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Generating Field-Field Quantum Correlations In Two-Mode Cavities

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Abstract

Quantum correlation is one of the most fascinating and conspicuous feature of quantum information theory. Particularly, entangled states are recently treasured as a physical resource for the performance of quantum information protocols. In this paper, we investigate theoretically the generating field-field quantum correlations dynamics using two-mode under closed cavity system. The system contains two two-level atoms and two-mode entangled coherent fields. Thus, the interaction between the system and the environment is considered, and the system is in closed cavity which there is no interaction between environment and cavity system. Under this conditions, the quantum correlation between two mode of entangled coherent fields were quantified through quantum correlation function of non-Hermitian operators. Furthermore, we use numerical simulations to verify the evolution quantum correlation between field-field, and discuss the influences of initial different kinds of atoms. We show that the entangled atoms can significantly enhance the field-field quantum correlations dynamics. Specifically, we examine, we study the transfer of quantum correlations between field-field of two-mode cavities system through the medium photon hopping process. Such quantum system serves as a promising platform for the determination of the quantum correlations between the mechanical and optical modes in quantum systems, and a fundamental resource for several quantum information protocols.

Keywords: Quantum Correlations, Entangled Coherent State, Two-Mode Cavities, Quantum Correlation Measurement, Continuous Variable System (CV)

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I. INTRODUCTION

Quantum mechanics is the most accurate and complete description of the world known. It is the precious scientific gifts of the last century and the theoretical background of an impressive range of applications and peculiar phenomena in complex systems. It is also the basis for an understanding of quantum computation and quantum information[1] and has an enormous technological impact. It offers a powerful method of encoding and manipulating information that is not possible within a classical framework[2].

Such quantum feature of correlations not only is the key to our understanding of quantum world, but also is essential for the powerful applications of quantum information and quantum computation[3]. In order to characterize the correlation in quantum state, many approaches have been proposed to reveal different aspects of quantum correlations, such as the various measures of entanglement and the various measures of discord and related measures[4]. It is believed that some aspects of quantum correlations could still exist without the presence of entanglement and these aspects could be revealed via local measurements with respect to some basis of a local system[5].

Both quantum correlations and entanglement are special types of quantum coherence[6]. In quantum information processing (QIP) both quantum correlation and entanglement are one of the basic properties of quantum optics and information that differentiates it from classical mechanics[7]. Quantum entanglement originated from nonlocal quantum correlation, is basic in quantum physics both for understanding the non-locality of quantum mechanics[8]. It plays an important role and generate much interest in QIP domain[9].

In quantum optical physics, quantum correlation and entanglement hold significant positions in quest for knowledge as power of quantum mechanics[10]. Since quantum correlations has rarely been investigated before, so many people take it for granted that quantum entanglement is quantum correlations[11]. However, recently, some studies have displayed that entanglement is not the only type of measure of quantum correlation because there exist other types of non-classical correlations which are not captured by entanglement[12]. Subsequently, many peoples have drawn their attention to the definitions of quantum correlation and presented a variety of new computable measures such as quantum discord, geometric discord, measurement-induced disturbance[13].

Nowadays, the quantum correlation for two mode entangled coherent fields has been intensively studied in different researchers, because,they have wide practical application to QIP and it has been active and open research area in the quantum optical physics.[4,6,8,14].

Therefore, in a multipartite quantum system one can identify different kinds of quantum correlations such as, entanglement and genuine quantum correlations, each with characteristics useful as resources for quantum information processing. Currently, taking the advantage from properties like correlations, entanglement, non-locality and coherence, photons and atoms are the key elements for quantum technology applications. In particular, both theoretical and experimental research is thriving, and leading the way to new technological developments. This shows that the quantum correlation measurements between the two coherent field modes for continuous variable system is still active research area. Mainly, it is a crucial basic problem to characterize such complex of many degrees of freedom of continuous variable system[6,14]

With these considerations in mind, we theoretically study on the dynamics of quantum correlation of field-field interacting with two modes entangled coherent fields. Particulary, we consider two cases. In the first case, we consider when the two mode of entangled coherent fields (Cavity fields) initially interacts with the non-entangled atoms (separable ground states). While, in the second case, we consider both the cavity fields and the two atoms are initially in the entangled states. Thus, using these both situational cases, and tracing over the degree of freedom of the atomic states. Then, we utilize a quantum correlation measurement (δ')[14] by the means of photon correlation function. Consequently, we generating field-field quantum correlations dynamics. Specifically, we use numerical simulations to verify the evolution quantum correlation between field-field, and discuss the influences of initial for different initial atomic states. Furthermore, as a motivation of the paper the following questions is highlights;

- Is the entangled atoms enhance the field-field quantum correlations of the system?
- Is the separable atoms enhance the field-field quantum correlations of the system?
- How to measure the quantum correlation of field-field quantum correlations dynamics?

