GSJ: Volume 10, Issue 1, January 2022, Online: ISSN 2320-9186
www.globalscientificjournal.com

Enhancing the Dynamical Response of Elastic Plates Using Homogenization

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Plate refers to a structural element which is used in many applications in our life. Following its significance, it is important to study this application and to explore a modified case of plate with an aim to improve the application efficiency. This study intends to see the effect of Homogenization in the elastic plates in three different cases. In addition a modified plate at specific mode will be analyzed to make sure that the performance of such plates will be enhanced. Homogenization was found as a useful technique to minimize the transverse deflection of elastic plates. Findings showed the enhancement of the dynamic performance of elastic plates.
Keywords: ANSYS, Dynamical Response, Elastic Plates, Homogenization.

1. INTRODUCTION

A plate is a structural element characterized by two key properties. First includes its geometric configuration which is in the form of three-dimensional solid with minimum thickness when compared with other dimensions. Second includes the effects of the loads that are required to be applied on it, only to generate stresses whose resultants are; in practical terms, exclusively normal to the element's thickness [1-5]. The plates are widely used and can be found in many mechanical structures, such as; aircrafts and ships. In continuum mechanics, plate theories are mathematical descriptions of the mechanics of flat plates that draws on the theory of beams.

Shahriar [5] defined plates as plane structural elements with a small thickness compared to the planar dimensions. The typical thickness to width ratio of a plate structure is less than 0.1. A plate theory takes advantage of this disparity in length scale to reduce the full three-dimensional solid mechanics problem to a two-dimensional problem. The plate theory calculates the deformation and stresses in a plate subjected to loads (Fig. 1).

Among different plate theories that have been developed up till now, two plates are widely accepted and used in engineering, which includes:

- The Kirchhoff–Love theory of plates (classical plate theory) [6].
- The Mindlin–Reissner theory of plates (first-order shear plate theory) [7].

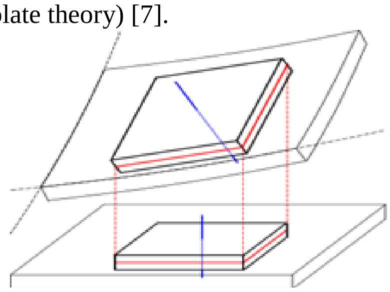

Fig.1: Deformation of a thin plate highlighting the displacement, the mid-surface (red) and the normal to the mid-surface (blue)

2. MATHEMATICAL FORMULATION

The Kirchhoff–Love theory is an extension of Euler–Bernoulli beam theory to thin plates. The theory was developed in 1888 by Love [2] using assumptions proposed by Kirchhoff. It was assumed that a mid-surface plane can be used to represent the three-dimensional plate in a two-dimensional form [6].

However, the kinematic assumptions made in this theory are as follows [3, 8-11]:

- Straight lines normal to the mid-surface remain straight after deformation.
- Straight lines normal to the mid-surface remain normal to the mid-surface after deformation.
- The thickness of the plate does not change during a deformation.

The Kirchhoff hypothesis implies that the displacement field has the given form:

$$u_\alpha(X) = u_\alpha^0(x_1, x_2) - x_3 \frac{\partial \omega^0}{\partial x_\alpha} = u_\alpha^0 - x_3 \omega_{,\alpha}^0 \; ; \; \alpha = 1, 2$$

$$u_3(X) = \omega^0(x_1, x_2)$$

Where x_1 and x_2 are the Cartesian coordinates on the mid-surface of the undeformed plate, x_3 is the coordinate for the thickness direction u_1^0, u_2^0 are the in-plane displacements of the mid-surface, and ω^0 is the displacement of the mid-surface in the x_3 direction.

If φ_α are the angles of rotation of the normal to the mid-surface, then in the Kirchhoff–Love theory $\varphi_\alpha = \omega_{,\alpha}^0$ (Fig. 2).

