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FINITE VOLUME ANALYSIS OF TEMPERATURE DISTRIBUTION IN A CIRCULAR FIN

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ABSTRACT

Heat transfer from a surface to surrounding fluid has a wide range of application in engineering. This is often achieved using extended surfaces or fins. Extended surfaces involve heat transfer by conduction within a solid coupled with heat transfer by convection from the boundaries of the solid. This study considers a one-dimensional heat transfer in a fin with circular cross-section. The governing equation was discretized by the finite volume approach. The temperature distribution along the length of fin was investigated. The effect of thermal properties of engineering materials on the temperature distribution was also studied. The study shows that temperature during convection is higher than that obtained considering an adiabatic condition. A higher thermal conductivity results in higher heat transfer to the surroundings.

NOMENCLATURE

$$K = thermal\ conductivity$$
$$A = Crosssectional\ Area$$
$$T = Temperature$$
$$h = Convective\ heat\ transfer\ co-efficient$$
$$T_a = Ambient\ Temperature$$
$$\dot{Q}_x = Rate\ of\ heat\ transfer$$

1.0 INTRODUCTION

In the study of heat transfer, a Fin is a surface that extends from an object to increase the rate of heat transfer to or from the environment by increasing convection. The amount of Heat conduction, convection, or radiation of an object determines the amount of heat it transfers. [1], [2]. Fins are probably the most common method of enhancing heat dissipation from a hot surface. The principle of operation is to provide a larger area over which convective heat transfer may occur than the original surface area. Pin fins are rather common for example, as cooling devices for micro chips.[1], [3]
A numerical solution of the coupled fin conduction equation and the laminar, forced convective boundary layer equations for a cylindrical fin has been carried out. The fin temperature becomes less uniform as R_0. decreases due to lower fin conductance. Thus, the fin effectiveness will decrease with decreasing R_0.[4]. Saheed [5] studied the analytical and numerical solution of one-dimensional rectangular fin with an additional heat source it was found out that If the heat generation is smaller the variation becomes small and

the percentage of the temperature variation becomes too small; however if the heat source larger the variation becomes larger and the percentage of the temperature gap becomes larger.

Tao et al. [6] studied the heat transfer of the rib in the internal cooling Rectangular channel, using separation of variables, analytical solutions of three dimensional steady-state heat conduction in rectangular ribs are given by solving three dimensional steady-state function of the rectangular ribs And the high dimensionless temperature field extends when Bi. The advanced mathematical schemes for analyzing the temperature profile of the Conductive-Convective rectangular linear fin of straight profile which is the solution of second order differential equation. Linear Differential fin equations are solved through Bessel functions, which gave standard Exact solution then solved by Approximated method, Numerical methods either iterative or non- iterative i.e. Power Series Solution, Finite Difference Technique and modern methods like Differential Transformation Method (DTM) further comparison is made from results obtained by different method in tabular and graphical form.[7].

A Fourier series approach to solve the two-dimensional rectangular fin. The temperature distribution in the rectangular fin with arbitrary variable heat transfer coefficient has been written in terms of a summation of series.[8]. Basri et al. [9]studied the temperature distribution in insulated-tip and convection-tip 1-D rectangular fin are computed numerically using FEM.

Moitsheki [10] did the exact solutions for the longitudinal fin of triangular and parabolic profiles. Both thermal conductivity and heat transfer coefficients are given as power law temperature dependent. In this study, the finite volume method was used to solve a circular fin subjected to adiabatic or convective condition.

2.0 MATHEMATICAL MODEL

Consider a circular fin as shown in Figure 1. The circular fin is fixed at base and free at the tip where it is subjected to either adiabatic or convection. At the fixed end, the Temperature is $T_B =$ the fin is surrounded by ambient temperatureT_a , the length of the fin varies from $x = 0\ to\ L$. Two conditions were considered for the tip that is adiabatic and convection.

