Global Scientific JOURNALS
GSJ: Volume 11, Issue 9, September 2023, Online: ISSN 2320-9186
www.globalscientificjournal.com

Existence and Uniqueness of Solution of an HIV/AIDS transmission dynamics Model Incorporating Screening and Highly Active Antiretroviral Therapy (HAART)

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KeyWords

Dynamics, Existence and uniqueness of solution, HIV/AIDS, HAART, Model, Transmission, Treatment, Voluntary counselling, Screening.

ABSTRACT

HIV/AIDS is one of the most deadly diseases human kind has ever faced, with profound social economic and public health consequences. The increasing trends of HIV pose a significant public health concern to the health sector, public health workers, the government and the whole world at large. As at today, there is yet no complete cure for HIV/AIDS. Many researchers and public health workers are still working hard to curtail and control the disease. In this study, a mathematical model for transmission model is formulated considering screening and treatment with highly active and antiretroviral therapy and the existence of its unique solution is rigorously investigated

INTRODUCTION

Human immunodeficiency Virus (HIV) causes Acquired immune deficiency syndrome (AIDS) and it is one of the deadliest epidemics in human history. It was first identified and reported among homosexual men and intravenous drug users in New York and California. Shortly after its detection in the United states, evidence of Aids grew among heterosexual men, women and children in sub-Saharan Africa and it quickly developed in to a worldwide epidemic affecting virtually every nation. HIV/AIDS is seen as a chronic disease nowadays, because HIV-positive people can live with infection for many years provided their immune systems are checked. It gradually destroys the immune system until it is unable to fight infections that would normally have been prevented. With deterioration of the immune system, the body develops opportunistic diseases that lead to AIDS [1]. As the immune system becomes compromised, the HIV opportunistic diseases such as meningitis, Cancers and Tuberculosis do easily attack the body [2].

The United Nations program on HIV/AIDS (UNAIDS) estimates that worldwide the number of new cases of HIV infection peaked in the later 1990s with more than 3 million people nearly infected each year. However, some regions of the world especially Vietnam, Indonesia and other countries in Southern Asia, began to experience an increase in the early 2000s. AIDS cases have been reported in every nation of the world, the disease affects some countries more than others. About 90 percent of all HIV-infected people live in the developing world. AIDS has struck sub-Saharan Africa severely and two-thirds of all people living with HIV infection reside in African countries (South of the Sahara), where AIDS is one of the leading causes of death. Infact, two-third of the world's HIV-Infected people live in Sub-Saharan Africa and more than 1.5 Million of them die annually [3]. The control of HIV/AIDS is not yet over [24]. Over 22million people have died from AIDS worldwide, and the United Nations estimate that, currently, there are 14million AIDS orphans and that there will be 25million by 2010. In 2008, early mortality rate in Sub-Saharan Africa is very high. Between 8% and 26% of patients of HIV/AIDS die in the first year of ART with most deaths occurring in the

first few months [3]. In 2007, an estimated 22.5 Million adults and children in the sub-Saharan region were living with HIV/AIDS and 1.6 Million of them equally died, representing 76% of global AIDS death [4]. More so, in the countries hardest hit, AIDS has sapped the population of young men and women who form the foundation of the labour force. Most die while in the peak of their reproductive years. Moreover, the epidemic has overwhelmed health care systems, increases the number of orphans, and caused life expectancy rates to plummet. It therefore constitutes a serious threat to future development in Africa.

Transmission of HIV/AIDS causing virus occurs most commonly through sexual intercourse. HIV can also be transmitted through transfusions of HIV-contaminated blood or by using a contaminated needle or syringe to inject drugs into the blood stream. Physicians prefer to use the term AIDS for cases where a person has reached the final, life-threatening stage of HIV infection[5].In a person infected with HIV, the virus steadily destroys $CD4^{+}$T cells over a period of years, diminishing the cells protective ability and weakening the immune system. HIV infects some other cells and wrecks the largest part of the $CD4^{+}$T cells and this causes the $CD4^{+}$T cells destruction and decline, hence reducing the confrontation of the susceptible system [6],[7].