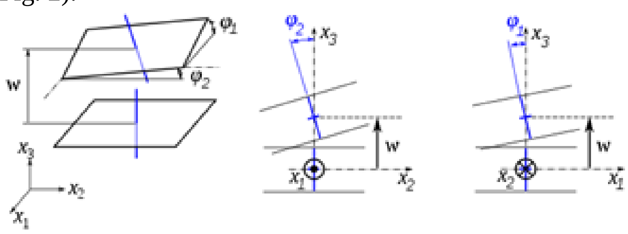

Fig. 2: Displacement of the mid-surface (left) and of a normal (right) [3]

3. MODELING AND ANALYSIS SETUP

Certain constraints were identified during the process. These constraints include (Fig. 3):

- Maximum thickness of the plate is 0.08 inches which cannot be exceeded further.
- Minimum thickness of layer of the plate is 0.02 inches.
- The volume of the new case should be reduced or retained the same as the plain case.
- Material is Aluminum alloy.
- Boundary condition: all clamped.
- Height 20 inches.
- Width 20 inches.

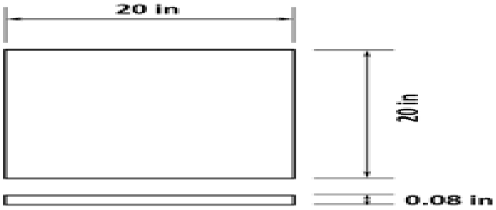

Fig. 3. Dimensions of the plate

Many assumptions were considerable, which include:

- The range of the frequency is 20, so the min freq. is the (frequency – 20) and the same way for the max (frequency + 20).
- The solutions interval is 80.
- The mesh size is same for new plate and original plate in all cases.
- Force = 1N in direction of the negative y-axis in the center of the plate.

3.1 Case 1: Mode Shape 1

3.1.1 Original Plate

A square plate with width (20 inches), height (20 inches), and (0.08 inches) thickness was created using ANSYS design modeler, as shown in Fig 4. Aluminum material was assigned for the plate. A point was created at the middle of the plate to exert the force on it later.

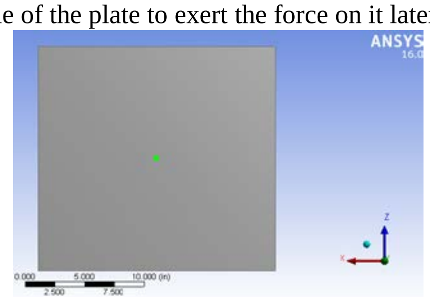

Fig. 4. The Original Plate

3.1.2 Modal

In order to find the fifth mode shape and its natural frequency for the original plate, ANSYS model was used. The mesh sizing was considered as 0.393 inches (10 mm), as indicated in Fig 5. The first natural frequency was found to be (70.265 HZ), its mode shape is shown in Fig 6.

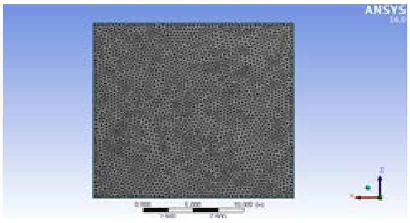

Fig. 5. The mesh for the original plate

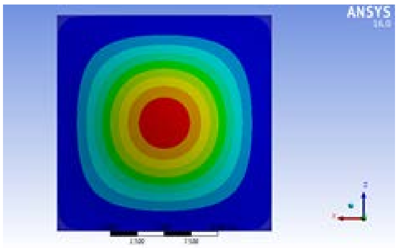

Fig. 6. The fifth mode shape for the original plate

3.1.3 Harmonic Model

In order to find the frequency response, ANSYS harmonic model was used. 1N excitation force was applied at the middle of the plate (Fig 7). In the analysis settings of the harmonic model, the minimum frequency range was 50.265 HZ, whereas the maximum frequency range was 90.265 HZ, and the solution intervals were 80. Consequently, the frequency response of exactly at the first natural frequency was 70.265 HZ. The amplitude at 70.265 Hz was found to be 176.55 in (Fig 8).

