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ECONOMIC AND MATHEMATICAL MODELING OF BANK LOAN PORTFOLIO CORSE WORK

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Introduction

The major part of banks' assets and loan interest income makes a significant share of banks' income. Inappropriate loan portfolio evaluation might have negative impact on a commercial bank's performance, the overall banking system, and the economic growth of the country. It is not enough for a bank to have a precise strategy, high lending culture, and observance of general principles to ensure the further growth of profitable loans. It is necessary to apply various evaluation methods of historical and present data, of ratios and factors enabling to implement coherent and comprehensive loan portfolio evaluation, and to encompass different factors as far as possible. Due to a complex business environment and intense competition between banks, it is not enough to evaluate a commercial bank loan portfolio only through the aspect of credit risk, i.e. loss probability level aspect, as is suggested by the scientists. As to every business subject striving for a successful performance and further development, it is essential for a bank to earn profit by financing the other subjects, and to establish the level of assets liquidity [15].

1. Lending and Loan: basic concepts

Landing detention

Lending (also known as "financing") in its most general sense is the temporary giving of money or property to another person with the expectation that it will be repaid. In a business and financial context, lending includes many different types of commercial loans [18].

Importance of loans [16]

The loans have a special importance in terms of being the most important component in the bank's assets and the most fertile because it generates attractive returns to the bank if managed efficiently and effectively. It includes various forms such as general capital loans, overdrafts, investment lending, asset backed loans and credit card accounts. Etc. As the technological developments and economic loans have given great importance can be summarized as follows.

1 - Increase production: Large industrial and agricultural projects, new ones, which require continuous financial resources, exceed the resources of the projects, as well as the technical and technical developments and the required replacement of machines and equipment. Therefore, these projects borrow from banks or issue bonds and put them on the public. Specialized banks play a greater role in providing such financial resources.

2 - Distribution of financial and credit resources on various economic activities: Lending plays an important role in the distribution of financial resources available to the banking system between different sectors and economic activities to ensure the efficient use of these resources through the distribution of all projects according to their needs.

3 - Increased consumption: Borrowing contributes to consumers of low-income earners acquiring some consumer durables and other commodities, thus having obligations to pay the value of that credit when their future income increases and also helps stimulate demand side of consumer goods and services leading to increased market share increasing production volume and supporting the national economy.

4 - Operating idle resources: The utilization of idle money can be exploited temporarily through short-term transfers. The borrower benefits from using these resources in temporary activities that earn him a profitable income. In return, the lender will receive the proceeds of these resources on an appropriate income.

5 - A tool for exchange: Bank lending is a suitable way to transfer the use of money from one person to another, that is, the medium of exchange, through lending can convert the savings of individuals and facilities and government to those who need or can invest in production and distribution.

Main types of loan in Banks:

1 - Personal Loans

Most banks provide some form of personal loans which consumers may use towards an expense such as buying a new TV or paying off a bill. Personal loans are typically unsecured and low in value. Lenders often require a form of identification from the consumer and proof of assets equal to or greater than the loan requested. The approval process only takes a few days, but interest rates are generally high.

Examples [7]:

Loan for 2 year with high interest like 16%	Unlimited Money
Loan for 1 year with high interest like 14%	Unlimited Money

Loan for 3 year with high interest like 16%	Up to 10,000 BYR
Loan for 10 year with high interest like 16%	Unlimited Money
Loan for 1 year with high interest like 12.5%	Up to 2500 BYR

2 - Credit Cards

Credit cards are one of the most widely accepted forms of payment, while essentially being a loan. The application process is usually quick and credit lines can vary from a few thousand to endless limits. One major advantage of credit cards is that funds are guaranteed and protected from the lender, this adds a level of protection for consumers. A major downside of credit cards is that interest rates can be high, and continual unpaid balances can rack up major interest fees.

3 - Home Equity Loans

Most home buyers are aware of home equity loans, most commonly known as mortgages. These large, long-term loans are usually secured and require some form of collateral often the property itself. Interest rates can be as low as a single percent, but as principal costs are high even low-interest rates can become huge fines when unpaid.

A similar form of home equity loan is a line of credit. The same general rules apply to lines of credit; however, they allow consumers to borrow more funds after a percentage of the original loan has been repaid.

4 - Small Business Loans

Small business loans are often provided by either the Small Business Administration (SBA) or a local bank. Loan terms vary in both value and length, and interest rates are often flexible. As business loans can range from thousands to millions of dollars, collateral is required. The approval process is strict and obtaining a loan can be difficult.

A loan can often make a life changing difference in a person's life. With dozens of options to fit your exact needs, taking out a loan should not be ignored. Just be sure to understand the conditions and choose what's right for you [17].

Classification of Bank loans on various grounds

The sign of classification	type of loan
By the nature of the borrower	- loans to individuals; - loans to legal entities .....
In the areas of use-consumer loans:	- trade credit; - investment loan; - industrial loans; - agricultural credit; - mortgage loans, etc.
By the terms provided	- short-term loans (provided for a period of up to one year); - medium-term loans (provided for a period of one to three years); - long-term loans (term of more than three years)
By size	- large loans (more than 5% of the Bank's capital); - average loans (the amount is determined by the Bank individually); - small loans (the amount is determined by the Bank individually)
Collateral	-secured loans; - unsecured loans (loans to insiders)
According to the method of providing funds	- loans of a compensatory nature (the loan is used for the implementation of the specified goals, for example-replenishment of funds); -payment credits (the credit received is the payment of the settlement documents, e.g. a credit line, an overdraft loan)
According to the method of interest payment-accrued interest is paid at the beginning of the loan term;	- accrued interest is paid at the end of the loan term; - accrued interest shall be paid in equal installments
By the method of loan repayment	- loans repaid at a time;

	- loans repaid in installments
By the date of return	-the loan repayment period is precisely defined, the date of return is fixed in the loan agreement; - the term of repayment is not defined in the loan agreement, but the term within which the borrower undertakes to repay the loan from the moment of its demand by the Bank is determined.

