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GENERAL FORMULA OF A MOD B

Keywords

a mod b, trigonometry, floor, digital root

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ABSTRACT

The power of trigonometric functions permits us to link them with modular arithmetic, so instead of blocking on the 180 trigonometric cycle, the exdended trigonometric functions give us all possible cycles. So we'll also see general formulas of a mod b formula.

INTRODUCTION

In mathematics, **modular arithmetic** is a system of arithmetic for integers, where numbers "wrap around" when reaching a certain value, called the **modulus.** The modern approach to modular arithmetic was developed by Carl Friedrich Gauss in his book *Disquisitiones Arithmeticae*, published in 1801.

A familiar use of modular arithmetic is in the 12-hour clock, in which the day is divided into two 12-hour periods. If the time is 7:00 now, then 8 hours later it will be 3:00. Simple addition would result in 7 + 8 = 15, but 15:00 reads as 3:00 on the clock face because clocks "wrap around" every 12 hours and the hour number starts over at zero when it reaches 12. We say that 15 is *congruent* to 3 modulo 12, written 15 ≡ 3 (mod 12), so that 7 + 8 ≡ 3 (mod 12). Similarly, 8:00 represents a period of 8 hours, and twice this would give 16:00, which reads as 4:00 on the clock face, written as 2 × 8 ≡ 4 (mod 12).

In this paper we'll see general formula of a mod b.

I. A mod B general formula

1. General formula for a mod 180, in comparative tables:

By using trigonometric functions, we can have a formula for a mod 180. The form of trigonometric function is linked to a cycle (180), that's why we can have a mod 180 by a general formula.

Given the function $Z(\ )$

$$Z(a)=\frac{|\sin(a)|}{\sin(a)}\times ar\cos(\cos(a))+\frac{1}{2}\left(1-\frac{|\sin(a)|}{\sin(a)}\right)\times 180$$

A	$Z(a)$	a mod 180
1	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	6	6
7	7	7
8	8	8
9	9	9
10	10	10
11	11	11
12	12	12
13	13	13
14	14	14
15	15	15
16	16	16
17	17	17
18	18	18
19	19	19
20	20	20
21	21	21

22	22	22
23	23	23
24	24	24
25	25	25
26	26	26
27	27	27
28	28	28
29	29	29
30	30	30
31	31	31
32	32	32
33	33	33
34	34	34
35	35	35
36	36	36
37	37	37
38	38	38
39	39	39
40	40	40
41	41	41
42	42	42
43	43	43
44	44	44
45	45	45
46	46	46
47	47	47
48	48	48
49	49	49
50	50	50
51	51	51
52	52	52
53	53	53
54	54	54
55	55	55
56	56	56
57	57	57
58	58	58
59	59	59
60	60	60
61	61	61
62	62	62
63	63	63
64	64	64
65	65	65
66	66	66
67	67	67

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68	68	68
69	69	69
70	70	70
71	71	71
72	72	72
73	73	73
74	74	74
75	75	75
76	76	76
77	77	77
78	78	78
79	79	79
80	80	80
81	81	81
82	82	82
83	83	83
84	84	84
85	85	85
86	86	86
87	87	87
88	88	88
89	89	89
90	90	90
91	91	91
92	92	92
93	93	93
94	94	94
95	95	95
96	96	96
97	97	97
98	98	98
99	99	99
100	100	100

Figure 1 : comparative table of a mod 180 and $Z(a)$ between 1 and 100

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A	$Z(a)$	a mod 180
101	101	101
102	102	102
103	103	103
104	104	104
105	105	105
106	106	106
107	107	107
108	108	108
109	109	109
110	110	110
111	111	111
112	112	112
113	113	113
114	114	114
115	115	115
116	116	116
117	117	117
118	118	118
119	119	119
120	120	120
121	121	121
122	122	122
123	123	123
124	124	124
125	125	125
126	126	126
127	127	127
128	128	128
129	129	129
130	130	130
131	131	131
132	132	132
133	133	133
134	134	134
135	135	135
136	136	136
137	137	137

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138	138	138
139	139	139
140	140	140
141	141	141
142	142	142
143	143	143
144	144	144
145	145	145
146	146	146
147	147	147
148	148	148
149	149	149
150	150	150
151	151	151
152	152	152
153	153	153
154	154	154
155	155	155
156	156	156
157	157	157
158	158	158
159	159	159
160	160	160
161	161	161
162	162	162
163	163	163
164	164	164
165	165	165
166	166	166
167	167	167
168	168	168
169	169	169
170	170	170
171	171	171
172	172	172
173	173	173
174	174	174
175	175	175
176	176	176
177	177	177
178	178	178
179	179	179
180	0	0
181	1	1
182	2	2
183	3	3

