Global Scientific JOURNALS

GSJ: Volume 11, Issue 2, February 2023, Online: ISSN 2320-9186
www.globalscientificjournal.com

# EVALUATION OF THE OPTIMAL INVESTMENT STRATEGY IN THE ACCUMULATION PHASE

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KeyWords
Accumulation Phase, Asset Allocation, Certainty Equivalent, Optimal Investment, Retirement Strategy, Ruin Probability.

## ABSTRACT

The importance of the individual asset management in the accumulation phase is becoming more crucial in recent years due to increasing longevity risk. This paper investigates the optimal static investment strategy of individuals in the accumulation phase under several assumptions on the labor income stream, pension contribution, and financial market. We employ two kinds of measures in the evaluation of different investment mix. First, we use certainty equivalent measure to evaluate welfare effects of using a certain investment strategy over others. Second, we also employ ruin probability which measures the chance of reaching zero wealth before the time of death. Our analyses based on the utility measure suggest that a risk-loving individual should optimally invest all the assets to stocks while a 25% risky investment is optimal for a very risk-averse individual. We also find that a full risk-free investment may lead to a disastrous situation if we consider the ruin probability during post-retirement period. Our study implies that government should make more efforts to boost the financial literacy of the people to invest more in stocks during their working period.

**MAIN PAPER STARTS HERE…**

## Introduction

In recent decades, individuals are facing longevity risk due to increasing life expectancy and exacerbating fiscal instability of the national pension system. This is particularly the case in developed countries with underdeveloped social security system where the living standard has significantly improved and hence the life expectancy has been markedly increased but the income stability in the post-retirement period is not sufficient to ensure the financial stability to retirees. Thus, the importance of private pension savings in the accumulation phase is becoming crucial to every individual in order to have enough wealth at retirement which will be used to finance the consumption in the decumulation phase. The fact that many countries are providing preferential pension tax benefits to pension savings also confirms that the government is making efforts to shift the main source of retirement income from social security expenses to individual pension savings.

In this context, the rapid demographic change and the diminishing generosity of the social security system have given rise to the growth of the asset management of the occupational pension plans in the second pillar. At the time when the interest rate was consistently high and the industry was growing fast, workers were able to earn higher rate of return without investing in risky assets. This implies that people did not care much about how to allocate their savings to risk-free assets (e.g., bank deposits) or to risky assets (e.g., bonds or stocks) since it was not needed as the country had not realized longevity risks to come. However, as people have experienced the low interest rate era and the national pension system is not as generous as it was in the past, we are now aware of the importance of managing their assets to maximize their pension wealth at retirement.

In this paper, we investigate the optimal investment strategy of individuals in the accumulation phase. Under several assumptions about the labor income stream, pension contribution, and financial market, we simulate the stochastic wealth paths to account for the investment uncertainty during the working period. To evaluate different strategies of the asset allocation, we employ two methods: certainty equivalent and ruin probability. The first measure, certainty equivalent, allows us to evaluate welfare effects of an investment strategy over others by comparing lifetime discounted utility over the investment horizon. Under this measure, the optimal investment strategy would depend on the risk attitude of the individual. The second measure, ruin probability, is also a useful standard to evaluate the optimal investment strategy as it provides a different perspective compared to the certainty equivalent. Specifically, a ruin probability measures the chance of reaching zero wealth before the time of death, implying that the accumulated retirement wealth is exhausted during the retirement phase. Hence, while the certainty equivalent measure provides the best investment strategy that gives the highest utility, the ruin probability measure gives the best investment strategy that gives the minimum risk of outliving assets.

Our study is related to the finance and economics literature on the optimal investment decision of an individual. Although many papers have attempted to find the dynamic optimal investment strategy over the life-cycle (Merton, 1969; Merton, 1971; Coco et al., 2005), we rather focus on the static optimal investment strategy in the accumulation phase only. That is, we do not allow for a time-varying asset allocation. Also, we do not actually solve for the optimal strategy; we set up a multiple set of possible investment choices and evaluate them under two different measures, namely the utility maximization method and the ruin probability minimization method. This is because, as Allen and Carroll (2001) have suggested, it is extremely difficult for an average person to perform an exact calculation of the financial decisions by incorporating a multiple of parameters to be considered in a dynamic programming model. This has also been verified by Hu and Scott (2007) and Salibury and Nenkov (2016) who have demonstrated that behavioral factors may restrict people’s ability to draw optimal solutions. Hence, similar to the spirit of Gomes et al. (2008), Winter et al. (2012), and Love (2013), this paper aims to investigate optimal “(conventional) rules of thumb” that are “alternatives to dynamic programming solutions” (Love, 2013).