Assumptions

1. One- dimensional
2. Steady state condition with no heat generation
3. thermal conductivity is constant
4. convective heat transfer coefficient is unifom over the surface
5. radiation from the surface is negligibl

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Governing Equation

$$\left(KA\frac{d^2T}{dx^2}\right) - hP(T - T_a) = 0 \quad (1)$$

Let $m^2 = \frac{hP}{KA}$ (2)

$$\frac{d^2T}{dx^2} - m^2(T - T_a) = 0 \quad (3)$$

Boundary Condition

At the base

$T_B = 10$ (4)

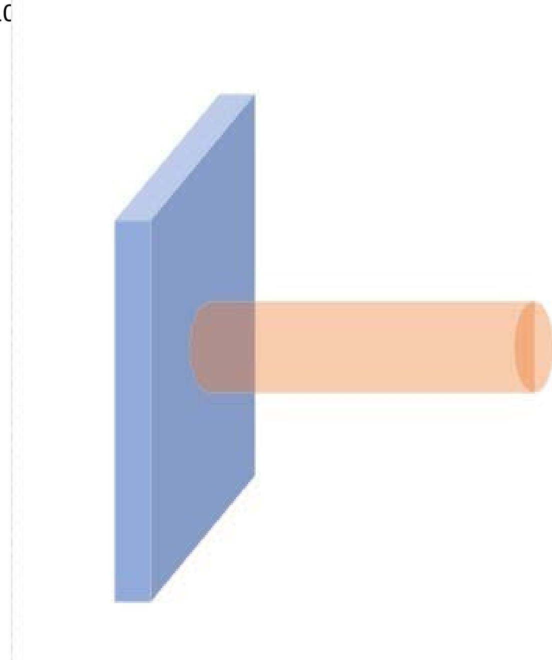

Figure 1 : A circular fin

At the tip

Adiabatic condition

$$\dot{Q}_x = -KA\frac{dT}{dx}\Big|_{x=l} = 0 \quad (5)$$

$$\frac{dT}{dx}\Big|_{x=l} = 0 \quad (6)$$

Convection condition

$$\dot{Q}_x = -KA\frac{dT}{dx}\Big|_{x=l} = hA(T - T_a) \quad (7)$$

$$\frac{dT}{dx}\Big|_{x=l} = -\frac{h(T-T_a)}{k} \quad (8)$$

The above equations were discretized using the finite volume approach.

Governing equations

$$\int_{\Delta v}\frac{d}{dx}\left(\frac{dT}{dx}\right)dv - \int_{\Delta v} m^2(T-T_a)\,dv = 0 \quad (9)$$

$$\int d\left(\frac{dT}{dx}\right)A - \int m^2(T-T_a)dv \quad (10)$$

$$\left[\left(\frac{dT}{dx}\right)A\right]_w^e - \left[m^2\left(T_p - T_a\right)\right]V \quad (11)$$

$$V = A\Delta x \quad (12)$$

$$\left[\left(\frac{dT}{dx}\right)A\right]_w^e - \left[m^2\left(T_p - T_a\right)A\Delta x\right] = 0 \quad (13)$$

$$\left[\left(\frac{dT}{dx}\right)A\right]_w^e - \left[m^2\left(T_p - T_a\right)A\Delta x\right] = 0 \quad (14)$$

Interior nodes

Nodal Point $i = 2 \ to \ N-1$

$$\left[\left(\frac{dT}{dx}\right)A\right]_e - \left[\left(\frac{dT}{dx}\right)A\right]_w - m^2\left(T_p - T_a\right)A\Delta x = 0 \quad (15)$$

$$\left[\left\{\frac{T_E - T_P}{\Delta x}\right\} - \left\{\frac{T_P - T_W}{\Delta x}\right\}\right] - m^2 T_p\Delta x + m^2 T_a \Delta x = 0 \quad (16)$$

$$\frac{2}{\Delta x}T_p = \frac{T_W}{\Delta x} + \frac{T_E}{\Delta x} + m^2 T_a\Delta x - m^2 T_p \Delta x \quad (17)$$

$$a_p T_p = a_w T_w + a_E T_E + S_u + S_p T_p \quad (18)$$

Table 2.1: The interior nodes

a_w	a_e	s_p	a_p	s_u
$\frac{1}{\Delta x}$	$\frac{1}{\Delta x}$	$-[m^2\Delta x]$	$a_w + a_e - s_p$	$[m^2\Delta x]T_a$