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184	4	4
185	5	5
186	6	6
187	7	7
188	8	8
189	9	9
190	10	10
191	11	11
192	12	12
193	13	13
194	14	14
195	15	15
196	16	16
197	17	17
198	18	18
199	19	19
200	20	20

Figure 2 : comparatif table of a mod 180 and $Z(a)$ between 101 and 200

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A	$Z(a)$	a mod 180
201	21	21
202	22	22
203	23	23
204	24	24
205	25	25
206	26	26
207	27	27
208	28	28
209	29	29
210	30	30
211	31	31
212	32	32

213	33	33
214	34	34
215	35	35
216	36	36
217	37	37
218	38	38
219	39	39
220	40	40
221	41	41
222	42	42
223	43	43
224	44	44
225	45	45
226	46	46
227	47	47
228	48	48
229	49	49
230	50	50
231	51	51
232	52	52
233	53	53
234	54	54
235	55	55
236	56	56
237	57	57
238	58	58
239	59	59
240	60	60
241	61	61
242	62	62
243	63	63
244	64	64
245	65	65
246	66	66
247	67	67
248	68	68
249	69	69
250	70	70
251	71	71
252	72	72
253	73	73
254	74	74
255	75	75
256	76	76
257	77	77
258	78	78

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259	79	79
260	80	80
261	81	81
262	82	82
263	83	83
264	84	84
265	85	85
266	86	86
267	87	87
268	88	88
269	89	89
270	90	90
271	91	91
272	92	92
273	93	93
274	94	94
275	95	95
276	96	96
277	97	97
278	98	98
279	99	99
280	100	100
281	101	101
282	102	102
283	103	103
284	104	104
285	105	105
286	106	106
287	107	107
288	108	108
289	109	109
290	110	110
291	111	111
292	112	112
293	113	113
294	114	114
295	115	115
296	116	116
297	117	117
298	118	118
299	119	119
300	120	120

Figure 3 : comparative table of a mod 180 and $Z(a)$ between 201 and 300

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a	$Z(a)$	a mod 180
301	121	121
302	122	122
303	123	123
304	124	124
305	125	125
306	126	126
307	127	127
308	128	128
309	129	129
310	130	130
311	131	131
312	132	132
313	133	133
314	134	134
315	135	135
316	136	136
317	137	137
318	138	138
319	139	139
320	140	140
321	141	141
322	142	142
323	143	143
324	144	144
325	145	145
326	146	146
327	147	147
328	148	148
329	149	149
330	150	150
331	151	151
332	152	152
333	153	153
334	154	154

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335	155	155
336	156	156
337	157	157
338	158	158
339	159	159
340	160	160
341	161	161
342	162	162
343	163	163
344	164	164
345	165	165
346	166	166
347	167	167
348	168	168
349	169	169
350	170	170
351	171	171
352	172	172
353	173	173
354	174	174
355	175	175
356	176	176
357	177	177
358	178	178
359	179	179
360	0	0

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Figure 4 : comparative table of a mod 180 and $Z(a)$ between 301 and 360

- **Conclusion :**
  According to the comparative tables, we can say that : a mod 180 = $Z(a)$

  $\Rightarrow \quad (a+2k\pi) \quad \text{mod} \quad 180 \quad = Z(a+2k\pi) = Z(a)$

  It means that for any positive integer a we have $a \quad \text{mod} \quad 180 = Z(a)$ is true.

  We can see that the normal cycle for trigonometric functions is 180.

2. General formula for a mod b:

In the previous formula the cycle was 180 that is the natural cycle of trigonometric functions.
We are now going to use extended trigonometric functions for any cycle different from 180. (cost, sint and tant).

If this cycle is b we have:

$$\cos t(a,b) = \cos\left(\frac{180a}{b}\right)$$

$$\sin t(a,b) = \sin\left(\frac{180a}{b}\right)$$

$$\tan t(a,b) = \tan\left(\frac{180a}{b}\right)$$

$$ar\cos t(c,b) = ar\cos(c) \times \frac{b}{180}$$

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Consequently, for any cycle b, the function become:

$$Z(a,b) = \frac{\left|\sin\left(\frac{180a}{b}\right)\right|}{\sin\left(\frac{180a}{b}\right)} \times ar\cos\left(\cos\left(\frac{180a}{b}\right)\right) \times \frac{b}{180} + \frac{1}{2}\left(1 - \frac{\left|\sin\left(\frac{180a}{b}\right)\right|}{\sin\left(\frac{180a}{b}\right)}\right) \times b$$

$a \mod b = Z(a,b)$, for the cycle b.

CONCLUSION

The hope is that the existence of a formula of a mod b would permit to more efficiency on solving modular arithmetic problems.

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