This paper proceeds as follows. Section 2 presents our theoretical model and the research methodology, and the results are reported in Section 3. Section 4 concludes with some policy implications.

## Methodology

### Investor’s Preference

We assume that an investor’s preference follows the Constant Relative Risk Aversion (CRRA) utility function. The utility function is defined as:

$$U(W_t) = \frac{W_t^{1-\gamma}}{1-\gamma}, \quad (1)$$

where we denote by $W_t$ the wealth at time t and by $\gamma > 1$ the risk aversion coefficient. Higher $\gamma$ implies that the investor is more risk-averse, which leads to stronger desire to smooth the wealth path and avoid uncertainty of the wealth over time. In other words, an individual with higher $\gamma$ tends to prefer a relatively lower expected value with a small volatility rather than a relatively higher expected value with a large volatility. Note that the utility derived from the equation (1) gives an ordinal number although the wealth itself is a cardinal one. This implies that we cannot simply interpret the level of utility and we need to evaluate the welfare by another measure. This will be discussed later.

## Labor Income and Pension Contribution

We consider a discrete-time model where an investor accumulates part of her labor income as retirement wealth. Specifically, we set up an investment horizon $t = 0, \dots, T$ as the accumulation phase in which an individual retires at time T. We do not consider uncertainty in lifetime, that is, no mortality rate is incorporated in our model and the investor survives until time T for sure. During the accumulation period, the investor receives labor income $Y_t$ and we assume that labor income grows over time with an inflation rate $i_t$. Hence, the labor income process follows:

$$Y_{t+1} = Y_t(1 + i_t), \text{ for } t = 0, \dots, T-1 \quad (2)$$

In Korea, the minimum contribution rate of the workers in DC plans are 8.33% (1/12) of the yearly income and we follow this rule in our paper. Therefore, the pension contribution $\mathrm{Contr}_t$ is determined as follows:

$$\mathrm{Contr}_t = Y_t/12. \quad (3)$$

## Financial Market Assumptions

The investor can either invest in a risk-free asset $B_t$ or a risky asset $S_t$. Each asset represents the risk-free government bond and the risky stock index in the financial market, respectively. The price dynamics of both assets are defined as in the standard literature:

$$dB_t = rB_t dt, \quad (4)$$
$$dS_t = \mu S_t dt + \sigma S_t dZ_t, \quad (5)$$

where r denotes the risk-free rate of return, $\mu$ the expected return of the risky asset, $\sigma$ the standard deviation of the risky asset, and $Z_t \sim N(0,1)$ the Wiener process.

## Investor’s Wealth Accumulation Process

We further assume that the investor can choose the asset allocation of her portfolio consisting of the risky asset with a fraction $f_t$ and the riskless asset with a fraction $(1 - f_t)$. Here, the portfolio is the accumulated retirement wealth in the pension account made by the pension contribution over the working period. The wealth dynamics in the accumulation phase are then given by:

$$W_{t+1} = W_t(1 + r + f_t\mu + f_t\sigma Z_t) + \mathrm{Contr}_{t+1}, \quad (6)$$

with the initial wealth condition $W_0 = Y_0$. That is, the wealth in the next period is the sum of the portfolio value based on the wealth in the previous period and the pension contribution saving based on the labor income received in the next period. Note that we abstain from other possible source of income, that is, the labor income is the unique source of the wealth accumulation. Moreover, we also assume that the remaining labor income after the pension contribution is used to finance the current consumption in each period. Hence, the investor only has the pension account as the wealth that is going to be used to finance the future consumption and no other wealth accumulation is allowed in our model.