Boundary conditions

At the base

At node $i = 1$

$$\left[\left(\frac{dT}{dx}\right)A\right]_w^e - \left[m^2\left(T_p - T_a\right)A\Delta x\right] = 0 \quad (19)$$

$$\left[\left(\frac{dT}{dx}\right)A\right]_e - \left[\left(\frac{dT}{dx}\right)A\right]_w - m^2\left(T_p - T_a\right)A\Delta x = 0 \quad (20)$$

$$\left[\left\{\frac{T_E - T_P}{\Delta x}\right\} - \left\{\frac{T_B - T_W}{\frac{\Delta x}{2}}\right\}\right] - m^2 T_p\Delta x + m^2 T_a\Delta x = 0 \quad (21)$$

$$\left(\frac{1}{\Delta x} + \frac{2}{\Delta x}\right)T_p = \frac{2T_B}{\Delta x} + \frac{T_E}{\Delta x} + m^2 T_a\Delta x - m^2 T_p\Delta x \quad (22)$$

$$a_p T_p = a_w T_w + a_E T_E + S_u + S_p T_p \quad (23)$$

Table 2.2 : At the base

a_w	a_e	s_p	a_p	s_u
0	$\frac{1}{\Delta x}$	$-\left[m^2\Delta x + \frac{2}{\Delta x}\right]$	$a_w + a_e - s_p$	$m^2\Delta x T_a + \frac{2T_B}{\Delta x}$

At the tip

ADIABATIC

At node $i = N$

$$\left[\left(\frac{dT}{dx}\right)A\right]_w^e - \left[m^2\left(T_p - T_a\right)A\Delta x\right] = 0 \qquad (24)$$

$$\left[\left(\frac{dT}{dx}\right)A\right]_e - \left[\left(\frac{dT}{dx}\right)A\right]_w - m^2\left(T_p - T_a\right)A\Delta x = 0 \qquad (25)$$

$$\left[0 - \left\{\frac{T_p - T_W}{\Delta x}\right\}\right] - m^2 T_p \Delta x + m^2 T_a \Delta x = 0 \qquad (26)$$

$$\left(\frac{1}{\Delta x}\right) T_p = \frac{T_w}{\Delta x} + m^2 T_a \Delta x - m^2 T_p \Delta x \qquad (27)$$

$$a_p T_p = a_w T_w + a_E T_E + S_u + S_p T_p \qquad (28)$$

Table 2.3 : At the tip adiabatic condition

a_w	a_e	s_p	a_p	s_u
$\frac{1}{\Delta x}$	`0	$-[m^2\Delta x]$	$a_w + a_e - s_p$	$m^2 \Delta x T_a$

CONVECTIVE

$$for\ i = N \qquad (29)$$

$$\frac{d^2T}{d^2x} - m^2\left(T_p - T_a\right) = 0 \qquad (30)$$

$$\int_{\Delta V} \frac{d}{dx}\left(\frac{dT}{dx}\right) dV - \int_{\Delta V} m^2\left(T_p - T_a\right) dV = 0 \qquad (31)$$

$$\int_{\Delta V} \frac{d}{dx}\left(\frac{dT}{dx}\right) A dx - \int_{\Delta V} m^2\left(T_p - T_a\right) dV = 0 \qquad (32)$$

$$A \left.\frac{dT}{dx}\right|_w^e - m^2\left(T_p - T_a\right)V = 0 \qquad (33)$$

$$A \left.\frac{dT}{dx}\right|_e - A \left.\frac{dT}{dx}\right|_w - m^2\left(T_p - T_a\right)V = 0 \qquad (34)$$

$$A \left.\frac{dT}{dx}\right|_e - A\left[\frac{T_p - T_w}{\Delta x}\right] - m^2\left(T_p - T_a\right)V = 0 \qquad (35)$$

$$\dot{Q}_{cond} = \dot{Q}_{conv} \qquad (36)$$

$$\dot{Q}_x = -KA\frac{dT}{dx} \qquad \dot{Q}_{conv} = hA\left(T_p - T_a\right) \qquad (37)$$

$$A \left.\frac{dT}{dx}\right|_e = \frac{-hA}{K}\left(T_p - T_a\right) \qquad (38)$$

$$\frac{-hA}{K}\left(T_p - T_a\right) - A\left[\frac{T_p - T_w}{\Delta x}\right] - m^2\left(T_p - T_a\right)V = 0 \qquad (39)$$

$$V = A\Delta x \qquad (40)$$

$$\frac{-hA}{K}\left(T_p - T_a\right) - A\left[\frac{T_p - T_w}{\Delta x}\right] - m^2\left(T_p - T_a\right)A\Delta x = 0 \qquad (41)$$

$$\frac{hT_a}{K} + m^2 T_a \Delta x + \frac{T_w}{\Delta x} = T_p\left(\frac{1}{\Delta x} + \frac{h}{k} + m^2\Delta x\right) \qquad (42)$$

$$\frac{hT_a}{K} + m^2 T_a \Delta x + \frac{T_w}{\Delta x} - T_p\left(\frac{h}{k} + m^2\Delta x\right) = T_p\left(\frac{1}{\Delta x}\right) \qquad (43)$$