1.1. The definition of loan portfolio

Loans that have been made or bought and are being held for repayment. Loan portfolios are the major asset of banks, thrifts, and other lending institutions, the value of a loan portfolio depends not only on the interest rates earned on the loans, but also on the quality or likelihood that interest and principal will be paid [5] and the simple definition of loan portfolio total of all loans held by a bank or finance company on any given day [13].

1.2. The structure of loan portfolio in Hoiupank 1996

The definition of loan structuring In very simple terms it's the way in which the loan product is set up to accommodate the borrower's best interest, either in short, medium or longer terms depending on the client's end goal, because we have to remember the goal of the loan is to secure a property, so a loan/debt product is really a short term means to a bigger end goal dream [8].

Portfolio structure –the percentage of groups of assets in the portfolio.

Structure of loan portfolio [14]

	31.12.1996	31.12.1995
Loans to individuals	34%	25%
Loans to corporate clients	61%	69%
Loans to municipalities	5%	6%
Total	100%	100%

In view of the rapid increase in the loan portfolio the Bank has firmly secured its loan portfolio by hedging future credit risks, following the principles of conservatism in provisioning loans. At the same time the risk exposure of the Bank's loan portfolio has been continuously diminishing which is another proof of the Bank's adherence to the so-called reasonable risk policy.

Quality of loan portfolio of loan portfolio [14]

	31.12.1996	31.12.1995
Estimated losses on loans and guarantees, 000 EEK	60,652	24,277
Percentage of doubtful loans (provisioning rate) in the loan portfolio	2.76	2.55
Percentage of overdue loans in the loan portfolio	1.61	1.38

An active co-operation with various financial institutions was performed in 1996 in order to find new crediting possibilities for clients. Also, new products have been launched in the domain of housing loans and financing factoring to corporate clients. In 1996, Hoiupank concluded co-operation agreements with Maaelu ja Põllumajanduse Krediteerimise Fond (Country-Life and Agriculture Crediting Fund), Väikeettevõtluse Krediteerimise Fond (Small Enterprises Crediting Fund) and Keskkonnafond (Environment Fund).

In 1997, Hoiupank intends to make a maximum use of the advantages of its organizational structure and IT level, enabling to provide more flexible services,

oriented to different client groups, and to offer the most universal services package in Estonia.

Loan portfolio by economic sectors [14]

	31.12.1996	31.12.1995
Industry	7%	10%
Trade	11%	16%
Power engineering	5%	7%
Services	6%	6%
Food industry	4%	6%
Hotels and tourism	3%	4%
Municipal sector	5%	6%
Financial institutions	8%	7%
Individuals	29%	18%
Student loans	6%	10%
Real estate	8%	0%
Others	8%	10%
Total	100%	100%

1.3. The profitable loan

Profitability is the degree to which an activity yields profit or financial gain. While this concept is simple to understand, in reviewing a bank's financial statements where profitability can be easily measured for past performance, bankers often don't measure the profitability of a loan at inception and certainly not with the same level of certainty. This is especially true when the loan profitability is measured ex-ante – meaning taking into account the forecasted versus actual results of a specific loan. We have written extensively on loan profitability and various ways to measure loan profits. In this post, we wanted to discuss the concept of lifetime value of net profit as an important tool for bankers to measure loan performance [19].

1.3.1. Customer Lifetime Value (CLTV)

"In marketing, Customer Lifetime Value (CLV or often CLTV), lifetime customer value (LCV), or Lifetime value (LTV) is a prediction of the net profit attributed to the entire future relationship with a customer."

But in a nutshell, Customer Lifetime Value a.k.a. LTV means the revenue generated by a customer in his entire lifetime.

This term became extremely popular from 1990 as it gives you a theoretical figure of how much a customer will generate revenue to the company. Based on prediction, marketing team can derive the rough amount to spend on customer acquisition process.

In a simple way, if the COCA (Cost of Customer Acquisition) is 120$, and the lifetime value of the customer is 150$, that's a profitable customer! Apart from that, here are some uses of LTV:

Designing customer acquisition strategies.
Determine levels of investments in marketing strategies.
Designing customer satisfaction strategies.
Deciding customer retention strategies for individual customers.
Measuring customer loyalty.
Improving Net Promoter Score [2].

1.3.2. How to calculate Customer Lifetime Value (LTV)

There are multiple methods of calculating LTV, but we are going to discuss these two methods which are widely popular:

1. Historic CLV
2. Predictive CLV

Historic CLV:

CLVH = (T1+T2+T3+T4……..TN) * AGM

Where,

T1 = Transaction 1
T2 = Transaction 2…
AGM = Average Gross Margin

Average Gross Margin =(Revenue – Cost of Goods Sold (COGS))/Revenue

For example if we have number of periods 1,2,3,4,5 and some transactions 2000,5000,6000,1000,8000 and AMG 5.5 so the CLVH will be :

T1 = 2000, T2 = 5000, T3 = 6000, T4 = 1000, T5 = 8000

AMG = 5.5

CLVH = (2000+5000+6000+1000+8000)*5.5

CLVH = 121000

This formula is the simple calculation of Customer Lifetime Value where previous transactions from the customer are considered. This formula will give you the actual numbers of a customer contributing in your business.