Therefore, we are interested to measure the dynamics of quantum correlation for two modes entangled coherent fields resonantly coupled to two-mode cavities by using a δ' measurement.

The paper is organized as follows. In Sec. II, we illustrate the physical model that describes the system. In Sec. III, we describe the quantum correlations for two mode of entangled coherent fields, while in Sec. IV, we generating field-field quantum correlations by utilizing the different initial atomic states. In Sec. V, we explicitly discuss the quantum correlations of the field-field under different initial atomic states. The conclusion is summarized in Sec. VI.

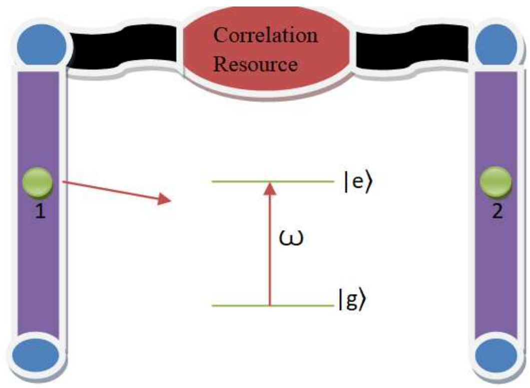

FIG. 1. Schematic model of two-level atoms interacting with two modes entangled coherent fields and coupled to two cavities under closed system.

II. MODEL AND HAMILTONIAN

In this section, we propose a quantum system containing two-level atoms, atom 1 with upper level excited state $\mid e_1\rangle$ and lower level ground state $\mid g_1\rangle$ and atom 2 with upper level excited state $\mid e_2\rangle$ and lower level ground state $\mid g_2\rangle$ which have frequencies of ω_1 and ω_2 couple with two closed cavities, contains two mode of entangled coherent fields with frequency of Ω_1 and Ω_2 initially prepare in coherent states. In this case, the interaction between the system and the environment is considered and the system is in closed cavity which implies that there is no interaction between environment and cavity system, as shown in Fig. (1). Specifically, we investigate the dynamics of quantum correlation for two modes entangled coherent fields resonantly coupled to two-mode cavities by using the quantum correlation measurement of δ'.

Such quantum system are used to capture the behaviour of quantum correlations between the two mode of entangled coherent fields. Recently, thus quantum system has been proposed with two continuous variable subsystems, two mode of entangled coherent fields have been most widely studied in Ref.[6,8,14,15]. Furthermore, these entangled continuous systems have desirable features which can be exploited in quantum teleportation, quantum communication, and theses two-mode coherent fields can be entangled by allowing them to interact with a two-level atom in a cavity have been investigated[4].

Therefore, the quantum correlation between two entangled coherent fields interact with two atoms in coupled closed cavities, as shown in Fig. (1), with the Hamiltonian of the system can be expressed as[6]

$$\hat{H} = \hat{H}_a + \hat{H}_f + \hat{H}_i, \tag{1}$$

this can be written as the form

$$\hat{H} = \frac{1}{2}\left\{\omega_1 \widehat{\sigma_1}^z + \omega_2 \widehat{\sigma_2}^z\right\} + \Omega_1 \hat{a}_1^\dagger \hat{a}_1 + \Omega_2 \hat{a}_2^\dagger \hat{a}_2 + G_1\{\widehat{\sigma}_1^\dagger \hat{a}_1 + \widehat{\sigma}_1^- \hat{a}_1^\dagger\} + G_2\{\widehat{\sigma}_2^\dagger \hat{a}_2 + \widehat{\sigma}_2^- \hat{a}_2^\dagger\}, \tag{2}$$