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$$a_p T_p = a_w T_w + a_E T_E + S_u + S_p T_p \quad (44)$$

Table 2.4: At the tip convective condition

a_w	a_e	s_p	a_p	s_u
0	$\frac{1}{\Delta x}$	$-\left[\frac{h}{k} + m^2 \Delta x\right]$	$a_w + a_e - s_p$	$\left[\frac{h}{k} + m^2 \Delta x\right] T_a$

2.0 RESULTS AND DISCUSSIONS

To check the accuracy of the finite volume method, the result was compared with the analytical solution of a circular fin as shown in figure 2. The values were closely related to the analytical results. The finite volume approach is okay for this analysis.
In Figures 3 and 4, the graph of the varying thermal conductivity in convective and adiabatic conditions, respectively. An increase in the thermal conductivity of the circular fin leads to an increase in the temperature distribution along the fin. The higher the thermal conductivity more heat is transfer, therefore, leading to a higher temperature along the length of the fin. An increase in the thermal conductivity results in a lower temperature gradient.

The graph of temperature distribution comparing the adiabatic and convection condition is shown in Figure 5. From the base to the midpoint of the fin, the temperature of both conditions is the same, but it changes slightly from the midpoint to the tip of the fin. In convective, heat is conduct away by the atmospheric air resulting in a lower temperature. But in the adiabatic condition, there is no external influence.

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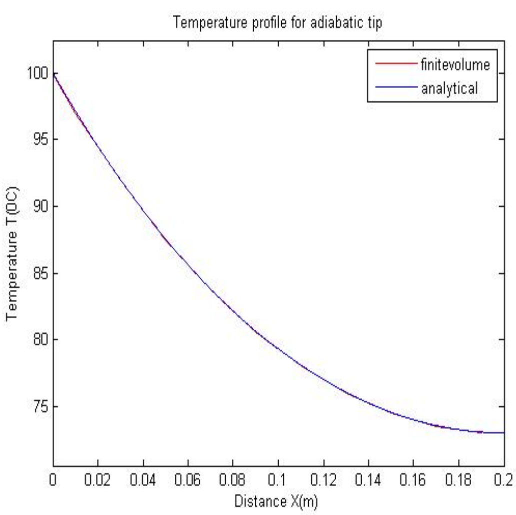

Figure 2: the graph of numerical validation

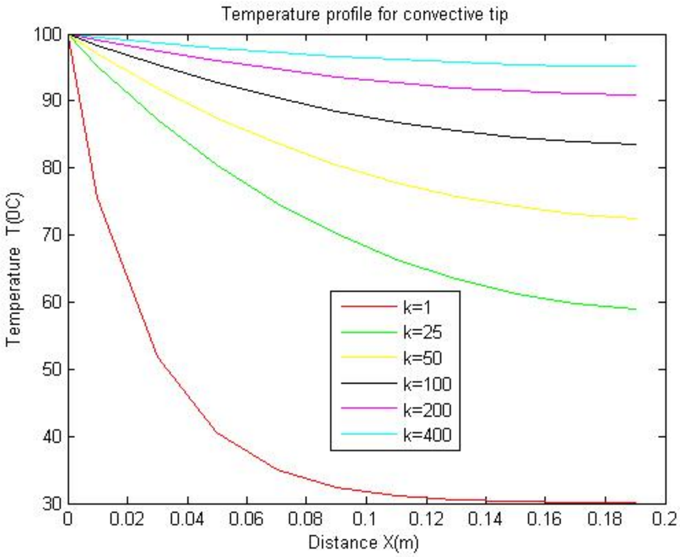

Figure 3 : the graph of the varying thermal conductivity in convective condition

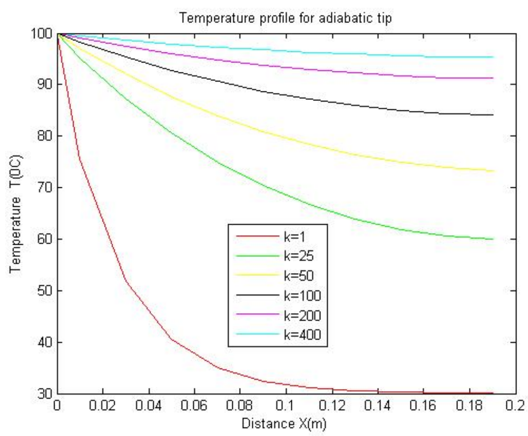

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Figure 4: The graph of varying thermal conductivity in adiabatic condition