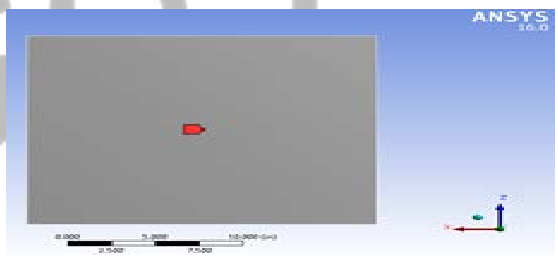

Fig. 7. Excitation force at the middle of the plate

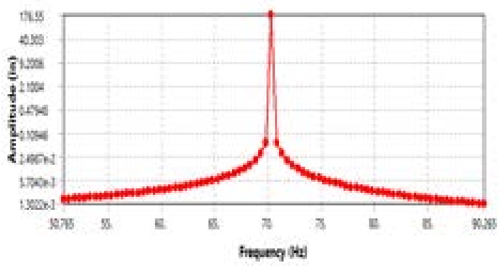

Fig. 8. Frequency response of the first mode shape for the original plate

3.1.4 Re-designed Plate

The re-designed plate consisted of three layers. The first layer was a square plate with width (20 inches), height (20 inches), and (0.02 inches) thickness. The second layer was a frame with (20 inches) width, (20 inches) height, offset entities (0.5 inches) to center of plate and thickness (0.06 inches). The second layout was above the first layout. While, the third layout was in complex shape with thickness (0.06in) and above the first layout. Fig 9 shows the re-designed plate, which was created by solidworks software and then imported to the ANSYS design modeler to add a point at the middle of the plate.

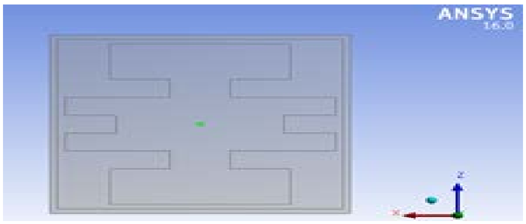

Fig. 9. The re-designed plate for the fifth mode shape

3.1.5 Modal Re-Design Plate

In order to find the first mode shape and its natural frequency for the re-designed plate, ANSYS model was used. The mesh sizing was 0.393 inches (10 mm) (Fig 10). The first natural frequency was found to be 57.841 HZ. Its mode shape is shown in Fig 11.

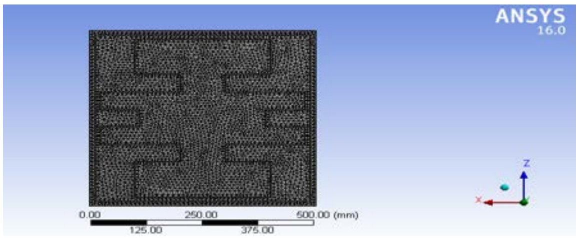

Fig. 10. The mesh for the re-designed plate

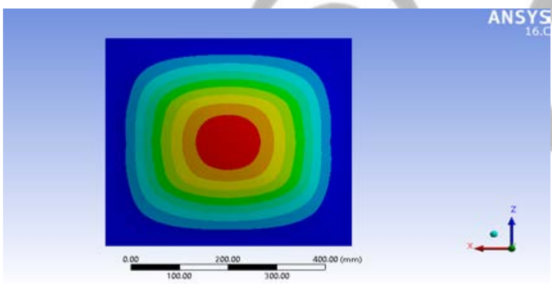

Fig. 11. The first mode shape for re-designed plate

3.1.6 Harmonic Model

Newton excitation force was applied at the middle of the plate (Fig 12). In the analysis settings of the harmonic model, the minimum frequency range was 37.841 Hz, whereas, the maximum frequency range was 77.841 Hz, and the solution intervals were 80. Consequently, the frequency response exactly at the first natural frequency was 57.841 HZ. The amplitude at 57.841 Hz was 77.61 in (Fig 13).