where, describes the Hamiltonian of atom is $\hat{H}_a = \frac{1}{2}\left\{\omega_1 \widehat{\sigma_1}^z + \omega_2 \widehat{\sigma_2}^z\right\}$, the Hamiltonian of the field $\hat{H}_f = \Omega_1 \hat{a}_1^\dagger \hat{a}_1 + \Omega_2 \hat{a}_2^\dagger \hat{a}_2$, is the Hamiltonian atom-field interactions $\hat{H}_i = G_1\{\widehat{\sigma}_1^\dagger \hat{a}_1 + \widehat{\sigma}_1^- \hat{a}_1^\dagger\} + G_2\{\widehat{\sigma}_2^\dagger \hat{a}_2 + \widehat{\sigma}_2^- \hat{a}_2^\dagger\}$, $\widehat{\sigma}_1^\dagger = |e_1\rangle\langle g_1|$ and $\widehat{\sigma}_1^- = |g_1\rangle\langle e_1|$ are the rising and lowering operators for the first atomic state and $\widehat{\sigma}_2^\dagger = |e_2\rangle\langle g_2|$ and $\widehat{\sigma}_2^- = |g_2\rangle\langle e_2|$ are the rising and lowering operators for the second atomic state , $\widehat{\sigma}_1^z =| e_1\rangle \mid g_1\rangle$ and $\widehat{\sigma}_2^z =| e_2\rangle \mid g_2\rangle$ represents the usual diagonal Pauli matrix that is two by two matrix for the first and second atoms respectively. While, the $\hat{a}_1^\dagger$ and $\hat{a}_1$ are the creation and annihilation operators for the first field and $\hat{a}_2^\dagger$ and $\hat{a}_2$ are the creation and annihilation operators for the second field respectively, G_1 and G_2 are atom-cavity coupling strengeth constant for atom 1 and atom 2, ω_i and Ω_i are frequency of atom and two mode of entangled coherent field respectively. For the system is resonant case $\omega_i = \Omega_i$ and the coupling strength of two atoms with cavities are the same (Symmetric atom cavities coupling strength with $G_1 = G_2 = G$).

III. FORMULATION OF TWO-MODE ENTANGLED COHERENT FIELDS

Quantum correlation exhibits better robustness than entanglement. This may be helpful for QIP. We analyze the degradation, creation, revival and enhancement of quantum correlation for different initial states and entangled or non entangled atoms. In the recent time, many different theoretical researchers have been proposed to dynamics and improvement of quantum correlations, and correlation criteria such as Hilery, Zubairy[16] Simon, Duan criterion that is prerequisite to judge whether or not quantum correlation exists in the systems. However, most of the criteria cannot tell us how much quantum correlations exist in the system.

Recently, Yang et.al[14] propose quantum correlation measurement by using photon correlations function for quantum correlation of CV systems. It is a flexible and simple measurement used to measure the quantum correlation of CV systems, which fits for most CV systems and also a convenient measurement for quantum correlation dynamic of two-mode CV systems.The normal measurements of CV systems namely quantum correlation measurement δ' observe and judge whether or not quantum correlation exists in the two-mode CV systems.

To describe the dynamics of quantum correlations for two mode of entangled coherent fields of the system more intuitively, we introduce the coherent fields that are initially defined in non separable coherent states and have frequency Ω_j, and an expansion in terms of two-mode Fock-States as[5].

$$| \quad \Psi_f\rangle = e\,\frac{-\mid \alpha \mid^2}{2}\frac{-\mid \beta \mid^2}{2}\sum_{mn=0}^{\infty}\frac{\alpha^n\beta^m}{\sqrt{n!m!}} \mid n,m\rangle, \tag{3}$$

this can be written as compact form

$$\mid \Psi_f\rangle = \sum_{nm}^{\infty} P_{nm} \mid n,m\rangle. \tag{4}$$

Here P_{nm} represents the probability of field in number state, and there is no orthogonality between two coherent states $\mid n,m\rangle$. The time evolution of operator of the total Hamiltonian is given as $U(t) = e^{-iHt}$, and there are many ways to solve the solution of Hamiltonian describing the system, but in this paper, we consider the dressed atomic basis state[17–19], which is given by the superposition of the uncoupled bare states

$$\begin{cases} |+,n\rangle = \cos\theta_n|e,n\rangle + \sin\theta_n|g,n+1\rangle \\ |-,n\rangle = -\sin\theta_n|e,n\rangle + \cos\theta_n|g,n+1\rangle, \end{cases} \tag{5}$$