## Evaluation Method 1: Certainty Equivalent

The investor's first measure to figure out the optimal investment strategy is the Certainty Equivalent (CE). Since we generate numerous possible wealth paths depending on the random process $Z_t$, it is not possible to simply conclude the dominance among the different investment strategies using the expected wealth at retirement. Moreover, as previously mentioned, our utility function only provides ordinal information on which wealth path dominates the other, but we cannot evaluate the welfare effect which must be based on cardinal values. Hence, the concept of CE has been widely accepted as a useful measure in welfare analysis among the researchers. The meaning of the CE, in our context, is a certain amount of wealth that gives the same level of utility based on the numerous wealth levels at retirement. The CE is defined as an inverse function of the expected utility derived by a certain random variable, which is $W_T$ in our case. Formally, the CE can be computed by the following equation:

$$\mathrm{CE}(f) = \left[(1-\gamma)\frac{W_T(f)^{1-\gamma}}{1-\gamma}\right]^{1-\gamma}, \tag{7}$$

where $W_T(f)$ denotes the wealth level at retirement with the investment strategy f, and hence the CE is also a function of f. By comparing two different CEs, the certainty equivalent utility loss $l_{CE}$ is obtained by:

$$l_{CE} = 1 - {}^{\mathrm{CE}_{sub}}\!/\!_{\mathrm{CE}_{opt}}, \tag{8}$$

where $\mathrm{CE}_{sub}$ is a CE based on a suboptimal investment strategy and $\mathrm{CE}_{opt}$ is a CE based on an optimal investment strategy.

## Evaluation Method 2: Ruin Probability

Until now, we only looked at the problem in the accumulation phase. However, what the investor accumulates during the accumulation phase is the retirement wealth that is going to be used as the consumption in the decumulation phase, i.e., after retirement. Hence, it is also important to evaluate the accumulated retirement wealth from the perspective of the usefulness in the decumulation phase. Albrecht and Maurer (2002) developed ruin probability, a novel measure to evaluate the retirement wealth combined with a consumption flow. Ruin probability is the probability of the wealth being less than zero, which implies that the wealth becomes insufficient to finance the consumption. We also employ this ruin probability as the investor's second measure to determine the optimal investment strategy. To generate the wealth path in the decumulation phase, we need to make an assumption on how the investor plans the consumption over time. Hence, for $t = T, \dots, D$ where D denotes the time of death, we assume that the investor consumes $C_t$ which is a fraction of the labor income in the last period in the accumulation phase, $Y_T$, and increases with the inflation rate. That is, the consumption process in the decumulation period is given by:

$$C_T = \alpha Y_{T-1}(1 + i_t), \tag{9}$$

$$C_{t+1} = C_t(1 + i_t), \text{ for } t = T, \dots, D-1, \tag{10}$$

where α is the replacement ratio based on the labor income in the last period before retirement. This leads to the wealth dynamics in the decumulation phase:

$$W_{t+1} = (W_t - C_t)(1 + r), \text{ for } t = T, \dots, D-1. \tag{11}$$

The equation (11) indicates that the remaining wealth after consumption is invested in the risk-free asset only. The reason why we do not include the risky asset is that retirees tend to invest most of the assets in the risk-free asset in practice, as the retirees do not receive labor income anymore and hence become more risk-averse in the decumulation phase. Based on the wealth process, we can calculate the ruin probability $RP_t$ using the following equation:

$$RP_t = \Pr(W_t < 0), \text{ for } t = T, \dots, D-1. \tag{12}$$

## Parameter Used in the Analysis

To simulate the wealth paths, we need to specify the parameters in our model. First of all, we test three different level of risk aversion $\gamma = \{2,5,8\}$. An investor with $\gamma = 2$, $\gamma = 5$, and $\gamma = 8$ implies a risk-lover, a moderate risk-averse individual, and a highly risk-averse individual, respectively. For the financial market parameters, we set $r = 0.02$, $\mu = 0.06$, and $\sigma = 0.2$ as in Bovenberg and Mehlkopf (2013). The parameters imply that the risk premium is 4%. We further assume that the inflation rate is 2% throughout the lifetime. Along with the different levels of risk aversion, we test five investment strategies: $f = 0\%, 25\%, 50\%, 75\%$, and 100%. Since we are more interested in the welfare analysis as well as the ruin probability measure, we normalize the initial labor income to one rather than use the absolute realistic income level. Finally, the replacement ratio in the decumulation is set as 50%, meaning that the investor requires half of the last salary as the initial consumption level after retirement. The parameters used in the simulation are presented in [Table 1].