For a small business, this can be extremely useful as there are limited customers to fulfill the product or service.

Predictive CLV:

Predictive CLV is more accurate compared to the Historic CLV. Here's the formula to calculate:

CLV1 = (T* AOV) AGM) ALT

Where,

T = Average monthly transactions
AOV = Average order value
AGM: Average gross margin
ALT: Average customer lifespan

This formula will give you predictive customer value for a decided life span. But, in actual scenarios, there are multiple discounts provided to the customers. Hence, here's more realistic formula to calculate Customer Lifetime Value.

We will use the above formula in this equation:

CLV = CLV1R/((1 + D – R))

Where,

R = Monthly retention rate
D = Monthly discount rate

For example if we have CLV1 = 5000 and R = 16% and D = 50% so the CLV will be

CLV = 5000*0.16 / (1 + 0.5 – 0.16)

CLV = 800/1.34

CLV = 597.01

1.3.3. Customer Lifetime Value is importunes

1 - Gives you insights of every marketing channel:

To acquire customers, you use various marketing and advertising channels. Finding out which channel gives you the best customers is extremely important, that's the reason in some feedback forms there are questions like "How did you find us?"

By finding out the best channels of marketing, you can focus more on such channels to acquire more customers. On the other hand, you can spend lesser money on the channels which are not profitable to your organizations.

Sorting out best customers by their marketing channels will also give you insights about how much to spend on such marketing channels.

2 - Retain your customers:

In the process of finding Customer Lifetime Value, you will find out the repetition of the customer to your brand.

How many times in a month the customer is coming back to us?
What amount of purchase does he do?
What kind of purchase does he making?

Such questions will be answered in the process of finding CLV. You can make individual strategies for such customers to retain them to your brand. With a personalized discount, a great customer relationship can be built.

3 - Improve your Net Promoter Score:

When you find the Lifetime Value, you can find three kinds of customers:

Promoters, Passives, Detractors

Promoters are the customers who are dedicated and loyal to your brand. They will stick with you brand for a long period of time, and you will realize it when you'll find their CLV. They will promote you brand, and spread good words.

Passives are the customers who will choose your brand for a certain period of time. But, they can be easily distracted! They may not spread a bad word of mouth, but they will choose a different product if there's a better opportunity.

Detractors are the customers who have had a bad experience with your product or service. They will speak ill about your brand, and may hurt your business. These are the customers whom you should worry about the most [2]

1.4. Risk of loan

For most banks, loans are the largest and most obvious source of credit risk. However, there are other pockets of credit risk both on and off the balance sheet, such as the investment portfolio, overdrafts, and letters of credit. Many products, activities, and services, such as derivatives, foreign exchange, and cash management services, also expose a bank to credit risk. The risk of repayment, i.e., the possibility that an obligor will fail to perform as agreed, is either lessened or increased by a bank's credit risk management practices.

A bank's first defense against excessive credit risk is the initial credit-granting process − sound underwriting standards, an efficient, balanced approval process, and a competent lending staff. Because a bank cannot easily overcome borrowers with questionable capacity or character, these factors exert a strong influence on credit quality. Borrowers whose financial performance is poor or marginal, or whose repayment ability is dependent upon unproven projections can quickly become impaired by personal or external economic stress. Management of credit risk, however, must continue after a loan has been made, for sound initial credit decisions can be undermined by improper loan structuring or inadequate monitoring.

Traditionally, banks have focused on oversight of individual loans in managing their overall credit risk. While this focus is important, banks should also view credit risk management in terms of portfolio segments and the entire portfolio.

The focus on managing individual credit risk did not avert the credit crises of the 1980s. However, had the portfolio approach to risk management augmented these traditional risk management practices, banks might have at least reduced their losses. Effective management of the loan portfolio's credit risk requires that the board and management understand and control the bank's risk profile and its credit culture.

To accomplish this, they must have a thorough knowledge of the portfolio's composition and its inherent risks. They must understand the portfolio's product mix, industry and geographic concentrations, average risk ratings, and other aggregate characteristics. They must be sure that the policies, processes, and practices implemented to control the risks of individual loans and portfolio segments are sound and that lending personnel adhere to them.

Banks engaged in international lending face country risks that domestic lenders do not. Country risk encompasses all of the uncertainties arising from a nation's economic, social, and political conditions that may affect the payment of foreigners' debt and equity investments. Country risk includes the possibility of political and social upheaval, nationalization and expropriation of assets, governmental repudiation of external indebtedness, exchange controls, and currency devaluation or depreciation. Unless a nation repudiates its external debt,

these developments might not make a loan uncollectible. However, even a delay in collection could weaken the lending bank.

Transfer risk, which is a narrower form of country risk, is the possibility that an obligor will not be able to pay because the currency of payment is unavailable. This unavailability may be a matter of government policy. For example, although an individual borrower may be very successful and have sufficient local currency cash flow to pay its foreign (e.g., U.S. dollar) debt, the borrower's country may not have sufficient U.S. dollars available to permit repayment of the foreign indebtedness. The transfer risk associated with banks' exposures in foreign countries is evaluated by the Interagency Country Exposure Review Committee (ICERC).

For examination purposes, the transfer risk rating assigned to a country by the ICERC applies to all bank assets in that country. However, examiners may classify individual loans and other assets more severely for credit risk reasons [6].