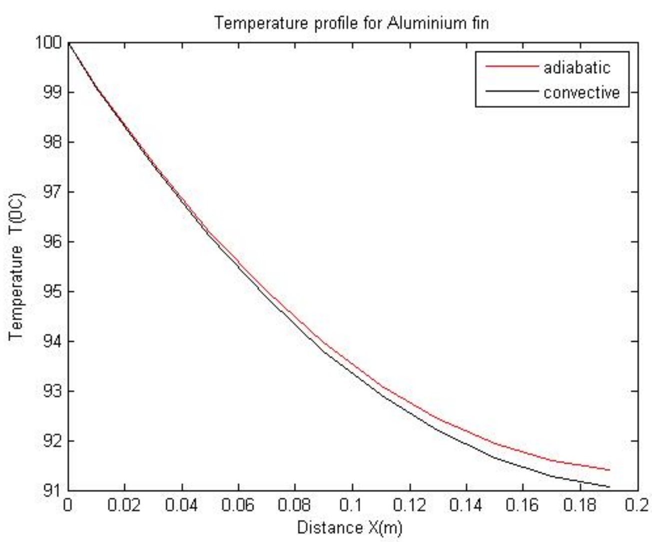

Figure 5 : the graph of temperature distribution comparing the adiabatic and convection condition

3.0 CONCLUSION

This paper considered the temperature distribution in a circular fin subjected to adiabatic or convection at the tip. The finite volume was used to discretize the governing equation. It was discovered that the gradient of the graph reduces with an increase in thermal conductivity. The value of the temperature at the base of the fin is more for adiabatic than convective because heat is conducted away in the convective condition by the atmospheric air.

REFERENCES

[1] G. Penumuchu and B. Bhukya, "Temperature Distribution Analysis of Circular Cross Section Fin by Analytical and Finite Difference Method using C ++ Coding," vol. 3, no. 8, pp. 22–27, 2016.

[2] J. B. Aparecido and R. M. Cotta, "Improved one-dimensional fin solutions," *Heat Transf. Eng.*, vol. 11, no. 1, pp. 49–59, 1990.

[3] S. Baba, S. Nagakishore, and P. M. B. Rao, "AN D E N G I N E E R I N G T E C H N O L O G Y (I J R A S E T) Thermal Analysis on a Finned Tube Heat Exchanger of a Two Stage Air Compressor," vol. 2, no. V, pp. 507–513, 2014.

[4] V. K. Garg and K. Velusamy, "On the heat transfer from a cylindrical fin," *Int. J. Heat Mass Transf.*, vol. 32, no. 1, pp. 187–192, 1989.

[5] A. Almunwwarah, "Analytical and Numerical Solution of One-Dimensional a Rectangular Fin with an Additional Heat Source," pp. 1–6, 2015.

[6] T. Nie and W. Liu, "Analytical Solutions of Three Dimensional Steady-State Heat Transfer in Rectangular Ribs," vol. 6, no. 90916018, pp. 1442–1445, 2012.

[7] V. Agarwal, G. Chaturvedi, and A. Agrawal, "Modern Mathematical Techniques for Analysis of Temperature Distribution in Straight Profile Fins.," vol. 3, no. 3, pp. 2156–2160, 2014.

[8] S. W. Ma, A. I. Behbahani, and Y. G. Tsueit, "Two-dimensional rectangular fin with variable heat transfer coefficient," vol. 34, no. I, pp. 79–85, 1991.

[9] S. BASRI, M. M. FAKIR, F. MUSTAPHA, D. L. A. MAJID, and A. A. JAAFAR, "Heat Distribution in Rectangular Fins Using Efficient Finite Element and Differential Quadrature Methods," *Engineering*, vol. 01, no. 03, pp. 151–160, 2009.

[10] R. J. Moitsheki, "Steady one-dimensional heat flow in a longitudinal triangular and parabolic fin," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 16, no. 10, pp. 3971–3980, 2011.

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