^2}\frac{\partial^2 \mathrm{L}}{\partial \mathrm{y}\,\partial \mathrm{x}}\mathrm{dy} - \frac{\partial \mathrm{L}}{\partial \mathrm{x}}\mathrm{dx} - \mathrm{Z}\frac{\partial^2 \mathrm{L}}{\partial \mathrm{x}\,\partial \mathrm{z}}\mathrm{dx} - \mathrm{Z}e^{z^2}\frac{\partial^2 \mathrm{L}}{\partial \mathrm{y}\,\partial \mathrm{z}}\mathrm{dy} - \frac{\partial \mathrm{L}}{\partial \mathrm{y}}\mathrm{dy}(21)$$

Taking care of $\mathrm{i}_\xi \phi_\mathrm{L} = \mathrm{dE_L}$, we have

$$-\mathrm{X}e^{z^2}\frac{\partial^2 \mathrm{L}}{\partial \mathrm{x}\,\partial \mathrm{y}}\mathrm{dx} - \mathrm{y}e^{z^2}\frac{\partial^2 \mathrm{L}}{\partial \mathrm{y}\,\partial \mathrm{y}}\mathrm{dx} - \mathrm{Z}e^{z^2}\frac{\partial^2 \mathrm{L}}{\partial \mathrm{y}\,\partial \mathrm{z}}\mathrm{dx} + \frac{\partial \mathrm{L}}{\partial \mathrm{x}}\mathrm{dx} + \mathrm{X}e^{z^2}\frac{\partial^2 \mathrm{L}}{\partial \mathrm{x}\,\partial \mathrm{x}}\mathrm{dy} + \mathrm{Y}e^{z^2}\frac{\partial^2 \mathrm{L}}{\partial \mathrm{y}\,\partial \mathrm{x}}\mathrm{dy} + \mathrm{Z}e^{z^2}\frac{\partial^2 \mathrm{L}}{\partial \mathrm{x}\,\partial \mathrm{z}}\mathrm{dy} + \frac{\partial \mathrm{L}}{\partial \mathrm{y}}\mathrm{dy} = 0 \qquad (22)$$

$$-\frac{\partial \mathrm{L}}{\partial \mathrm{y}}\left(\mathrm{X}e^{z^2}\frac{\partial}{\partial \mathrm{x}}\mathrm{dx}+\mathrm{y}e^{z^2}\frac{\partial}{\partial \mathrm{y}}\mathrm{dx}+\mathrm{Z}e^{z^2}\frac{\partial}{\partial \mathrm{z}}\mathrm{dx}\right)+\frac{\partial \mathrm{L}}{\partial \mathrm{x}}\mathrm{dx}$$
$$+\frac{\partial \mathrm{L}}{\partial \mathrm{x}}\left(\mathrm{X}e^{z^2}\frac{\partial}{\partial \mathrm{x}}\mathrm{dy}+\mathrm{Y}e^{z^2}\frac{\partial}{\partial \mathrm{y}}\mathrm{dy}+\mathrm{Z}e^{z^2}\frac{\partial}{\partial \mathrm{z}}\mathrm{dy}\right)+\frac{\partial \mathrm{L}}{\partial \mathrm{y}}\mathrm{dy}=0 \quad (23)$$

$$-\frac{\partial \mathrm{L}}{\partial \mathrm{y}}e^{z^2}\left(\mathrm{X}\frac{\partial}{\partial \mathrm{x}}+\mathrm{y}\frac{\partial}{\partial \mathrm{y}}+\mathrm{Z}\frac{\partial}{\partial \mathrm{z}}\right)\mathrm{dx}+\frac{\partial \mathrm{L}}{\partial \mathrm{x}}\mathrm{dx}+\frac{\partial \mathrm{L}}{\partial \mathrm{x}}e^{z^2}\left(\mathrm{X}\frac{\partial}{\partial \mathrm{x}}+\mathrm{Y}\frac{\partial}{\partial \mathrm{y}}+\mathrm{Z}\frac{\partial}{\partial \mathrm{z}}\right)\mathrm{dy}$$
$$+\frac{\partial \mathrm{L}}{\partial \mathrm{y}}\mathrm{dy}=0$$

$$\left[-e^{z^2}\frac{\partial \mathrm{L}}{\partial \mathrm{y}}\left(\mathrm{X}\frac{\partial}{\partial \mathrm{x}}+\mathrm{y}\frac{\partial}{\partial \mathrm{y}}+\mathrm{Z}\frac{\partial}{\partial \mathrm{z}}\right)+\frac{\partial \mathrm{L}}{\partial \mathrm{x}}\right]\mathrm{dx}+\left[e^{z^2}\frac{\partial \mathrm{L}}{\partial \mathrm{x}}\left(\mathrm{X}\frac{\partial}{\partial \mathrm{x}}+\mathrm{Y}\frac{\partial}{\partial \mathrm{y}}+\mathrm{Z}\frac{\partial}{\partial \mathrm{z}}\right)+\frac{\partial \mathrm{L}}{\partial \mathrm{y}}\right]\mathrm{dy}=0$$