**Table 1.** Parameters used in the simulation**.**

| Parameter | Description | Value |
|---|---|---|
| $\gamma$ | Risk aversion coefficient | 2,5,8 |
| $r$ | Riskless rate | 0.02 |
| $\mu$ | Expected return of the risky asset | 0.06 |
| $\sigma$ | Volatility of the risky asset | 0.2 |
| $i_t$ | Inflation rate | 0.02 |
| $f$ | Risky investment fraction | 0, 0.25, 0.5, 0.75, 1 |
| $Y_0$ | Initial labor income | Normalized to 1 |
| $\alpha$ | Replacement ratio | 0.5 |
| $T$ | Retirement period | 40 |
| $D$ | Time of death | 80 |

## Discussion

## Wealth Path Simulation

Before we investigate the optimal investment strategies for individuals with different risk tolerance levels, we first show the effect of stochastic process embedded in the wealth path $W_t$ in the accumulation phase. This is to demonstrate the need of CE as a measure of welfare effect based on the numerous wealth path with the market risk of investment which cannot be solely evaluated by the expected wealth value. To show this, we simulate the wealth path in the accumulation phase by generating 100,000 random variables for the Wiener process $Z_t$. Hence, each scenario contains 40 random variable following the standard normal distribution. Using the wealth dynamics in the equation (6), we then simulate the wealth path of each scenario and therefore we obtain 100,000 different wealth paths over the accumulation phase. In [Figure 1], we illustrate the expected wealth path (left panel) and the 100,000 stochastic wealth paths when the investor invests 25% of her portfolio in the risky asset.

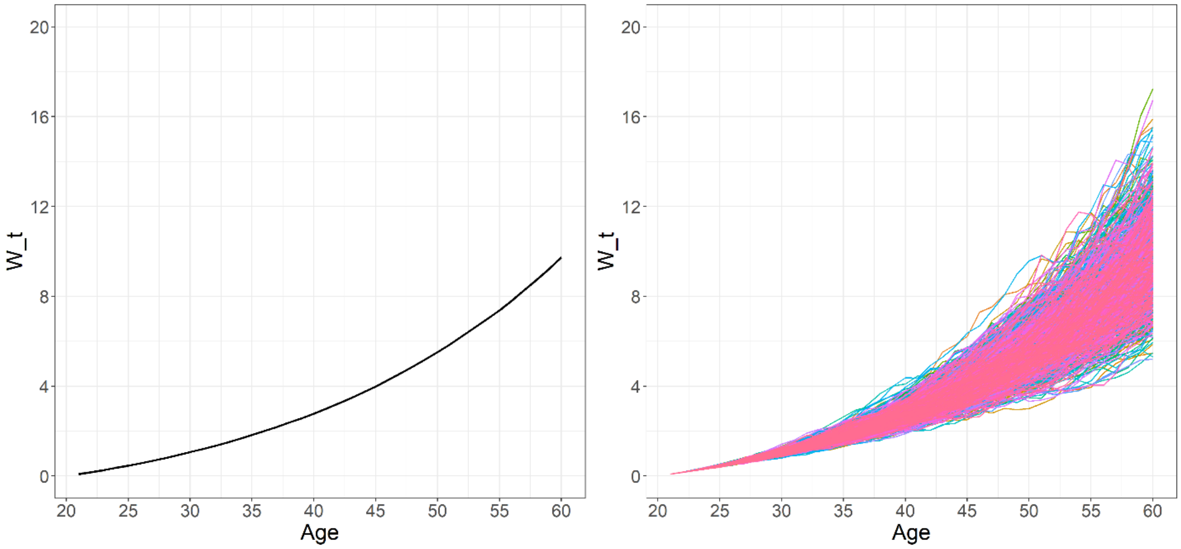

**Figure 1.** Expected wealth path vs. stochastic wealth paths ($\boldsymbol{f} = \mathbf{25}\%$)

We see that the investment risk entails significant amount of uncertainty over the horizon behind the expected wealth path. For instance, the expected value of the portfolio in the pension plan at the age of 40 is 2.78, meaning that the investor will accumulate on average about 2.8 times of her annual labor income after 20 years of investment when $f = 25\%$. However, the maximum possible wealth is 4.37 and the minimum possible wealth is 1.89 at 40 in our simulation, suggesting that it could also be that the investor's portfolio value is 1.57 times higher than the expected wealth level or only 67.9% compared to the expected wealth, depending on the market condition. If the investment horizon gets longer, the effect of uncertainty also increases substantially. At the time of retirement, the expected wealth value is 9.74 while the maximum possible wealth is 17.25 and the minimum possible wealth 5.2. This implies that the investor can also have the portfolio value that is 1.77 times higher than the expected wealth level or only 53.4% compared to the expected wealth. This uncertainty band reveals the investment risk the investor faces during the accumulation phase, and the effect is stronger when the investor decides to take more risks, as illustrated in [Figure 2].