1.5. Measures of Risk - Variance and Standard Deviation

Risk reflects the chance that the actual return on an investment may be very different than the expected return. One way to measure risk is to calculate the variance and standard deviation of the distribution of returns.

Consider the probability distribution for the returns on stocks **A** and **B** provided below.

State	Probability	Return on Stock A	Return on Stock B
1	20%	5%	50%
2	30%	10%	30%
3	30%	15%	10%
3	20%	20%	-10%

The expected returns on stocks A and B were calculated on the Expected Return page. The expected return on Stock A was found to be 12.5% and the expected return on Stock B was found to be 20%.

Given an asset's expected return, its variance can be calculated using the following equation:

$$\mathrm{Var}(R) = \sigma^2 = \sum_{i=1}^{N} p_i (R_i - E[R])^2$$

where

- N = the number of states,
- p_i = the probability of state i,
- R_i = the return on the stock in state i, and
- E[R] = the expected return on the stock.

The standard deviation is calculated as the positive square root of the variance.

$$\mathrm{SD}(R) = \sigma = \sqrt{\sigma^2} = \left(\sigma^2\right)^{\frac{1}{2}}$$

Variance and Standard Deviation on Stocks A and B

Note: $E[R_A] = 12.5\%$ and $E[R_B] = 20\%$

Stock A

$$\sigma_A^2 = .20(.05 - .125)^2 + .30(.10 - .125)^2 + .30(.15 - .125)^2 + .20(.20 - .125)^2 = .00263$$

$$\sigma_A = \sqrt{.00263} = .0512 = 5.12\%$$

Stock B

$$\sigma_B^2 = .20(.50 - .20)^2 + .30(.30 - .20)^2 + .30(.10 - .20)^2 + .20(-.10 - .20)^2 = .04200$$

$$\sigma_B = \sqrt{.04200} = .2049 = 20.49\%$$

Although Stock B offers a higher expected return than Stock A, it also is riskier since its variance and standard deviation are greater than Stock A's. This, however, is only part of the picture because most investors choose to hold securities as part of a diversified portfolio.

Example for solving the risk problem using Wolfram:

The investor can create a portfolio of 5 types of assets, the historical data on the profitability of which for 10 periods are given in the table

№ period	shares 1	shares 2	shares 3	shares 4	shares 5
1	8,1	1,9	2,3	10,3	18
2	3	7	8	8	10
3	5,3	4,7	3,7	18,6	19,1
4	1	9	10	15	15
5	-3,1	9,9	8,9	4,8	2,7
6	3	7	8	-0,9	1,1
7	5	5	4,5	14,6	10,3
8	3,2	6,8	7,8	12,8	15
9	1,2	8,8	7,8	9,3	11,5
10	1,3	8,7	9,7	12,1	12,4

[source: Author`s construction]

To solve the Markowitz problem with the required portfolio efficiency equal to the arithmetic mean of the mathematical expectations of the returns of all shares.

Use P as the value of 10 periods for 5 types of assets P={{8.1,3,5.3,1,-3.1 ,3 ,5 ,3.2 ,1.2 ,1.3 }
,{1.9,7,4.7,9,9.9,7,5,6.8,8.8,8.7},{2.3,8,3.7,10,8.9,8,4.5,7.8,7.8,9.7},{10.3,8,18.6,15,4.8,-0.9,14.6,12.8,9.3,12.1}, {18,10,19.1,15,2.7,1.1,10.3,15,11.5,12,4}}

Finding the main of P by using

Mean [Transpose [P]]

The answer is {5.46667, 4.53333, 4.66667, 12.3, 15.7}

Now find the standard deviation

SD1=StandardDeviation[P[[1]]] the answer 2.55408

SD2=StandardDeviation[P[[2]]] the answer 2.55408

SD3=StandardDeviation[P[[3]]] the answer 2.97041

SD4=StandardDeviation[P[[4]]] the answer 5.57584

SD5=StandardDeviation[P[[5]]] the answer 4.96689

Find the variance

Var1=Variance[P[[1]]] the answer 6.52333

Var2=Variance[P[[2]]] the answer 6.52333

Var3=Variance[P[[3]]] the answer 8.82333

Var4=Variance[P[[4]]] the answer 31.09

Var5=Variance[P[[5]]] the answer 24.67

Now Find the square of SD

SD1^2 the answer 6.52333

SD2^2 the answer 6.52333

SD3^2 the answer 8.82333

SD4^4 the answer 966.588

SD5^5 the answer 3022.89

S=StandardDeviation[Transpose[P]]

The answer {2.55408,2.55408,2.97041,5.57584,4.96689}

Now we put values to A and B

A=X^2 , B=S^2

Finally we minimalize the risk

MinimizeMinimize[{A.B,X1+X2+X3+X4+X5\[Equal]1,5.46667` X1+4.53333` X2+4.66667` X3+12.3` X4+15.7` X5\[Equal]8.533333333333335`},{X1,X2,X3,X4,X5}]

And

MinimizeMinimize[{SP[X1,X2,X3,X4, X5],X1+X2+X3+X4+X5\[Equal]1,5.46667` X1+4.53333` X2+4.66667` X3+12.3` X4+15.7` X5\[Equal]8.533333333333335`},{X1,X2,X3,X4,X5}