If the curve $\alpha : \mathrm{I} \subset \mathrm{R} \rightarrow M^3$ be integral curve of ξ,

$$\alpha=\frac{\partial}{\partial \mathrm{t}}=\mathrm{X}\frac{\partial}{\partial \mathrm{x}}+\mathrm{y}\frac{\partial}{\partial \mathrm{y}}+\mathrm{Z}\frac{\partial}{\partial \mathrm{z}} \quad (24)$$

which satisfies

$$\left[-e^{z^2}\frac{\partial}{\partial \mathrm{t}}\frac{\partial \mathrm{L}}{\partial \mathrm{y}}+\frac{\partial \mathrm{L}}{\partial \mathrm{x}}\right]\mathrm{dx}+\left[e^{z^2}\frac{\partial}{\partial \mathrm{t}}\frac{\partial \mathrm{L}}{\partial \mathrm{x}}+\frac{\partial \mathrm{L}}{\partial \mathrm{y}}\right]\mathrm{dy}=0 \quad (25)$$

it follows equations

$$e^{z^2}\frac{\partial}{\partial \mathrm{t}}\frac{\partial \mathrm{L}}{\partial \mathrm{y}}-\frac{\partial \mathrm{L}}{\partial \mathrm{x}}=0 \quad , \quad e^{z^2}\frac{\partial}{\partial \mathrm{t}}\frac{\partial \mathrm{L}}{\partial \mathrm{x}}+\frac{\partial \mathrm{L}}{\partial \mathrm{y}}=0 \quad (26)$$

Finally one may say that the triple $(M^3,\phi.\xi,\eta,g)$ is Lagrangian mechanical system on 3- Almost f-Cosymplectic Manifolds$(\mathrm{M}^3,\phi.\xi,\eta,\mathrm{g})$.

5.Humiliation Dynamical System Equationsthree- Almost f-Cosymplectic Manifolds

Definition 5.1

The kinetic energy $T=\frac{1}{2}m(\dot{x}^2+\dot{y}^2+\dot{z}^2)$ and $P=m_i gh$ is Potential energy .then $L=T+P$ is called the Hamiltonian function.

Definition 5.2

Let $X_H is$ unique vector field on H is Hamiltonian function and ϕ^* is dual of ϕ and ω from on T^*M and

$$i_{X_H}=dH \quad (27)$$

Is called Hamiltonian dynamical equation .

Here, we present Hamiltonian equations on 3- Almost f-Cosymplectic Manifolds$(\mathrm{M}^3,\phi.\xi,\eta,\mathrm{g})$such that

$$\omega=\frac{1}{2}(\mathrm{Xdx}+\mathrm{Ydy}+\mathrm{Zdz}) \quad (28)$$

1-form on

Let ϕ^* be an almost product structure defined by λLiouville form determined by

$$\lambda=\phi^*(\omega)=\phi^*\left[\frac{1}{2}(\mathrm{Xdx}+\mathrm{Ydy}+\mathrm{Zdz})\right] \quad (29)$$

$$\lambda=\phi^*(\omega)=\frac{1}{2}[\mathrm{X}\phi^*(\mathrm{dx})+\mathrm{Y}\phi^*(\mathrm{dy})+\mathrm{Z}\phi^*(\mathrm{dz})]$$

$$\lambda = \phi^*(\omega) = \frac{1}{2}\left[\mathrm{X}(e^{z^2}\mathrm{dy}) + \mathrm{Y}(-e^{z^2}\mathrm{dx}) + \mathrm{Z}(0)\right]$$

$$\lambda = \phi^*(\omega) = \frac{1}{2}\left[\mathrm{X}e^{z^2}\mathrm{dy} - \mathrm{Y}e^{z^2}\mathrm{dx}\right] \qquad (30)$$

Differential of λ

$$\varphi = -\mathrm{d}\lambda = -\mathrm{d}\left(\frac{1}{2}\left[\mathrm{X}e^{z^2}\mathrm{dy} - \mathrm{Y}e^{z^2}\mathrm{dx}\right]\right)$$

It is known that ifφ is a closed 2- form on T^*M^3, then φ_H is also a Symplectic structure onT^*M^3

$$\varphi = -\mathrm{d}\lambda = e^{z^2}\mathrm{dy}\Lambda\mathrm{dx} \qquad (31)$$