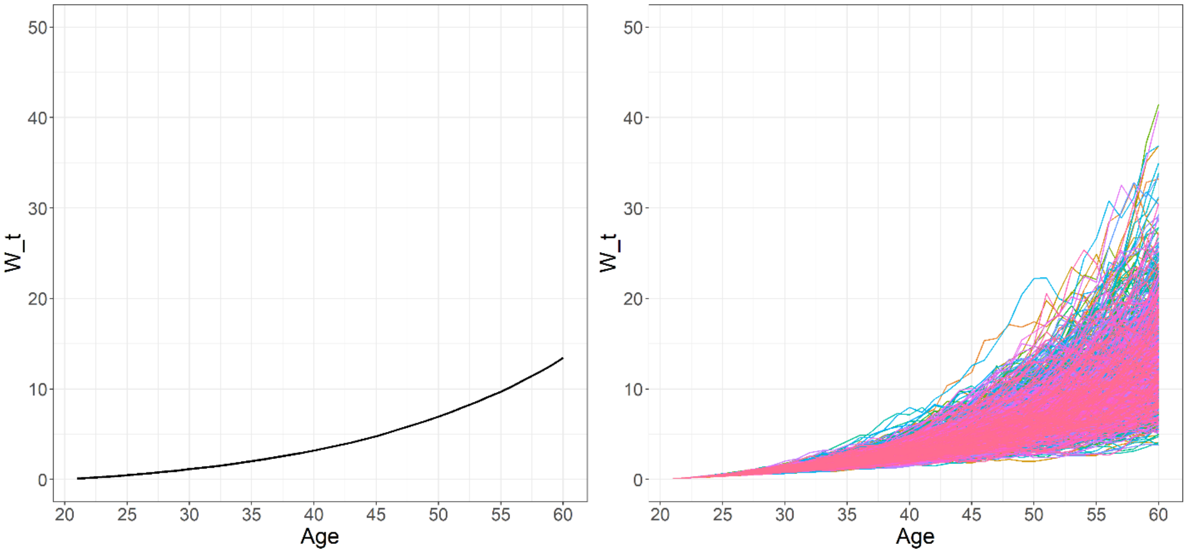

**Figure 2.** Expected wealth path vs. stochastic wealth paths ($\boldsymbol{f} = \mathbf{50}\%$)

In [Figure 2], it can be seen that the uncertainty band is enlarged and the wealth paths cover larger area than the previous illustration. For instance, the expected retirement wealth (at age 60) is 13.43, meaning that the investor will accumulate on average about 13.4 times of her annual labor income after 40 years of investment when $f = 50\%$. However, the maximum possible wealth is 41.47

and the minimum possible wealth is 3.77 at 60 in our simulation, suggesting that it could also be that the investor's portfolio value is 3.09 times higher than the expected wealth level or only 28.1% of the expected wealth, depending on the market condition. Compared to the case with $f = 25\%$, this indicates that a higher risk-taking can either lead to a significant return or a significant loss of the portfolio value over the investment horizon. Note that we test both a risk-loving and a risk-averse investor with different risk aversion coefficient of the CRRA utility function. Therefore, although the expected wealth path with $f = 50\%$ is always higher than the case with $f = 25\%$ in the accumulation phase, we cannot judge whether an investment strategy with a higher expected wealth would be optimal to a risk-averse individual. This provides a justification of using the CE as a measure of evaluating the welfare implication.

## Welfare Analysis

In this section, we explore the optimal investment strategy of investors by comparing utility derived by the wealth levels at retirement. First, we compare the expected utility of wealth at retirement, $E[W_T]$, under different risky investment fractions. That is, we use the 100,000 simulated values of $W_T$ and compute each utility using the equation (1). We then compute the mean of the utility values to obtain $E[W_T]$. The results are presented in [Figure 3].