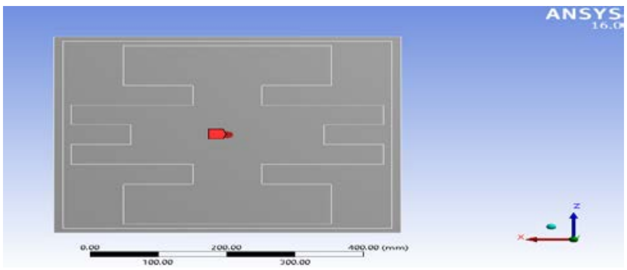

Fig. 12. Excitation force at the middle of the plate

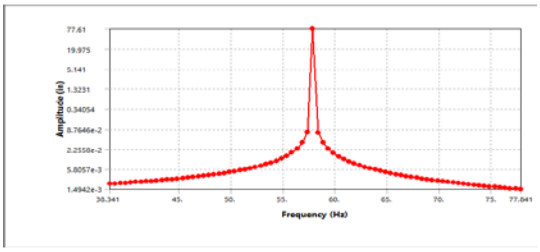

Fig. 13. Frequency response of the first mode shape for the re-design plate

3.2 Case 2: Mode Shape 4

The geometry of the plate was applied to the Modal. Afterward, the plate draws the geometry with the given dimensions and added the material "aluminum alloy" from engineering data. Fig 14 shows the geometry of the Plain Plate with the given dimensions.

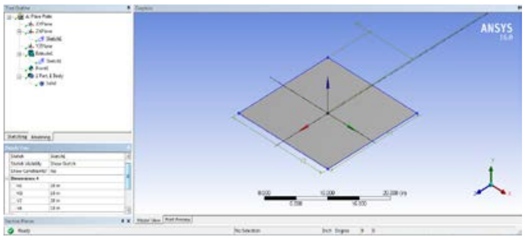

Fig. 14. Geometry of Plain Plate

Fig 15 clearly shows addition of the required material i.e. "aluminum alloy" from the engineering data and its properties.

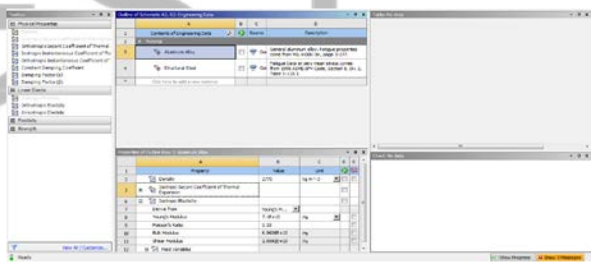

Fig. 15. Adding the material "aluminum alloy"

Afterward, the plate was added to the Model to assign the material with a mesh element = 0.39 in = 10 mm. Fig. 16 shows the mesh element in the plain plate.

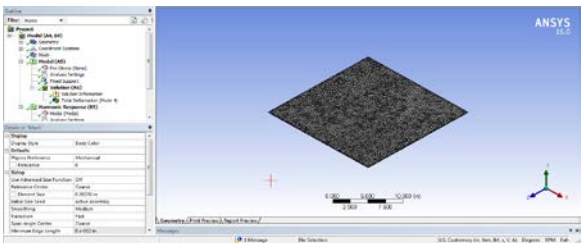

Fig. 16. The mesh on plain plate

Then, offering support for the four around faces (Fig 17-19).