Table of classification of credit risk in Intel soft wear [1]

	duration	credit_amount	installment_commitment	residence_since	age	existing_credits	num_dependents
count	1000.000000	1000.000000	1000.000000	1000.000000	1000.000000	1000.000000	1000.000000
mean	20.903000	3271.258000	2.973000	2.845000	35.546000	1.407000	1.155000
std	12.058814	2822.736876	1.118715	1.103718	11.375469	0.577654	0.362086
min	4.000000	250.000000	1.000000	1.000000	19.000000	1.000000	1.000000
25%	12.000000	1365.500000	2.000000	2.000000	27.000000	1.000000	1.000000
50%	18.000000	2319.500000	3.000000	3.000000	33.000000	1.000000	1.000000
75%	24.000000	3972.250000	4.000000	4.000000	42.000000	2.000000	1.000000
max	72.000000	18424.000000	4.000000	4.000000	75.000000	4.000000	2.000000

2. Mathematical modeling of bank loan portfolio

2.1. Markowitz Theory

Harry M. Markowitz is credited with introducing new concepts of risk mea-surement and their application to the selection of portfolios. He started with the idea of risk aversion of average investors and their desire to maximize the expected return with the least risk [9].

Markowitz model is thus a theoretical framework for analysis of risk and return and their inter-relationships. He used the statistical analysis for measurement of risk and mathematical programming for selection of assets in a portfolio in an efficient manner. His framework led to the concept of efficient portfolios. An efficient portfolio is expected to yield the highest return for a given level of risk or lowest risk for a given level of return [9].

2.1.1. Assumptions of Markowitz Theory

The Portfolio Theory of Markowitz is based on the following assumptions:

1- Investors are rational and behave in a manner as to maximize their utility with a given level of income or money.

2- Investors have free access to fair and correct information on the returns and risk.

3- The markets are efficient and absorb the information quickly and perfectly.

4- Investors are risk averse and try to minimize the risk and maximize return.

5- Investors base decisions on expected returns and variance or standard deviation of these returns from the mean.

6- Investors choose higher returns to lower returns for a given level of risk [9].

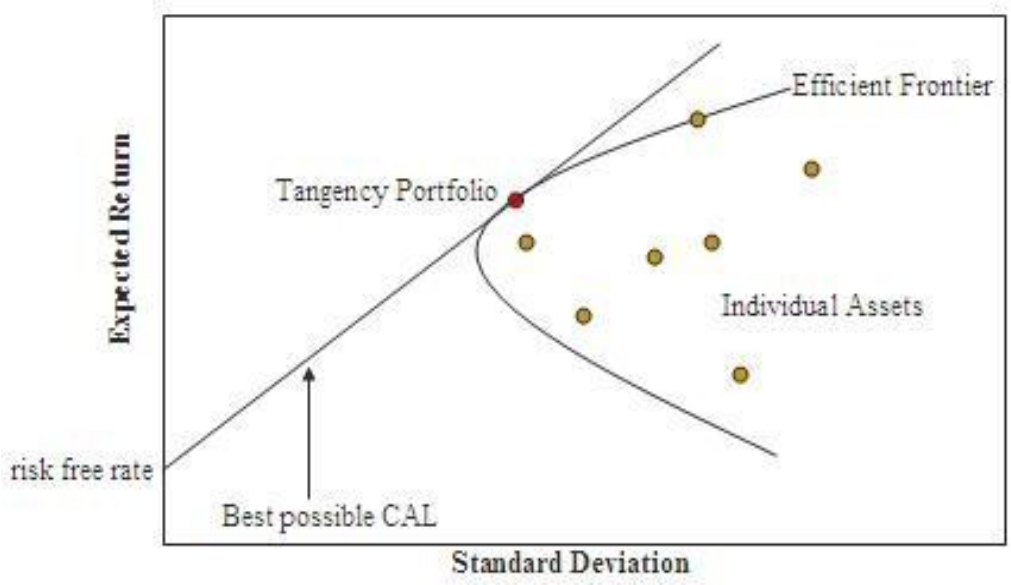
2.1.2. Mathematical model

Risk and expected return.

MPT assumes that investors are risk adverse, meaning that given two portfolios that offer the same expected return, investors will prefer the less risky one. Thus, an investor will take on increased risk only if compensated by higher expected returns. Conversely, an investor who wants higher expected returns must accept more risk. The exact trade-off will be the same for all investors, but different investors will evaluate the trade-off differently based on individual risk aversion characteristics. The implication is that a rational investor will not invest in a portfolio if a second portfolio exists with a more favorable risk-expected return profile – i.e., if for that level of risk an alternative portfolio exists that has better expected returns [10].

2.1.3. Markowitz Efficient Frontier

The concept of Efficient Frontier was also introduced by Markowitz and is easier to understand than it sounds. It is a graphical representation of all the possible mixtures of risky assets for an optimal level of Return given any level of Risk, as measured by standard deviation.



Graph for Efficient Frontier [11]

The chart above shows a hyperbola showing all the outcomes for various portfolio combinations of risky assets, where Standard Deviation is plotted on the X-axis and Return is plotted on the Y-axis.

The Straight Line (Capital Allocation Line) represents a portfolio of all risky assets and the risk-free asset, which is usually a triple-A rated government bond.

Tangency Portfolio is the point where the portfolio of only risky assets meets the combination of risky and risk-free assets. This portfolio maximizes return for the given level of risk.

Portfolio along the lower part of the hyperbole will have lower return and eventually higher risk. Portfolios to the right will have higher returns but also higher risk.

Markowitz Portfolio Theory (Modern Portfolio Theory or Passive Investment Approach) is the base idea of the Ways2Wealth concept.

Read more in the other articles to understand the Ways2Wealth Investment Approach [11].

