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model is constructed for deteriorating items with instantaneous replenishment, exponential decay rate and a time varying linear demand without shortages under permissible delay in payments.

Himanshu and Ashutosh, (2014), studied an Optimum Inventory Policy for Exponentially Deteriorating Items considering multivariate consumption rate with partial backlogging. They developed a partial backlogging inventory model for exponential deteriorating items considering stock and price sensitive demand rate in fuzzy surroundings.

2 Assumptions and Notation

- The inventory system involves only one single item and one stocking point.
- Amelioration occurs when the items are effectively in stock.
- Deterioration occurs when the items are effectively in stock.
- The cycle length is T.
- The initial inventory level is I_o.
- The unit cost of the item is a known constant C, and the replenishment cost is also a known constant C_o per replenishment.
- The demand rate, $R(t) = e^{at}$ increases exponentially with time.
- The level of on-hand inventory at any time t is I(t).
- The ordering quantity per cycle which enters into inventory at t = 0 is I_o.
- The rate of amelioration α is a constant.
- The rate of deterioration β is a constant.
- The total number of ameliorated amount over the cycle T, when considered in terms of value (0, T) is given by A_T.

- The total number of deteriorated amount over the cycle T, when considered in terms of value (0, T) is given by D_T.
- The total number of on-hand inventory within the cycle T is I_T.
- The inventory holding cost $C_h = \lambda_1 + t\lambda_2$ is linearly dependent on time

3. Model Formulation

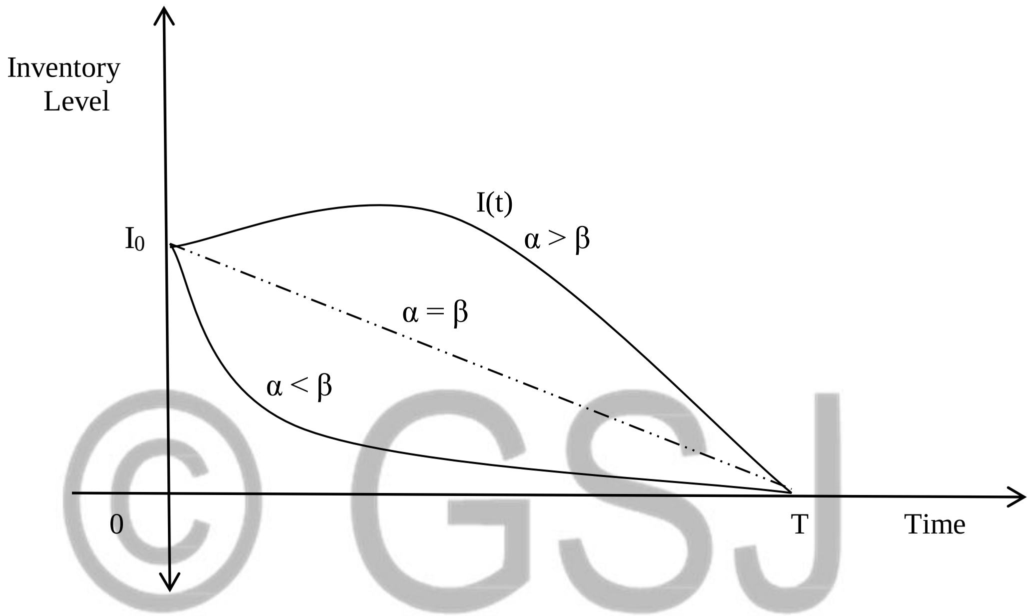

Figure 1: Inventory movement in an item that is both ameliorating and deteriorating with exponentially increasing demand and linear time dependent holding cost.

From Figure 1, I(t) be the on-hand inventory at time t ≥ 0, then at time t + Δt, the inventory level in the interval (0 , T) is given by:

$$I(t+\Delta t) = I(t) + (\alpha - \beta)I(t)\Delta t - \Delta t e^{at} \tag{1}$$