Screening is the process of performing the HIV-antibody test to all individuals within a defined population. Routine screening of unaware infective has now become an integral part of programs in low and middle income countries. People can get HIV tests done at a health clinic, at special HIV voluntary counselling and testing (VCT) sites [8]. Access to antiretroviral treatment is scaled up in low and middle income countries and there is critical opportunity to simultaneously expand access to HIV treatments as it continues to be the main stray preventions of the response to HIV epidemic [8]. Antiretroviral medication has made it possible for many individuals who have been very sick with HIV/AIDS to become fully functioning again, with a low, or even undetectable viral load. Without antiretroviral therapy, someone who has AIDS typically dies within a year [9]. In 2022, 86% [73-98%] of all people living with HIV knew there HIV status. Among people who knew their status, 89% [75-98%] were accessing treatment. And among people accessing treatment, 93% [79-98%] were virally suppressed. Among women, the 95-95-95 targets were 90% [76-98%], among men the 95-95-95 targets were 83% [70-98%] and among children ages 0-14 years the 95-95-95 targets were 63%[49-86%]. Among all people living with HIV, 86% [73-98%] knew their status, 76% [65-89%] were accessing treatment and 71% [60-83%] were virally suppressed in 2022. Infact, at the end of December 2022, 29.8 million people were accessing antiretroviral therapy, up from 7.7 million in 2010. In 2022, around 630,000 [480,000-880,000] people died from AIDS-related illnesses worldwide, compared to 2.0 million [1.5-2.8million] people in 2004 and 1.3 million [970,000-1.8million] people in 2020. [25] estimates that US$29billion (in constant 2019 united states dollars) will be required for the AIDS response in low-and middle-income countries, including countries formally considered to be upper-income countries, in 2025 to get on track to end AIDS as a public health threat.

Globally, median HIV prevalence among the adult population (ages 15-49) was 0.7%. However median prevalence was higher among key populations. 25% among sex workers, 77% among gay men and other men who have sex with men, 5.0 % among people who inject drugs, 10.3% among transgender persons, and 1.4% among people in prisons. Every week, 4000 adolescent girls and young women aged 15-24 years became infected with HIV globally in 2022..3100 of these infections occurred in sub-Saharan Africa [25].

Mathematical models for transmission dynamics have given better understanding of epidemiological patterns for disease control as they provide short and long term predictions for HIV/AIDS [29]. A number of different Mathematical models of HIV/AIDS have been developed, ranging from simple extrapolation of past curves to complex transmission models [10],[11],[12],[13],[14],[15],[16],[17],[22],[24],[26],[27],[28] among others. Transmission and prevention of HIV/AIDS, symptoms and stages of HIV infection and AIDS, screening and treatment of HIV/AIDS have also been included. Although no cure has been found for AIDS, new drugs are available that can prolong the life span and improve the quality of life of infected people [18].

The study is organized as follows. The formulation of the model and the basic properties are presented. Also, the proof of the existence and uniqueness of solutions is carried out, and the concluding remark.

FORMULATION OF THE MODEL

In this research, the susceptible and infectious epidemic model (SI) is considered as presented by [17],[23]. A population size of $N(t)$ was partitioned into 4 subclasses of individuals who were susceptible, asymptomatic infective, symptomatic infective, treated infective, and AIDS with sizes denoted by $S(t), I_1(t), I_2(t),$ and $A(t)$, respectively, as shown in figure 1.

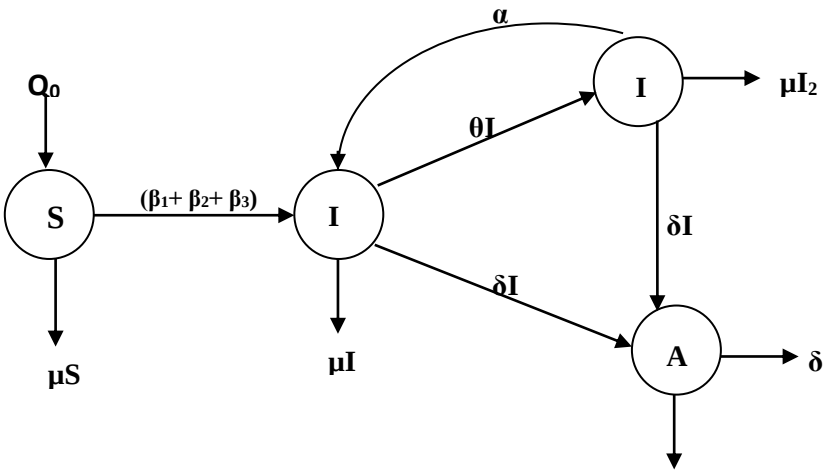

Figure 1: *Transmission diagram for susceptible-infected model .*

Model Equation:

$$\frac{ds}{dt} = Q_0 - (\beta_1 I_1 + \beta_2 I_2 + \beta_3 A)S - \mu S$$
$$\frac{dI_1}{dt} = (\beta_1 I_1 + \beta_2 I_2 + \beta_3 A)S - (\theta + \mu + \delta_1)I_1 + \alpha I_2$$
$$\frac{dI_2}{dt} = \theta I_1 - (\mu + \delta_2)I_2 - \alpha I_2$$
$$\frac{dA}{dt} = \delta_1 I_1 + \delta_2 I_2 - (\mu + d)A$$
$$S(0) = S_0, I_0(0) = I_0, I_2(0) = I_0, A(0) = A_0$$

Model Parameters :

The parameters used in the model are defined as follows:

Q_0 is the constant number of recruitment of susceptible, $\beta_1, \beta_2, \beta_3,$ are the transmission rates for susceptible individuals with asymptomatic infective, susceptible individuals with symptomatic infective, and the susceptible individuals with AIDS respectively, μ is the natural mortality rate unrelated to AIDS, at which treated individual develops to full blown AIDS, α is the rate at which AIDS patients get treatment with highly active antiretroviral Therapy (haart), δ_1 is rate of movement from asymptomatic class I_1 (t) to full blown AIDS A (t). δ_2 is rate of movement from symptomatic class I_2 (t) and develop to AIDS class, *d* is the AIDS related or Disease related death rate and θ is the rate of screening from screening rate from I_1 (t) to

I_2 (t).

EXISTENCE AND UNIQUENESS OF SOLUTION OF THE MODEL

In this section, we establish conditions for the existence and uniqueness of a solution of our model. We shall rigorously employ Picard theorem to achieve this.

Theorem 1: Picard Theorem

Suppose

$$y' = f(t, y), y(t_0) = y_0 \qquad 2.1$$

Is given system of ordinary differential equations and suppose $f(t,x)$ is continuous and satisfies a Lipschitz condition in the closed and bounded domain $\|x-x_0\| \le \varphi, \|t-t_0\| \le \tau$. Let $\|f(t,x)\| < M$

Then the IVP (2.1) has a unique solution in the interval $\|t-t_0\| \le h$, where h=Min $\{t, \varphi / M\}$.

For more on Picard theorem, see for example, HU and Li (2004) and Muscat (2008).

Consider our system of equations

$$\begin{aligned} f_1(t,x) &= \lambda - (\beta_1 I_1 + \beta_2 I_2 + \beta_3 I_3)S - \mu S \\ f_2(t,x) &= (\beta_1 I_1 + \beta_2 I_2 + \beta_3 I_3)S - (\theta + \mu + \delta_1)I_1 + \alpha I_2 \\ f_3(t,x) &= \theta I_1 - (\mu + \delta_2)I_2 - \alpha I_2 \\ f_4(t,x) &= \delta_1 I_1 + \delta_2 I_2 - (\mu + d)A \\ S(0) = S_0, I_0(0) &= I_0, I_2(0) = I_0, A(0) = A_0 \end{aligned} \quad 2.2$$

So that our system of equations has the form

$$x' = f(t,x) = f(x,) \qquad x(t_0) = x_0 \quad 2.3$$

Define

$$D = \{x = (S, I_1, I_2, A) : S, I_1, I_2 A \le 1\}, \quad 2.4$$

And let

$\|x-x_0\| \le \varphi, \|t\| \le \tau$, with $t_0 = 0, x_0 = (S, I_1, I_2 A)$

Now, we shall show or proof using Picard theorem that (2.3) has a unique solution, by proving the following:

1. f is continuous
2. f satisfies a Lipschitz condition, and
3. $|f| \le M$

Now, the fuction $f(t,x)$ is continuous as each component $f_i, i = 1,2,3,4$ of $f(t,x)$ is a continuous function of the variable
$x = (S, I_1, I_2, A)^T$.