2.1.4. Markowitz Portfolio Theory using WOLFRAM

Problem statement.

The investor can make a portfolio of three types of assets A,B, C, the return R_A, R_B, R_C of which are uncorrelated random variables and have the following parameters:

- expected rate of return: $m_A = 8,\ m_B = 10,\ m_C = 12$,
- $\sigma_A = 1,\ \sigma_B = 2,\ \sigma_C = 4$.

The investor chooses assets with the lowest variability of returns (minimal risk).

To determine the optimal portfolio with minimal risk, when expected rate of return of portfolio is $m_p = 10$.

Instructions for task.

1. To create a mathematical model of finding the optimal structure of the portfolio, minimizing the risk at the desired efficiency, we define the parameters of the problem:

Number of different assets from which the portfolio is formed – $n = 3$

The profit of assets $m_A = 8,\ m_B = 10,\ m_C = 12$

Variance (square of risk) of assets $\sigma_A^2 = 1,\ \sigma_B^2 = 4,\ \sigma_C^2 = 16$

2. We introduce variables to denote the required shares of assets x_A, x_B, x_C:

3. Let's write down the objective function (portfolio variance taking into account the independence of assets returns) and the purpose of its optimization

$$\sigma_p^2 = \sigma_1^2 x_1^2 + \sigma_2^2 x_2^2 + \sigma_3^2 x_3^2 = x_1^2 + 4x_2^2 + 16x_3^2 \to \min$$

4. Write down the constraints of the task:

Limitation on portfolio profitability

$$m_A x_A + m_B x_B + m_C x_C = 8x_A + 10x_B + 12x_C = m_p = 10$$

Restrictions on the structure of the portfolio

$$x_A + x_B + x_C = 1,$$
$$x_A, x_B, x_C \geq 0.$$

Thus, the mathematical model of the problem has the form:

$$x_1^2 + 4x_2^2 + 16x_3^2 \to \min$$
$$8x_A + 10x_B + 12x_C = 10,$$
$$x_A + x_B + x_C = 1,$$
$$x_A, x_B, x_C \geq 0.$$

Example 1.

$$m_A = 0.4,\ m_B = 0.5,\ m_C = 0.7,$$
$$\sigma^2{}_A = 0.01,\ \sigma^2{}_B = 0.02,\ \sigma^2{}_C = 0.04.$$
$$m_p = 0.4705.$$

For the expected rate of return of a portfolio mp=0.4705 find the structure of optimal portfolio with a minimum

So we will use an equation to minimalize the risk in our example portfolio with a desired portfolio profit mp=0.4705 by using Wolfram:

Minimize[{0.01 x1^2+0.02 x2^2+0.04 x3^2,x1+x2+x3==1,0.4 x1+0.5 x2+0.7 x3==0.4705,x1>=0,x2>=0,x3>=0},{x1,x2,x3}]

The optimal structure of the portfolio with the expected rate of return mp=0.4705 is

X1=0.575, X2=0.285, X3=0.14

The minimum risk of the portfolio with the profit 0.405 is equal 0.0057

As we see the risk is minimalized and that is very good for our portfolio.

Now let's use the desired profit of 0.4, 0.5 & 0.7 and find the appropriate optimal portfolios with the expected rate of return mp=0.4 with minimal risk.

Minimize[{0.01 x1^2+0.02 x2^2+0.04 x3^2,x1+x2+x3==1,0.4 x1+0.5 x2+0.7 x3==0.4,x1>=0,x2>=0,x3>=0},{x1,x2,x3}]

The results of the risk in the portfolio on the profit 0.4 and the page

X1→ 0.999, X2→0, X3→0

Minimize[{0.01 x1^2+0.02 x2^2+0.04 x3^2,x1+x2+x3==1,0.4 x1+0.5 x2+0.7 x3==0.5,x1>=0,x2>=0,x3>=0},{x1,x2,x3}]

The results of the risk in the portfolio on the profit 0.5 and the values X1→0.461538, X2→0.307692, X3→0.230769

We see it's minimized to but it's not equal any of the variables.

Minimize[{0.01 x1^2+0.02 x2^2+0.04 x3^2,x1+x2+x3==1,0.4 x1+0.5 x2+0.7 x3==0.7,x1>=0,x2>=0,x3>=0},{x1,x2,x3}]

The results of the risk in the portfolio on the profit 0.7 and the values

X1→0, X2→0, X3→1.0000000000000004

Expected Portfolio Value	Optimized Portfolio structure	Minimum Value of risk
0.4	X1=0.999 X2=0 X3=0	0.01
0.5	X1=0.461538 X2=0.307692 X3=0.230769	0.006
0.7	X1=0 X2=0 X3=1.0000000000000004	0.04

The Optimized Portfolio for Deference Value of the Expected Value [source: Author`s construction]

Now we will try some more variables like 0.45, 0.6 and we check the result

(Just for test):-

Minimize[{0.01 x1^2+0.02 x2^2+0.04 x3^2,x1+x2+x3==1,0.4 x1+0.5 x2+0.7 x3==0.45,x1>=0,x2>=0,x3>=0},{x1,x2,x3}]