Divide by Δt and taking limit as Δt → 0, we have:

$$\frac{d}{dt}\left[I(t)\right] = (\alpha - \beta)I(t) - e^{at}$$

$$\frac{d}{dt}[I(t)] - (\alpha - \beta)I(t) = -e^{at} \tag{2}$$

The solution of equation (2) is given by;

$$I(t) = -\frac{e^{at}}{a - \alpha + \beta} + Ke^{(\alpha-\beta)t} \tag{3}$$

Now applying the boundary condition at t = 0 and I(t) = I_0, I_0 is obtained as;

$$I_0 = -\frac{1}{a - \alpha + \beta} + K$$

and K is obtained as;

$$K = I_0 + \frac{1}{a - \alpha + \beta} \tag{4}$$

Substituting equation (4) into equation (3) gives;

$$I(t) = -\frac{e^{at}}{a - \alpha + \beta} + \left(I_0 + \frac{1}{a - \alpha + \beta}\right) e^{(\alpha-\beta)t}$$

$$= -\frac{e^{at}}{a - \alpha + \beta} + \frac{1}{a - \alpha + \beta} e^{(\alpha-\beta)t} + I_0 e^{(\alpha-\beta)t} \tag{5}$$

Apply the boundary conditions, $t = T, I(t) = 0$, equation (5) becomes

$$0 = -\frac{e^{aT}}{a - \alpha + \beta} + \frac{1}{a - \alpha + \beta} e^{(\alpha-\beta)T} + I_0 e^{(\alpha-\beta)T}$$

I_0 is thus obtained as follows;

$$I_0 = \frac{e^{(a-\alpha+\beta)T}}{a-\alpha+\beta} - \frac{1}{a-\alpha+\beta}$$

Substitute the value of I_0 into equation (5) to get;

$$I(t) = -\frac{e^{at}}{a-\alpha+\beta} + \frac{1}{a-\alpha+\beta}e^{(\alpha-\beta)t} + \left[\frac{e^{(a-\alpha+\beta)T}}{a-\alpha+\beta} - \frac{1}{a-\alpha+\beta}\right]e^{(\alpha-\beta)t}$$

$$= -\frac{e^{at}}{a-\alpha+\beta} + \frac{1}{a-\alpha+\beta}e^{(\alpha-\beta)t} + \frac{e^{(a-\alpha+\beta)T}}{a-\alpha+\beta}e^{(\alpha-\beta)t} - \frac{1}{a-\alpha+\beta}e^{(\alpha-\beta)t}$$

$$= \frac{1}{a-\alpha+\beta}\left[e^{(a-\alpha+\beta)T+(\alpha-\beta)t} - e^{at}\right] \tag{6}$$

4 Total Amount of On-Hand Inventory during the Complete Cycle:

$$I_T = \int_0^T I(t)dt$$

$$= \int_0^T \left[\frac{1}{a-\alpha+\beta}\left[e^{(a-\alpha+\beta)T+(\alpha-\beta)t} - e^{at}\right]\right]dt$$

$$= \frac{e^{(a-\alpha+\beta)T}}{a-\alpha+\beta}\int_0^T e^{(\alpha-\beta)t}dt - \frac{1}{a-\alpha+\beta}\int_0^T e^{at}dt$$

$$= \frac{e^{(a-\alpha+\beta)T}}{a-\alpha+\beta}\left[\frac{e^{(\alpha-\beta)T}}{\alpha-\beta} - \frac{1}{\alpha-\beta}\right] - \frac{1}{a-\alpha+\beta}\left[\frac{e^{aT}}{a} - \frac{1}{a}\right]$$

$$= \frac{1}{a(\alpha-\beta)(a-\alpha+\beta)}\left[a(e^{aT} - e^{(a-\alpha+\beta)T}) - (\alpha-\beta)(e^{aT} - 1)\right]$$

(7)

5 Total Demand within (0, T):

$$R_T = \int_0^T e^{at} I(t)dt$$

$$= \frac{e^{(a-\alpha+\beta)T}}{a-\alpha+\beta}\int_0^T e^{(a+\alpha-\beta)t}dt - \frac{1}{a-\alpha+\beta}\int_0^T e^{2at}dt$$

Integrating the above equation we get;

$$=\frac{e^{(a-\alpha+\beta)T}}{a-\alpha+\beta}\left(\frac{e^{(a+\alpha-\beta)T}-1}{a+\alpha-\beta}\right)-\frac{1}{a-\alpha+\beta}\left(\frac{e^{2aT}-1}{2a}\right)$$

$$=\frac{e^{(a-\alpha+\beta)T}}{(a-\alpha+\beta)(a+\alpha-\beta)}\left(e^{(a+\alpha-\beta)T}-1\right)-\frac{1}{2a(a-\alpha+\beta)}\left(e^{2aT}-1\right) \quad (8)$$