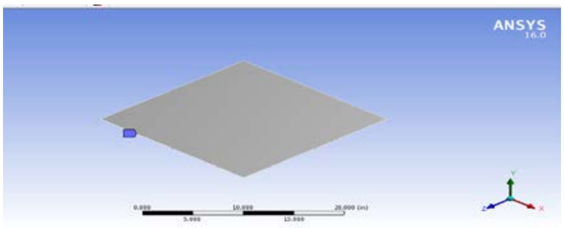

Fig. 17. Fixing support for the four around faces

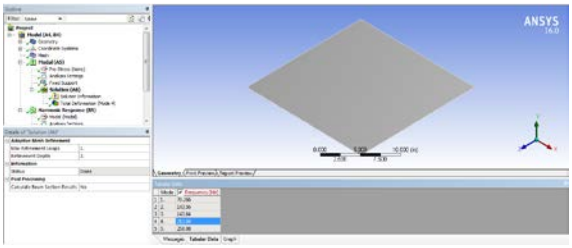

Fig. 18. The Frequency of mode shape 4

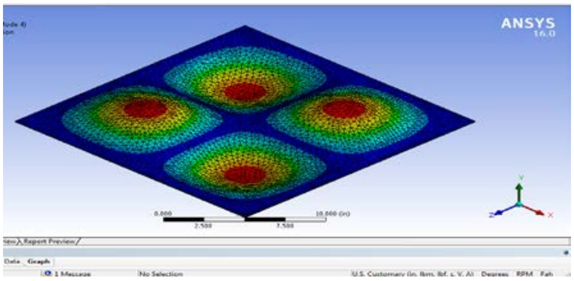

Fig. 19. Mode shape 4

3.2.1 Set up of Harmonic model (Plain Plate)

The Analysis Settings were edited with the minimum range of 190 Hz, while the max range was 230 Hz. Thus, the mean range was around 20 and the solution interval was 80 (Fig 20).

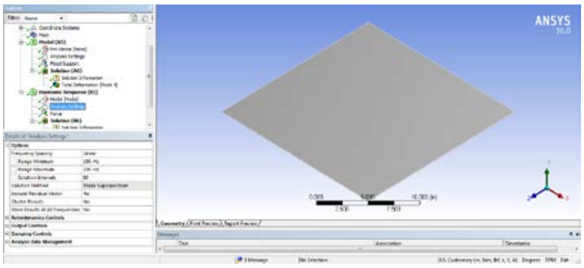

Fig. 20. Analysis Settings options

Certain amount of force was applied in the middle of the upper face by adding a point load with value of 1 N = 0.22481 Ibf, which directed the negative Y axis (Fig 21).

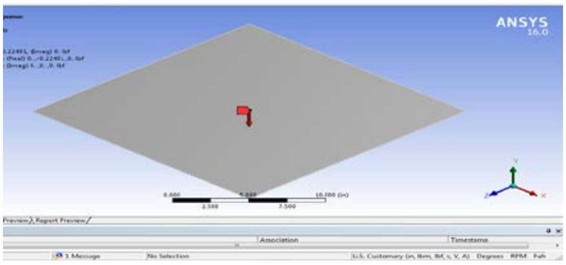

Fig. 21. Force acting in the middle of the upper face

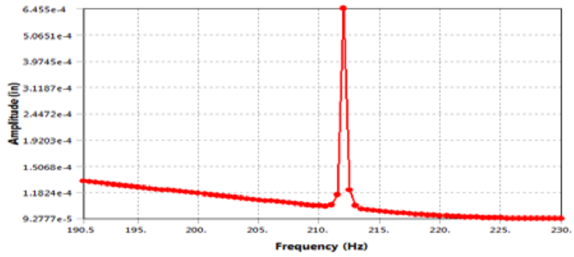

Fig. 22. Frequency response for plain plate

As evident from Fig. 22, the frequency was 212.04 Hz. While, the amplitude was 6.455e-4 inches. After finishing the analysis for the plain plate, it was the time to think of a new configuration that gives lower amplitude and improves it. So, the following configuration takes place where the material was cut from the upper face in circular shapes (Fig 23).