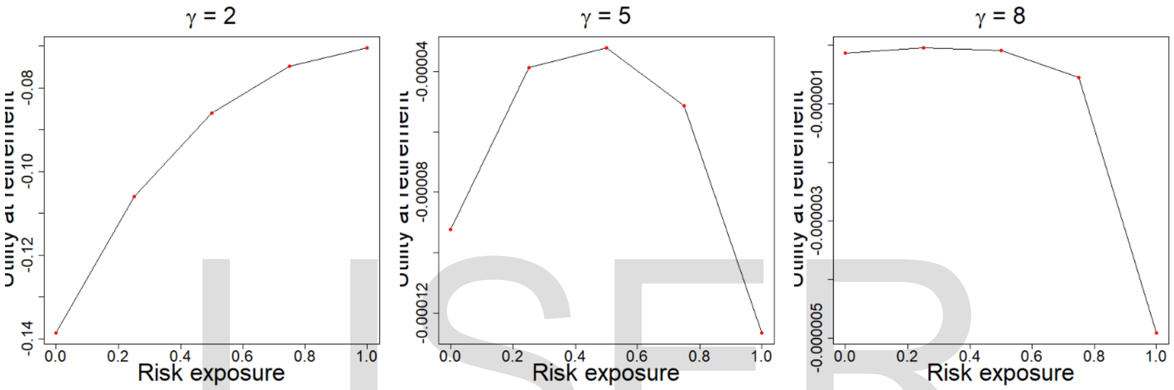

**Figure 3.** Expected utility of wealth at retirement ($\boldsymbol{\gamma = 2, 5, 8}$)

The panel in the left, middle, and right illustrate the expected utility of the retirement wealth for investors with $\gamma = 2, \gamma = 5$, and $\gamma = 8$, respectively, accumulated under the investment strategies $f = \{0\%, 25\%, 50\%, 75\%, 100\%\}$ as described in [Table 1]. We see that the optimal investment strategy varies depending on the risk attitude of the investors. For instance, investing 100% of the portfolio into the risky assets gives the highest utility to a risk-loving investor, however, this strategy turns out to be the worst one to the other two investors who are moderately risk-averse and highly risk-averse. For an investor with $\gamma = 5$, it is optimal to retain 50% of risky exposure in the accumulation phase while 25% is optimal for an investor with $\gamma = 8$. The overall pattern suggests that an investor with low risk aversion coefficient should optimally allocate all her pension contributions into risky assets, whereas high risk aversion leads to less aggressive investment strategies. Furthermore, it is observed that a small deviation from the optimal investment strategy does not give significant difference in utility for the most risk-averse investor. However, the utility itself cannot say more than ordinal implication and therefore the interpretation of the results are limited. Hence, we compute the certainty equivalent utility loss as in the equation (8) to evaluate the welfare effect of suboptimal investment decisions for investors with different risk tolerance levels. The welfare effects are reported in [Table 2].

**Table 2.** Certainty equivalent utility loss from suboptimal investment

| **Risk Tolerance** | **Risky Investment Fraction** | | | | |
|---|---|---|---|---|---|
| | $f = 0\%$ | $f = 25\%$ | $f = 50\%$ | $f = 75\%$ | $f = 100\%$ |
| $\gamma = 2$ | 99.1% | 94.3% | 75.3% | 34.5% | 0.0% |
| $\gamma = 5$ | 99.9% | 72.4% | 0.0% | 96.2% | 100.0% |
| $\gamma = 8$ | 100.0% | 0.0% | 99.3% | 100.0% | 100.0% |

The welfare analysis in [Table 2] confirms the optimal risky investment fractions we find previously in the utility comparison in [Figure 3]. The utility loss of zero percent implies that the corresponding $f$ results in the highest CE for each investor compared to other investment strategies. In addition to this, we can also see how much the investor would be worse off by choosing a suboptimal risk exposure. For example, allocating 75% of the pension wealth to stocks lead to a welfare loss of 34.5% to a risk-loving investor, whereas the same risk exposure entails significant welfare losses of 96.2% and 100% to investors with $\gamma = 5$ and $\gamma = 5$, respectively. Also, reducing the risky investment fraction to one step below from the optimal one the among the investment options lead to different magnitude of welfare losses. In particular, the welfare loss by a marginal deviation from the optimal strategy is larger for more risk-averse investor, suggesting that tailoring the optimal investment fraction is more important to investors with high risk aversion in terms of certainty equivalent utility loss. Reducing the risk exposure by one interval (i.e., 25 percent point) from the optimal investment strategy leads to the welfare loss of 34.5%, 72.4%, and 100% for each investor with different risk tolerances, respectively. In other words, an investor with $\gamma = 8$ must stick to the optimal investment fraction which is 25% out of the portfolio since her welfare loss can easily increase up to 100% otherwise. All in all, our utility comparison as well as the welfare analysis reveal the importance of finding optimal investment strategies to the individual, suggesting that the “almost zero” risky investment of the pension assets in practice is now causing significant welfare losses to the Korean workers.