Let us establish the Lipshitz condition. We do this by showing that each component of $f_i, i = 1...4$ satisfies a Lipschitz condition.

Let $x = (S, I_1, I_2, A)^T$,
then $f(x) = (f_1(x), f_2(x), f_3(x), f_4(x)^T$,

Now, noting that $(S, I_1, I_2, A)^T \le 1$, we have that

$$f_1(x) - f_1(y) = \left| \begin{array}{l} (Q_0 - Q_0) + (-\beta_1)\left[S(I_1 - I_1^*) + I_1(S - S^*)\right] + (-\beta_2)\left[S(I_2 - I_2^*) + I_2(S - S^*)\right] + \\ (-\beta_3)\left[S(A - A^*) + A(S - S^*)\right] + (-\mu)(S - S^*) \end{array} \right|$$

$$\leq \beta_1\left|I_1 - I_1^*\right| + \beta_1\left|S - S^*\right| + \beta_2\left|I_2 - I_2^*\right| + \beta_2\left|S - S^*\right| + \beta_3\left|A - A_1^*\right| + \beta_3\left|S - S^*\right| + \mu\left|S - S^*\right|$$

$$= \left(\mu + \beta_1 + \beta_2 + \beta_3\right)\left|S - S^*\right| + \beta_1\left|I_1 - I_1^*\right| + \beta_2\left|I_2 - I_2^*\right| + \beta_2\left|S - S^*\right| + \beta_3\left|A - A_1^*\right|$$

$$= L_{11}\left|S - S^*\right| + L_{21}\left|I_1 - I_1^*\right| + L_{31}\left|I_2 - I_2^*\right| + L_{41}\left|A - A_1^*\right|$$

Therefore,

$$\left|f_1(x) - f_1(y)\right| \leq L_1\left\|x - y\right\| \quad 2.5$$

Where L_1 =max $\left(L_{11}, L_{21}, L_{31}, L_{41}\right]$ and

$$L_{11} = \left(\mu + \beta_1 + \beta_2 + \beta_3\right)$$

$$L_{21} = \beta_1, L_{31} = \beta_2, L_{41} = \beta_3$$

Similarly,

$$f_2(x) - f_2(y) = \begin{vmatrix} \beta_1\left[S(I_1 - I_1^*) + I_1(S - S^*)\right] + \beta_2)\left[S(I_2 - I_2^*) + I_2(S - S^*)\right] + \\ \beta_3\left[S(A - A^*) + A(S - S^*)\right] + \left[-(\theta + \mu + \delta_1)(I_1 - I_1^*) + \alpha\left(I_2 - I_2^*\right)\right] \end{vmatrix}$$

$$\leq \beta_1\left|I_1 - I_1^*\right| + \beta_1\left|S - S^*\right| + \beta_2\left|I_2 - I_2^*\right| + \beta_2\left|S - S^*\right| + \beta_3\left|A - A^*\right| + \beta_3\left|S - S^*\right| + (\theta + \mu + \delta_1)\left|I_1 - I_1^*\right|$$
$$+ \alpha\left|I_2 - I_2^*\right|$$

$$= (\beta_1 + \beta_2 + \beta_3)\left|S - S^*\right| + \left[\beta_1 + (\theta + \mu + \delta_1)\right]\left|I_1 - I_1^*\right| + (\beta_2 + \alpha)\left|I_2 - I_2^*\right| + \beta_3\left|A - A^*\right|$$

$$= L_{12}\left|S - S^*\right| + L_{22}\left|I_1 - I_1^*\right| + L_{32}\left|I_2 - I_2^*\right| + L_{42}\left|A - A^*\right|$$

Therefore,

$$\left|f_2(x) - f_2(y)\right| \leq L_2\left\|x - y\right\| \quad 2.6$$

Where L_2 =max $\left(L_{12}, L_{22}, L_{32}, L_{42}\right]$ and

$$L_{12} = \beta_1 + \beta_2 + \beta_3; L_{22} = \beta_1 + (\theta + \mu + \delta_1); L_{32} = \beta_2 + \alpha; L_{42} = \beta_3$$

are constant depending on the parameter values of the model.