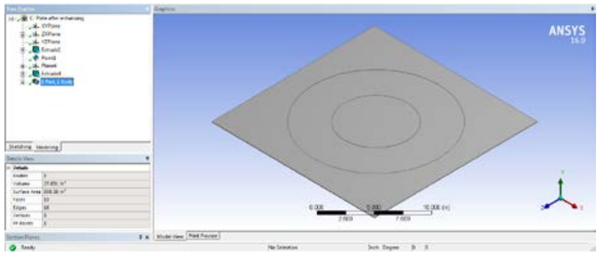

Fig. 23. New configuration for the plate

The steps were then repeated in the plain plate with the same mesh sizing 0.39 in (10 mm), frequency range by 20, and solution intervals by 80 (Fig 24).

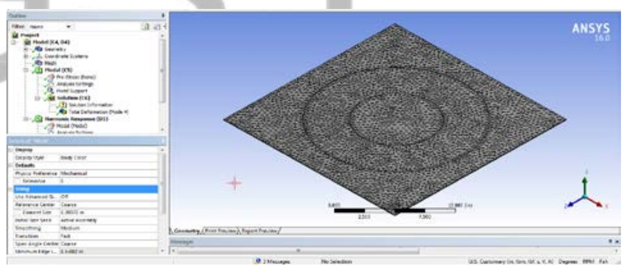

Fig. 24. The mesh in the new configuration of the plate

3.2.2 Result of Modal analysis (New Configuration)

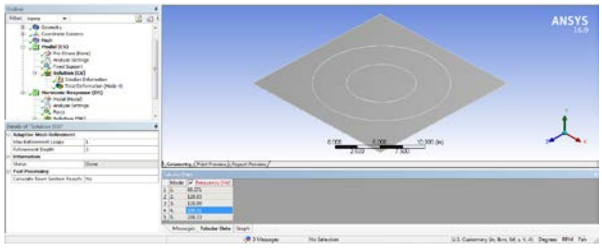

Fig. 25. The Frequency of mode shape 4 “New Configuration”

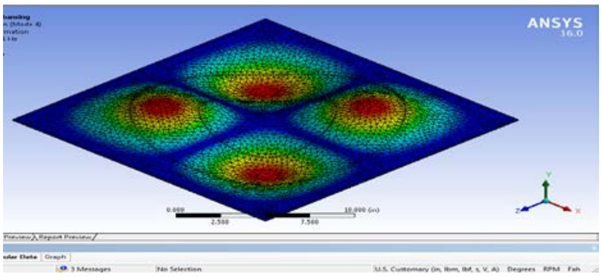

Fig. 26. Mode shape 4 “New Configuration”

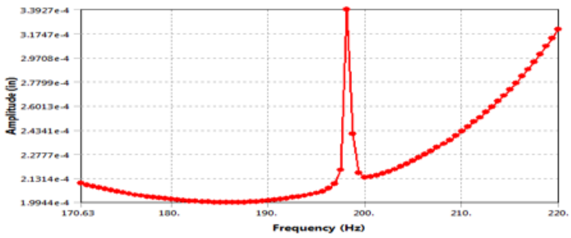

Fig. 27. Frequency response “New Configuration”

Fig 25-27 illustrated the “New Configuration” with a frequency of 198.31 Hz. where the amplitude was 3.3927e-4 inches. On comparing with the plain plate, the improvement for this plate was found by cutting material from the upper face in the shape of circle. No constraints were observed in the dimensions or material, and the new volume equals the new one.

3.3 Case 3: Mode Shape 5

3.3.1 Original Plate

Aluminum material was assigned for the plate. A circular imprint face with 0.03 inches (0.76 mm) diameter was created at the middle of the plate to exert the force on it later (Fig 28).