6 The Ameliorated Amount within (0, T):

$$A_T=\alpha I_T$$

$$=\frac{\alpha}{a(\alpha-\beta)(a-\alpha+\beta)}\left[a(e^{aT}-e^{(a-\alpha+\beta)T})-(\alpha-\beta)(e^{aT}-1)\right] \quad (9)$$

7 Deteriorated Amount within (0, T):

$$D_T=\beta I_T$$

$$\frac{\beta}{a(\alpha-\beta)(a-\alpha+\beta)}\left[a(e^{aT}-e^{(a-\alpha+\beta)T})-(\alpha-\beta)(e^{aT}-1)\right] \quad (10)$$

Inventory Holding Cost in a Cycle:

$$C_h(t)=\int_0^T(\lambda_1+t\lambda_2)I(t)dt$$

$$=\lambda_1\int_0^T I(t)dt+\lambda_2\int_0^T tI(t)dt$$

$$=\frac{\lambda_1}{a(\alpha-\beta)(a-\alpha+\beta)}\left[a(e^{aT}-e^{(a-\alpha+\beta)T})-(\alpha-\beta)(e^{aT}-1)\right]+\frac{\lambda_2 e^{(a-\alpha+\beta)T}}{(\alpha-\beta)^2(a-\alpha+\beta)}((\alpha-\beta)T-1)e^{(a-\alpha)T}$$

$$-\frac{\lambda_2}{a^2(a-\alpha+\beta)}(aT-1)e^{aT}+\frac{\lambda_2}{(a-\alpha)^2(a-\alpha+\beta)}-\frac{\lambda_2}{a^2(a-\alpha+\beta)} \quad (11)$$

Total Variable Cost:

$$TVC(T)=\frac{C_0}{T}+\frac{C_h(t)}{T}-\frac{CA_T}{T}+\frac{CD_T}{T}$$

$$=\frac{1}{T}\left[C_0+\left(\begin{array}{l}\frac{\lambda_1}{a(\alpha-\beta)(a-\alpha+\beta)}\left[a(e^{aT}-e^{(a-\alpha+\beta)T})-(\alpha-\beta)(e^{aT}-1)\right]\\ +\frac{\lambda_2 e^{(a-\alpha+\beta)T}}{(\alpha-\beta)^2(a-\alpha+\beta)}((\alpha-\beta)T-1)e^{(a-\alpha)T}-\frac{\lambda_2}{a^2(a-\alpha+\beta)}(aT-1)e^{aT}\\ +\frac{\lambda_2}{(a-\alpha)^2(a-\alpha+\beta)}-\frac{\lambda_2}{a^2(a-\alpha+\beta)}\end{array}\right)\right.$$
$$\left.+\frac{C(\beta-\alpha)}{a(\alpha-\beta)(a-\alpha+\beta)}\left[a(e^{aT}-e^{(a-\alpha+\beta)T})-(\alpha-\beta)(e^{aT}-1)\right]\right]$$

To obtain the value of T which minimizes the total variable cost per unit time, we differentiate the above equation with respect to T

$$\frac{d(TVC(T))}{dT}=\left[\begin{array}{l}-\frac{C_0}{T^2}+\frac{\lambda_1}{a(\alpha-\beta)(a-\alpha+\beta)}\left(\begin{array}{l}\frac{a}{T^2}\left((aT-1)e^{aT}-((a-\alpha+\beta)T-1)e^{(a-\alpha+\beta)T}\right)\\ -\frac{(\alpha-\beta)}{T^2}\left((aT-1)e^{aT}+1\right)\end{array}\right)\\ +\frac{\lambda_2}{(\alpha-\beta)^2(a-\alpha+\beta)}\left((\alpha-\beta)(2a-2\alpha+\beta)Te^{(2a-2\alpha+\beta)}-\frac{(2a-2\alpha+\beta)T-1}{T^2}e^{(2a-2\alpha+\beta)}\right)\\ -\frac{\lambda_2}{a^2(a-\alpha+\beta)}\left(a^2e^{aT}-\frac{(aT-1)}{T^2}e^{aT}\right)-\frac{\lambda_2}{(a-\alpha)^2(a-\alpha+\beta)T^2}-\frac{\lambda_2}{a^2(a-\alpha+\beta)T^2}\\ +\frac{C(\beta-\alpha)}{a(\alpha-\beta)(a-\alpha+\beta)}\left(\begin{array}{l}\frac{a(aT-1)}{T^2}e^{aT}-\frac{a((a-\alpha+\beta)T-1)}{T^2}e^{(a-\alpha+\beta)T}\\ -\frac{(\alpha-\beta)(aT-1)}{T^2}e^{aT}+\frac{(\alpha-\beta)}{T^2}\end{array}\right)\end{array}\right]$$