. Let $(M^3, \phi. \xi, \eta, g)$3- Almost f-Cosymplectic Manifoldsform φ. Suppose that Hamiltonian vector field $\mathrm{X_H}$ associated to Hamiltonian energy H is given by

$$\mathrm{X_H} = \mathrm{X}\frac{\partial}{\partial \mathrm{x}} + \mathrm{Y}\frac{\partial}{\partial \mathrm{y}} + Z\frac{\partial}{\partial \mathrm{z}} \qquad (32)$$

Calculates a value $\mathrm{X_H}$ and φ

$$\varphi(\mathrm{Z_H}) = \left(e^{z^2}\mathrm{dy}\Lambda\mathrm{dx}\right)\left(\mathrm{X}\frac{\partial}{\partial \mathrm{x}} + \mathrm{Y}\frac{\partial}{\partial \mathrm{y}} + Z\frac{\partial}{\partial \mathrm{z}}\right)$$

$$\varphi(\mathrm{Z_H}) = -e^{z^2}(\mathrm{dy}\Lambda\mathrm{dx})\left(\mathrm{x}\frac{\partial}{\partial \mathrm{x}}\right) + \mathrm{dy}\Lambda\mathrm{dx}\left(\mathrm{y}\frac{\partial}{\partial \mathrm{y}}\right) + \mathrm{dy}\Lambda\mathrm{dx}\left(z\frac{\partial}{\partial \mathrm{z}}\right)$$

$$\varphi(\mathrm{Z_H}) = -\mathrm{X}e^{z^2}\frac{\partial}{\partial \mathrm{y}} + \mathrm{Y}e^{z^2}\frac{\partial}{\partial \mathrm{y}} - \mathrm{Y}e^{z^2}\frac{\partial}{\partial \mathrm{x}} + \mathrm{Xz}\frac{\partial}{\partial \mathrm{x}} + \mathrm{Z}e^{z^2}\frac{\partial}{\partial \mathrm{x}} - Ze^{z^2}\frac{\partial}{\partial \mathrm{y}} (33)$$

Otherwise, one may calculate the differential of Hamiltonian energy as follows:

$$\mathrm{dH} = \frac{\partial \mathrm{H}}{\partial \mathrm{x}}\mathrm{dx} + \frac{\partial \mathrm{H}}{\partial \mathrm{y}}\mathrm{dy} + \frac{\partial \mathrm{H}}{\partial \mathrm{z}}\mathrm{dz} \ (34)$$

From (33) and (34) with respect to $\mathrm{i_{X_H}}\varphi = \mathrm{dH}$, we find Hamiltonian vector field on 3- Almost f-Cosymplectic Manifolds space be

$$\mathrm{X_H} = \frac{\partial \mathrm{H}}{\partial \mathrm{y}}\frac{\partial}{\partial \mathrm{x}} - \frac{\partial \mathrm{H}}{\partial \mathrm{y}}\frac{\partial}{\partial \mathrm{y}} \qquad (35)$$

Suppose that the curve

$$\alpha: \mathrm{I} \subset \mathrm{R} \rightarrow \mathrm{R_n^{2n}}$$

be an integral curve of Hamiltonian vector field $\mathrm{Z_H}$, i.e.,

$$\mathrm{Z_H}\big(\alpha(\mathrm{t})\big) = \dot{\alpha} \quad , \mathrm{t} \in \mathrm{I}.$$

In the local coordinates we have

$$\alpha(\mathrm{t}) = (\mathrm{x(t)}, \mathrm{y(t)}, \mathrm{z(t)}),$$

$$\dot{\alpha}(\mathrm{t}) = \frac{\mathrm{dx}}{\mathrm{dt}}\frac{\partial}{\partial \mathrm{x}} + \frac{\mathrm{dy}}{\mathrm{dt}}\frac{\partial}{\partial \mathrm{y}} + \frac{\mathrm{dz}}{\mathrm{dt}}\frac{\partial}{\partial \mathrm{z}} \qquad (36)$$

Now, by means of (26), from (27) and (28), we deduce the equations so-called Hamiltonian equations

$$\frac{\mathrm{dx}}{\mathrm{dt}} = e^{z^2}\frac{\partial \mathrm{H}}{\partial \mathrm{y}} \quad , \frac{\mathrm{dy}}{\mathrm{dt}} = -e^{z^2}\frac{\partial \mathrm{H}}{\partial \mathrm{x}} \qquad (37)$$

6. Conclusion:

Finally, we concluded that the triple $(M^3, \phi. \xi, \eta, g)$ is Hamiltonianmechanical system on 3- Almost f-Cosymplectic Manifolds$(\mathrm{M}^3, \phi. \xi, \eta, g)$ from the above deductions.

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