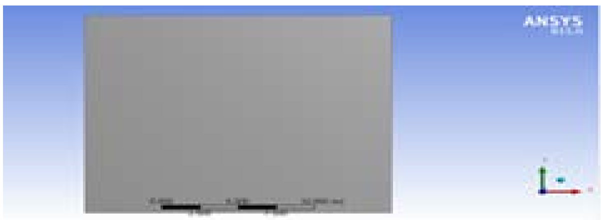

Fig. 28. The Original Plate

3.3.2 Modal Model

The mesh sizing was 0.015 m (Fig 29). The fifth natural frequency was found to be 266.42 HZ. Fig. 30 provides the virtual representation of its mode shape.

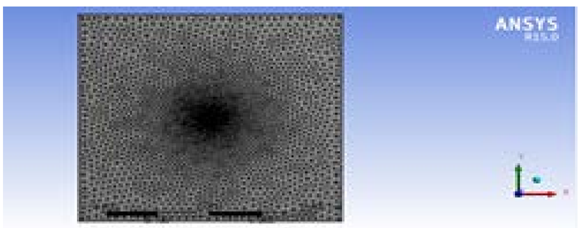

Fig. 29. The mesh for the original plate

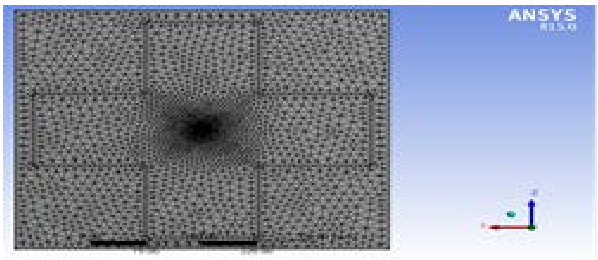

Fig. 30. The fifth mode shape for the original plate

3.3.3 Harmonic Model

One newton excitation force was applied at the middle of the plate. In the analysis settings of the harmonic model, the minimum frequency range was 246.42 HZ, whereas, the maximum frequency range was 286.42 HZ, and the solution intervals were 80. In particular, the frequency responses were exactly obtained at the fifth natural frequency (266.42 HZ). The amplitude at 266.42 Hz was found to be 6.9648e-002 m (Fig 31).

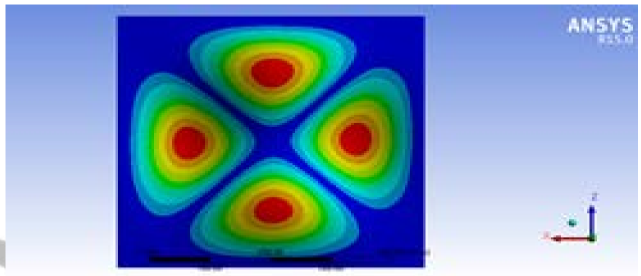

Fig. 31. Frequency response of the fifth mode shape for the original plate

3.3.4 Re-designed Plate

The re-designed plate consisted of two layers. The first layer was a square plate with width 20 inches, height 20 inches, and thickness 0.02 inches. The second layer was a cross symbol with thickness 0.06 inches. The cross symbol consists of two rectangles, each rectangle with width 6 inches and height 18 inches. Fig 32 shows the re-designed plate, which was created by ANSYS design modeler.

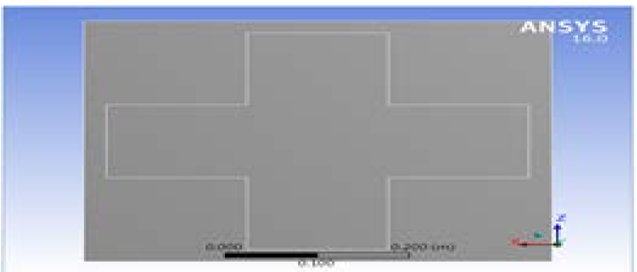

Fig. 32. The re-designed plate for the fifth mode shape

3.3.5 Modal Model

The mesh sizing was 0.015 m (Fig 33). The fifth natural frequency was found to be 244.23 HZ. Its mode shape was shown in Fig 34.