Also,

$$\left|f_3(x) - f_3(y)\right| = \left|\theta(I_1 - I_1^*) + \left[-(\mu + \delta_2)\right]\left(I_2 - I_2^*\right) + (-\alpha)(I_2 - I_2^*)\right|$$

$$\leq \theta\left|I_1 - I_1^*\right| + (\mu + \delta_2)\left|I_2 - I_2^*\right| + \alpha\left|I_2 - I_2^*\right|$$

$$= 0\left|S - S^*\right| + \theta\left|I_1 - I_1^*\right| + \left[(\mu + \delta_2) + \alpha\right]\left|I_2 - I_2^*\right| + 0\left|A - A^*\right|$$

$$= L_{13}\left|S - S^*\right| + L_{23}\left|I_1 - I_1^*\right| + L_{33}\left|I_2 - I_2^*\right| + L_{43}\left|A - A^*\right|$$

$$\left|f_3(x) - f_3(y)\right| \leq L_3\left\|x - y\right\|$$

Where L_3 = max $\left(L_{13}, L_{23}, L_{33}, L_{43}\right)$;

And

$$L_{13} = 0; L_{23} = \theta; L_{33} = (\mu + \delta_2) + \alpha; L_{43} = 0$$

are constant depending on the parameter values of the model.

$$|f_4(x) - f_4(y)| \le |\delta_1(I_1 - I_1^*) + \delta_2(I_2 - I_2^*) + [-(\mu + d)](A - A^*)|$$

$$\le \delta_1|I_1 - I_1^*| + \delta_2|I_2 - I_2^*| + (\mu + d)|A - A^*|$$
$$= 0|S - S^*| + \delta_1|I_1 - I_1^*| + \delta_2|I_2 - I_2^*| + (\mu + d)|A - A^*|$$
$$= L_{14}|S - S^*| + L_{24}|I_1 - I_1^*| + L_{34}|I_2 - I_2^*| + L_{44}|A - A^*|$$
$$= |f_4(x) - f_4(y)| \le L_4\|x - y\|$$

Where $L_4 = \max\ (L_{14}, L_{24}, L_{34}, L_{44})$;

And

$L_{14} = 0; L_{24} = \delta_1; L_{34} = \delta_2; L_{44} = \mu + d$

are constants depending on the parameter values of the model.

Therefore, $|f_4(x) - f_4(y)| \le L_4\|x - y\|$

To obtain the bound for $f(t, x)$:

nothing that $(S, I_1, I_2, A) \le 1$

$$|f_1(x)| \le Q_0 + \beta_1|I_1\|S| + \beta_2|I_2\|S| + \beta_3|A\|S| + \mu|S|$$

$$\le Q_0 + \beta_1 + \beta_2 + \beta_3 + \mu$$
$$= M_1$$

Similarly,

$$|f_1(x)| \le Q_0 + \beta_1|I_1\|S| + \beta_2|I_2\|S| + \beta_3|A\|S| + (\theta + \mu + \delta_1)|I_1| + \alpha|I_2|$$
$$\le \beta_1 + \beta_2 + \beta_3 + (\theta + \mu + \delta_1) + \alpha$$
$$= M_2$$
$$|f_3(x)| \le \theta|I_1| + (\mu + \delta_2)|I_2| + \alpha|I_2|$$
$$= \theta + (\mu + \delta_2) + \alpha$$
$$= M_3$$
$$|f_4(x)| \le \delta_1|I_1| + \delta_2|I_2| + (\mu + \delta_2)|A|$$
$$\le 2\delta + (\mu + d)$$
$$= M_4$$

Therefore, $\|f(x)\| = \max\ \{M_1, M_2, M_3 M_4\}$

$$\le M$$

2.7

Thus, there exists a unique solution for the IVP (2.3) in the domain, $|x - x_0| \le \varphi$ and $|t| \le h$, where $h = \min\left\{t, \frac{\varphi}{M}\right\}$

This completes the proof.

CONCLUSION

In this work, a mathematical model of HIV/AIDS transmission dynamics incorporating screening, treatment with highly and active retroviral therapy is formulated. It has been shown based on the results of the study that our model equations represent a useful model of a physical system by rigorously carrying out by analysing a classical qualitative proof of the existence and uniqueness of a solution. Hence the title of the paper.

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