## Ruin Probability

Although accumulated using the pension contribution which is part of the labor income during the working period, the purpose of saving the retirement wealth is to finance the future consumption in the decumulation phase. Therefore, it is also important to evaluate the investment strategies incorporating the consumption flow after retirement. To do this, we use the 100,000 scenarios of the retirement wealth per each risky investment fraction and set up the consumption flow in the decumulation phase with the replacement ratio of 50% as in the equation (9) and (10). The wealth decumulation path is then simulated by the wealth dynamics in the equation (11). In each period, we compute the ruin probability by counting the number of negative wealth out of the 100,000 scenarios. Here, negative wealth implies that the remaining retirement wealth is insufficient to finance the consumption and thus the investor is at the risk of ruin. [Figure 4] illustrates the overall ruin probabilities under different investment strategies. Note that since the ruin probability is a function of retirement wealth and the amount of consumption in each period, the results are irrelevant to the utility function and the risk aversion coefficient. Thus, the following analysis is generally applicable to any individual with different risk tolerance levels.

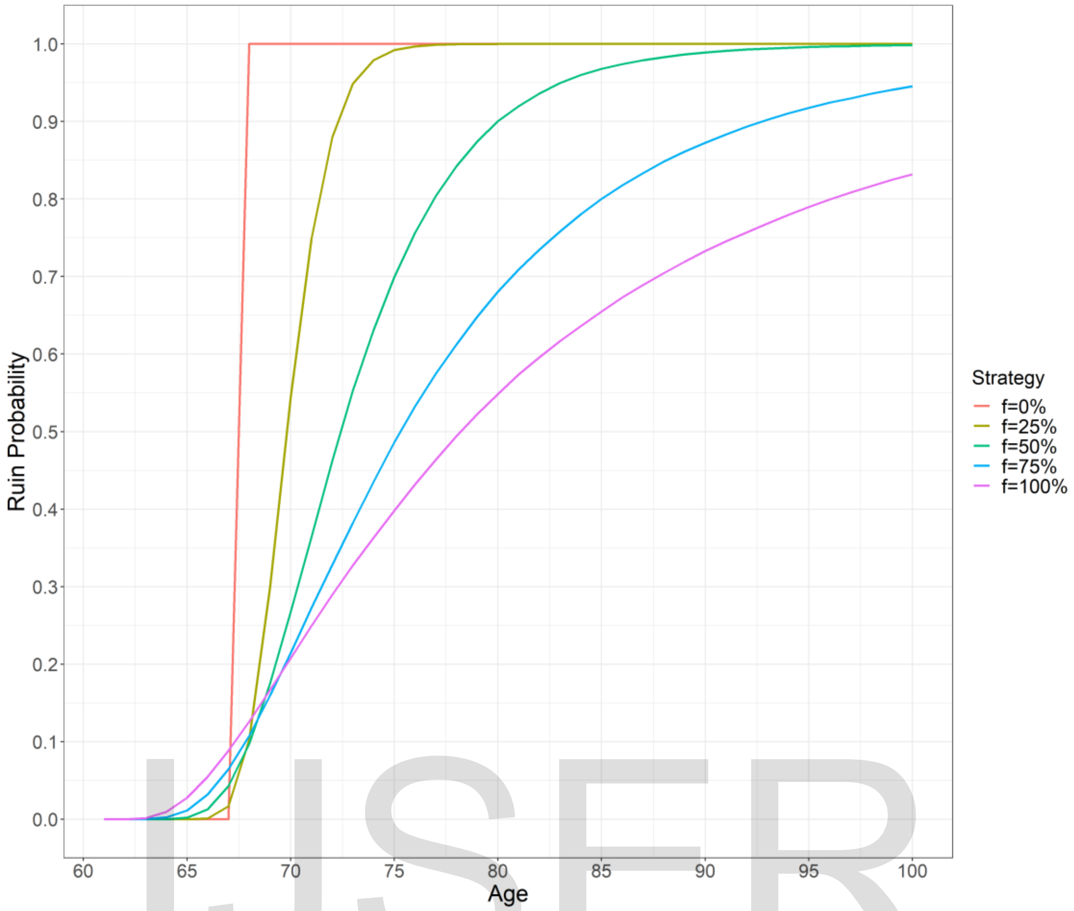

**Figure 4.** Ruin probability in decumulation phase for retirement wealth under different investment strategies

It is commonly observed that the ruin probability increases over the horizon but with different speed. In [Figure 4], we see that the ruin probability increases to 100% at age 67 for if the retirement wealth had been accumulated by a 0% risky investment strategy during the working period. With higher risky fractions, the speed of the increase in the ruin probabilities gets slower, and for some cases, the probabilities do not reach 100% until the terminal period at age 100. For instance, the ruin probability at age 100 is 94.6% and 83.2% for the retirement wealth under 75% and 100% risk exposure during the working period, respectively. Nevertheless, we cannot judge the dominance among different wealth levels at retirement since more uncertain wealth levels at retirement lead to higher ruin probabilities in the very early ages after retirement. To see this more clearly, we report the ruin probabilities in specific ages in [Table 3].

**Table 3.** Ruin probability in decumulation phase for retirement wealth under different investment strategies

| Age | Risky Investment Fraction | | | | |
|---|---|---|---|---|---|
| | $f = 0\%$ | $f = 25\%$ | $f = 50\%$ | $f = 75\%$ | $f = 100\%$ |
| 61 | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| 65 | 0.0% | 0.0% | 0.2% | 1.1% | 2.8% |
| 70 | 100.0% | 54.5% | 26.8% | 21.5% | 20.8% |
| 80 | 100.0% | 100.0% | 90.0% | 68.0% | 54.9% |

| | | | | | |
|---|---|---|---|---|---|
| 90 | 100.0% | 100.0% | 98.9% | 87.3% | 73.3% |
| 100 | 100.0% | 100.0% | 99.9% | 94.6% | 83.2% |