The result of the risk = 0.0059

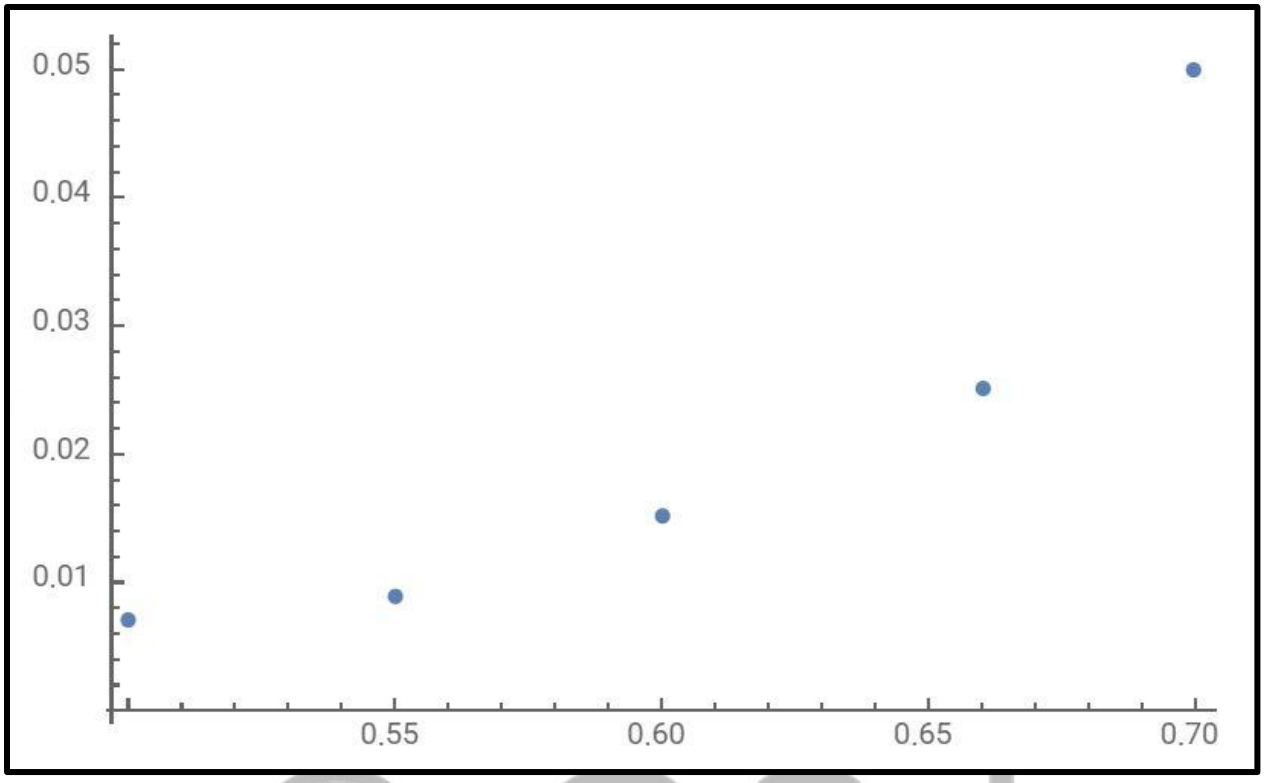
Minimize[{0.01 x1^2+0.02 x2^2+0.04 x3^2,x1+x2+x3==1,0.4 x1+0.5 x2+0.7 x3==0.6,x1>=0,x2>=0,x3>=0},{x1,x2,x3}]

The result of the risk = 0.015

From this we see that Markowitz theory is so great to minimalize the risk of the loan portfolio and with good loan profit

This is the command of the graph in wolfram:-

ListPlot[{{0.50,0.00696154},{0.55,0.00886154},{0.60,0.015154},{0.66,0.025},{0.7,0.05}}]



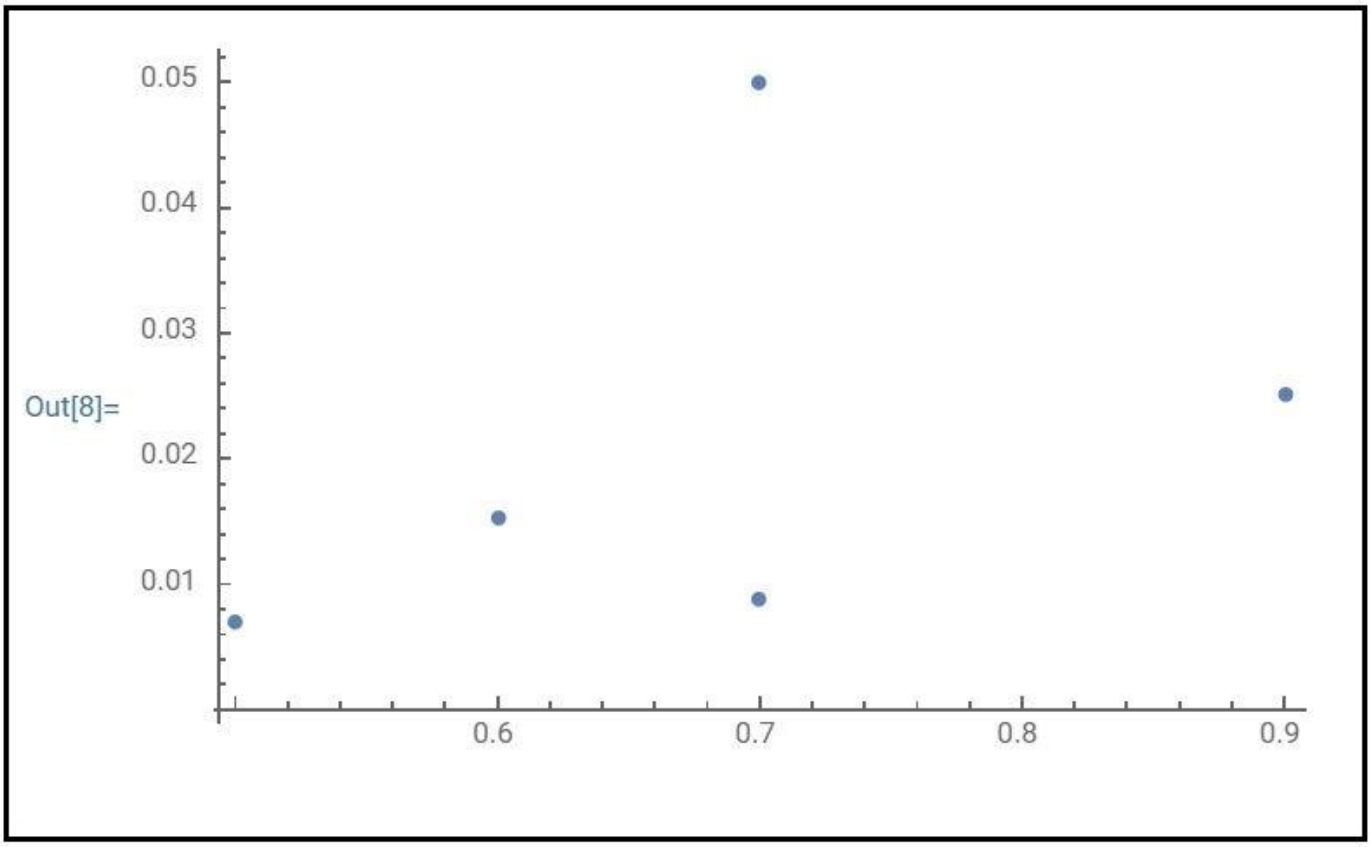
Graph for the above results [Source: Author`s construction]