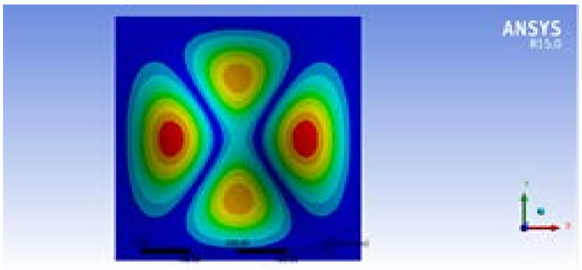

Fig. 33. The mesh for the re-designed plate

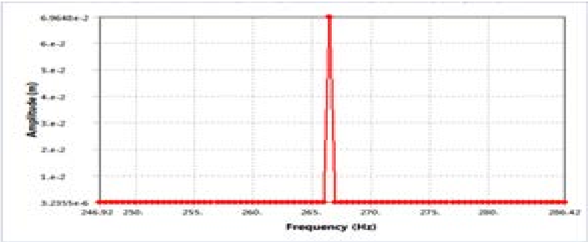

Fig. 34. The fifth mode shape for re-designed plate

3.3.6 Harmonic Model

One newton excitation force was applied at the middle of the plate. In the analysis settings of the harmonic model, the minimum frequency range was 224.23 Hz, while, the maximum frequency range was 264.23 Hz, and the solution intervals put 80. In particular, the frequency response was exactly at the fifth natural frequency (244.23 HZ). The amplitude at 244.23 Hz was found to be 1.7145e-002 m (Fig 35).

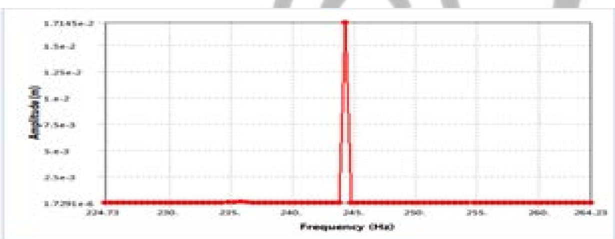

Fig. 35. Frequency response of the fifth mode shape for re-designed plate

4. DISCUSSION

A comparison between the original plate and re-designed plate in terms of volume, mass, frequency and amplitude is shown in Table 1.

Table 1: A comparison between the original plate and re-designed plate

	Original Plate	Re-Designed Plate	Percentage of Improving
Volume	32 in^3	22.725 in^3	28.98 %
Mass	3.2023 lbm	2.2742 lbm	28.98 %
Frequency	72.265 Hz	57.841 Hz	17.68 %
Amplitude	176.55 in	77.61 in	56.04 %

The amplitude at frequency in mode 1 forms both original plate and re-designed plate in large (Table 2).

Table 2: Amplitudes of original plate and re-designed plate

Original Plate			Re-Designed Plate		
No.	Frequency (Hz)	Amplitude (in)	No.	Frequency (Hz)	Amplitude (in)
39	69.765	5.9693e-002	39	57.341	7.0472e-002
40	70.265	176.55	40	57.841	77.61
41	70.765	5.9309e-002	41	58.341	6.9739e-002

To summarize this work, Table 3 provides the comparative analysis between the plain plate and the new configuration for a clearer view regarding the developed improvements in the plate.

Table 3. Comparing between Plain Plate and New Configuration

	Plain Plate	New Configuration	Percentage of Improving
Volume (in3)	32	27.831	13.03 %
Mass (lbm)	3.2023	2.7851	13.03 %
Frequency (Hz)	212.04	198.31	6.47 %
Amplitude (in)	6.455e-4	3.3927e-4	47.44 %

5. CONCLUSION

In conclusion, homogenization was found as a useful technique to minimize the transverse deflection of elastic plates. Results indicated the enhancement in the dynamic performance of elastic plates. Therefore, it can be concluded it is a good technique to reduce the weight of the plates, which is an important issue in many applications, such as aircrafts.

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