Interestingly, the optimal investment strategy in the accumulation based on the ruin probability in the decumulation phase is not consistent over time. That is, the ruin probability is zero until the age 65 only for the retirement wealth accumulated by risk-free investment and 25% risk exposure in the accumulation phase, while the other three investment strategies lead to ruin probabilities higher than zero. For instance, the ruin probability at age 65 is 2.8% if the retirement wealth has been accumulated with a full risky investment in stocks. This is because of the investment risk we have seen in [Figure 1] and [Figure 2] where the minimum possible wealth is far less than the expected value at retirement. Hence, under the pessimistic market condition during the investment horizon, the wealth level at retirement might not be sufficient to support the consumption stream even for five years after retirement. In the long run, however, aggressive investment during the accumulation phase provides better consumption opportunities on average in terms of ruin probability. At age 80, for example, the ruin probability for $f = 75\%$ increases up to 68%, indicating that in 68,000 cases out of 100,000 scenarios the remaining wealth reaches zero and no more wealth is left to be consumed if the investor retained 75% of her financial wealth to risky assets during the working period. In contrast, the ruin probabilities are 100% for the case with $f = 25\%$ and $f = 50\%$. Therefore, we conclude that both the level of accumulated wealth and the uncertainty in the amount of wealth are important aspects to be considered when determining the optimal investment strategy.

## Conclusion

In this study, we investigate the optimal investment strategy of an individual during the accumulation phase. To evaluate the optimal strategies from different perspectives, we employ two measures: certainty equivalent and ruin probability. First, in our welfare analysis, we find that the optimal investment strategy is a function of risk tolerance of the individuals. Specifically, our analysis suggests that a risk-loving individual should optimally invest all the assets to stocks while 25% risky investment is optimal for a very risk-averse individual. This indicates that the financial advisors in practice should correctly identify the risk attitudes of their clients before managing their portfolios, as a significant welfare loss can occur otherwise. Moreover, our results also suggest the importance of financial education. Many people are not investing in stocks for even once in their lifetime while our analysis shows that the minimum stock investment should be 25% even for the most risk-averse individual. Given the historical risk and return profile of riskless bonds and risky stocks, it is well-known that a long-term investment in stocks always outcompetes bonds and therefore a certain degree of stock investment is essential in our life. Therefore, our result implies that the government can enhance the welfare of the society by inducing people to invest some portion of their wealth into stocks during their working period.

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