We see that there is no dominant variable and it's great because if there is a dominant variable it will be not good for the investors of this portfolio because of the high risk or the low profit.

For the investors in any loan portfolio the low risk of the portfolio and the high profit is mean goal because if the risk is high the chance to get profit is to low and if the profit is low it will be west of time to invest in the portfolio, In deriving the CAPM, Sharpe, Lintner and Mossin assumed expected utility (EU) maximization in the face of risk aversion. Legendary article of Markowitz (1952) then gives rise to MPT. To avoid problems such as difficulty in input data, educating portfolio managers and time-cost consideration, using single index model and generating mean Variance structure have become famous (Elton, Gruber and Padberg,1976 and 2003).

Command for Other graph in Wolfram:

ListPlot[{{0.50,0.00696154},{0.70,0.00886154},{0.60,0.015154},{0.90,0.025},{0.7,0.05}}]



As we see we have dominant variables and that affects the portfolio mostly in bad way and it became not good for investments

[[Source: Author`s construction]

2.2. The Literature review about Sharpe's Theory:

Sharpe has received a Nobel Prize in 1990 for the model which empirical evidence is less than poor. Fama and French (2004) argue the reason could be many simplifying assumptions. To better understanding these assumptions we should break down the model and see its segmented portions.

Many academics have applied single index model on real world data and have tried to construct optimal portfolio. Debasish Dutt (1998) found that all the stocks selected are bank stocks. He used Sharpe single index model in order to optimize a portfolio of 31 companies from BSE (Bombay Stock Exchange) for the period October 1, 2001 to April 30, 2003 and used BSE 100 as market index.

Later on Asmita Chitnis (2010) optimized two portfolios using single index model, compared them, and he found out that portfolios tend to spread risk over many securities and thus help to reduce the overall risk involved. "The greater the portfolio's Sharpe's ratio, the better is its performance."

A bubble in stock price may occur due to behavioural finance responses of individuals Werner de Bondt found that behavioural finance has already proved to be a productive, pragmatic, and intuitive approach to asset pricing research. With its requirements for realism in assumptions, behavioural finance also brings discipline to market modeling.

According to Barley Rosser (200) a speculative bubble exists when the price of something does not equal its market fundamentals for some period of time for reasons other than random shock.

The latest situation of the extremely inflated asset prices during early 2010 and up to 2011 has been indicated as bubble (Rahman, 2010) because DSE (Dhaka Stock Exchange) had risen by 125 percent over the period from March 2009 and February 2010 [19].

Sharpe's single index model:

he single-index model (SIM) is a simple asset pricing model to measure both the risk and the return of a stock. The model has been developed by William Sharpe in 1963 and is commonly used in the finance industry. Mathematically the SIM is expressed as:

$$r_{it} - r_f = \alpha_i + \beta_i(r_{mt} - r_f) + \epsilon_{it}$$
$$\epsilon_{it} \sim N(0, \sigma_i)$$

where:

r_{it} is return to stock *i* in period *t*

r_f is the risk free rate (i.e. the interest rate on treasury bills)

r_{mt} is the return to the market portfolio in period *t*

α_i is the stock's alpha, or abnormal return

β_i is the stocks's beta, or responsiveness to the market return

Note that $r_{it} - r_f$ is called the excess return on the stock, $r_{mt} - r_f$ the excess return on the market

ϵ_{it} are the residual (random) returns, which are assumed independent normally distributed with mean zero and standard deviation σ_i

These equations show that the stock return is influenced by the market (beta), has a firm specific expected value (alpha) and firm-specific unexpected component (residual). Each stock's performance is in relation to the performance of a market index (such as the All Ordinaries). Security analysts often use the SIM for such functions as computing stock betas, evaluating stock selection skills, and conducting event studies [12].

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