$$=-\frac{1}{T^2}\left[\begin{array}{l}-C_0+\frac{\lambda_1}{a(\alpha-\beta)(a-\alpha+\beta)}\begin{pmatrix}a\left((aT-1)e^{aT}-((a-\alpha+\beta)T-1)e^{(a-\alpha+\beta)T}\right)\\-(\alpha-\beta)\left((aT-1)e^{aT}+1\right)\end{pmatrix}\\+\frac{\lambda_2}{(\alpha-\beta)^2(a-\alpha+\beta)}\left((\alpha-\beta)(2a-2\alpha+\beta)T^2-((2a-2\alpha+\beta)T-1)\right)e^{(2a-2\alpha+\beta)}\\-\frac{\lambda_2}{a^2(a-\alpha+\beta)}\left(a^2T^2-(aT-1)\right)e^{aT}-\frac{\lambda_2}{a^2(a-\alpha)^2(a-\alpha+\beta)T^2}\left(a^2-(a-\alpha)^2\right)\\+\frac{C(\beta-\alpha)}{a(\alpha-\beta)(a-\alpha+\beta)}\begin{pmatrix}\left((aT-1)(a-\alpha+\beta)\right)e^{aT}-a\left((a-\alpha+\beta)T-1\right)e^{(a-\alpha+\beta)T}\\+(\alpha-\beta)\end{pmatrix}\end{array}\right]$$

For optimal cycle period T which minimizes the total variable cost per unit time,

$$\frac{d}{dT}TVC(T)=0$$

Therefore;

$$0=-\frac{1}{T^2}\left[\begin{array}{l}-C_0+\frac{\lambda_1}{a(\alpha-\beta)(a-\alpha+\beta)}\begin{pmatrix}a\left((aT-1)e^{aT}-((a-\alpha+\beta)T-1)e^{(a-\alpha+\beta)T}\right)\\-(\alpha-\beta)\left((aT-1)e^{aT}+1\right)\end{pmatrix}\\+\frac{\lambda_2}{(\alpha-\beta)^2(a-\alpha+\beta)}\left((\alpha-\beta)(2a-2\alpha+\beta)T^2-((2a-2\alpha+\beta)T-1)\right)e^{(2a-2\alpha+\beta)}\\-\frac{\lambda_2}{a^2(a-\alpha+\beta)}\left(a^2T^2-(aT-1)\right)e^{aT}-\frac{\lambda_2}{a^2(a-\alpha)^2(a-\alpha+\beta)}\left(a^2-(a-\alpha)^2\right)\\+\frac{C(\beta-\alpha)}{a(\alpha-\beta)(a-\alpha+\beta)}\begin{pmatrix}\left((aT-1)(a-\alpha+\beta)\right)e^{aT}-a\left((a-\alpha+\beta)T-1\right)e^{(a-\alpha+\beta)T}\\+(\alpha-\beta)\end{pmatrix}\end{array}\right]$$

Multiplying through by $T^2a^2(\alpha-\beta)^2(a-\alpha+\beta)$ we obtain;

$$\begin{array}{l}0=-a^2(\alpha-\beta)^2(a-\alpha+\beta)C_0+a(\alpha-\beta)\lambda_1\begin{pmatrix}a\left((aT-1)e^{aT}-((a-\alpha+\beta)T-1)e^{(a-\alpha+\beta)T}\right)\\-(\alpha-\beta)\left((aT-1)e^{aT}+1\right)\end{pmatrix}\\+a^2\lambda_2\left((\alpha-\beta)(2a-2\alpha+\beta)T^2-((2a-2\alpha+\beta)T-1)\right)e^{(2a-2\alpha+\beta)}\\-(\alpha-\beta)^2\lambda_2\left(a^2T^2-(aT-1)\right)e^{aT}-\lambda_2\left(a^2-(a-\alpha)^2\right)\\+a(\alpha-\beta)C(\beta-\alpha)\begin{pmatrix}\left((aT-1)(a-\alpha+\beta)\right)e^{aT}-a\left((a-\alpha+\beta)T-1\right)e^{(a-\alpha+\beta)T}\\+(\alpha-\beta)\end{pmatrix}\end{array}\qquad(12)$$

10 Economic Order Quantity:

$$EOQ = \frac{1}{a-\alpha+\beta}\left(e^{(a-\alpha+\beta)T} - 1\right) = I_0 \tag{13}$$

11 Numerical Examples

We use equation (12) to obtain the numerical examples below.

Table 1: Input parameter values for the five numerical examples;

a	α	β	C	C_0	λ_1	λ_2
6	0.23	0.01	200	7000	20	6,000
10	0.4	0.2	200	100,000	2	2,000
13	0.6	0.3	200	50,000	10	1,500
15	0.5	0.3	200	100,000	6	2,500
20	0.8	0.6	150	60,000	20	4,000

Table 2: Output parameter values for the five numerical examples showing the optimal solution obtained;

T*	TVC(T)*	EOQ*
0.4575 (167 days)	19159	2.26
0.5178 (189 days)	224390	16.21
0.3479 (127 days)	169178	6.46
0.3507 (128 days)	331714	12.06
0.2384 (81 days)	294196	5.61

12 Sensitivity Analysis

Now we carryout sensitivity analysis to see the effect of parameter changes on the decision variables. This has been carried out on second example by changing (that is increasing/decreasing) the parameters by ±1%, ±5% and ±25% and taking one parameter at a time, keeping the remaining parameters constants.

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Table 3: Sensitivity analysis of the second example from Table 3.11.1

Parameters	% change in the parameter value	% change in results		
		T*	TVC(T)*	EOQ*
a	-25	30.16	-23.08	14.93
	-5	4.76	-4.5	2.31
	-1	1.06	-0.90	1.19
	1	-1.06	0.90	-1.23
	5	-4.23	4.47	-1.60
	25	-18.52	22.03	-10.53
α	-25	20.11	-19.06	193.50
	-5	2.65	-3.08	15.48
	-1	0.53	-0.60	2.94
	1	-0.53	0.59	-2.80
	5	-2.65	2.88	-13.33
	25	-10.05	12.94	-42.36
β	-25	-5.28	6.88	-27.15
	-5	-1.59	1.46	-8.14
	-1	-0.53	0.30	-2.72
	1	0	-0.30	0.11
	5	1.06	-1.51	6.03
	25	7.94	-8.20	53.42
C	-25	0	0.06	0.02
	-5	0	0.012	0.02
	-1	0	0.002	0.02
	1	0	-0.003	0.02
	5	0	-0.01	0.02
	25	0	-0.06	0.02
C_0	-25	-3.70	-21.91	-17.22
	-5	-0.53	-4.32	-2.64
	-1	0	-0.86	0.02
	1	0	0.86	0.02
	5	0.53	4.29	2.76
	25	2.65	21.23	14.48
λ_1	-25	0	0.005	0.024
	-5	0	0.0009	0.024
	-1	0	8.69	0.024
	1	0	-0.00031	0.024
	5	0	-0.001	0.024
	25	0	-0.005	0.024
λ_2	-25	3.70	-3.94	20.84
	-5	0.53	-0.72	2.76
	-1	0	-0.14	0.24
	1	0	0.14	0.24
	5	-0.53	0.69	-2.64
	25	-2.65	3.23	-12.62

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13 Discussion of Results

We use equation 12 to obtain the numerical examples as Table 1 shows the input parameters while Table 2 shows the output values. We then discuss the effect of changes in the values of the parameters on decision variables as contained in Table 3 above. The Table shows that all the decision variables are sensitive to changes in all the parameters. We also notice the following from the table:

- As T* increases, β and C_0 increases.
- As TVC(T)* increases, a, α, C_0 and λ_2 increases.
- As EOQ* increases, β and C_0 increases.
- As T* decreases, a, α and λ_2 decreases.
- As TVC(T)* decreases, β, C and λ_1 decreases.
- As EOQ* decreases, a, α and λ_2 decreases.

The analysis above shows that the cycle period, T* increases as the replenishment cost and rate of deterioration increase and this conforms to common expectation since the higher ordering cost increase the quantity of the inventory and hence takes longer time to be disposed of. The analysis also shows, as expected, that the total variable cost, TVC(T)*, increases as the rate of demand, the rate of amelioration, the replenishment cost and holding cost increase. Lastly, we noticed in the analysis that the EOQ* increases with increase in the rate of deterioration and replenishment cost. However, the change in T* has no effect on C and λ_1, and the change in EOQ* has a constant effect on C and λ_1.

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