where

$$\begin{aligned}\cos\theta_n =&\frac{R_n-\delta}{\sqrt{(R_n-\delta)^2+4g_i^2(n+1)}},\\ \sin\theta_n =&\frac{2g_i\sqrt{n+1}}{\sqrt{(R_n-\delta)^2+4g_i^2(n+1)}},\end{aligned} \tag{6}$$

and states $|e,n\rangle$ and $|g,n+1\rangle$ are often called the bare states, while the eigenstates $|\pm,n\rangle$ are the dressed-atom states.The energy eigenvalues are,

$$E_{n\pm}=\Omega_i(n+\frac{1}{2})\pm\frac{R_n}{2}, \tag{7}$$

we have introduced the quantized generalized Rabi-flopping frequency $R_n=\sqrt{\delta^2+4g_i^2(n+1)}$ and $\delta=\omega_i-\Omega_i$, is the detuning of the radiation frequency from the atomic resonance.

Therefore, to explain our idea more clearly about the measure of quantum correlation between the two mode of entangled coherent fields, we use QC measurement (δ') which is defined as the correlation function of two non Hermitian operators. Because it is a flexible and simple measurement used to measure the QC of CVS, which fits and the convenient measurement of the quantum correlation dynamic of two-mode CVS[6,14].

$$\delta'=|\ \langle\hat{a}_1\hat{a}_2\ \rangle-\langle\hat{a}_1\rangle\langle\hat{a}_2\ \rangle\ | \tag{8}$$

This QC of the field-field can be a non Hermitian extension of the second-order correlation function that is also used to determine the effective QC of the field-field for the mesoscopic quantum system.

IV. INITIAL STATES AND FIELD-FIELD QUANTUM CORRELATIONS

To clearly explain field-field quantum correlations more and measure quantum correlation between the subsystems, we utilize quantum correlation measurement, which is defined as the correlation function of two non-Hermitian operators. So, let consider a two-mode coherent state, by which we describe the two cavity fields with frequency Ω_1 and Ω_2. The two-mode coherent state defined as

$$| \Psi f\rangle = N\{| \alpha\rangle \mid \beta\rangle - \mid -\alpha\rangle \mid -\beta\rangle\} \tag{9}$$

The quantum correlation for the two-mode coherent state is given by $\delta' =| \alpha || \beta |$ which is non-zero; this indicates that there is QC between the two mode fields. In order to confirm the QC exist between the two-mode entangled coherent fields before interact with non entangled two level of atoms we can utilize Eq. (8) and it can be determine by taking the expectation values of each non Hermitian operators of fields. i.e, we first the formulate the expectation value for $\hat{a}_1$ as $\langle \hat{a}_1\rangle = \langle \Psi_f \mid \hat{a}_1 \mid \Psi_f\rangle$. By substituting Eq. (9), and after straightforward calculation, we obtained

$$\langle \hat{a}_1\rangle = \alpha \frac{[(\langle \alpha \mid \langle \beta \mid -\langle -\alpha \mid \langle -\beta \mid) \ (\mid \alpha \ \rangle_1 \mid \beta \ \rangle_2 + \mid -\alpha \ \rangle_1 \mid -\beta \ \rangle_2)]}{[(\langle \alpha \mid \langle \beta \mid -\langle -\alpha \mid \langle -\beta \mid) \ (\mid \alpha \ \rangle_1 \mid \beta \ \rangle_2 + \mid -\alpha \ \rangle_1 \mid -\beta \ \rangle_2)]}. \tag{10}$$

Similarly, the expectation value for $\hat{a}_2$ can be formulate as $\langle \hat{a}_2\rangle = \langle \Psi_f \mid \hat{a}_2 \mid \Psi_f\rangle$, after some step of calculation, we obtained as

$$\langle \hat{a}_2\rangle = \beta \frac{[(\langle \alpha \mid \langle \beta \mid -\langle -\alpha \mid \langle -\beta \mid) \ (\mid \alpha \ \rangle_1 \mid \beta \ \rangle_2 + \mid -\alpha \ \rangle_1 \mid -\beta \ \rangle_2)]}{[(\langle \alpha \mid \langle \beta \mid -\langle -\alpha \mid \langle -\beta \mid) \ (\mid \alpha \ \rangle_1 \mid \beta \ \rangle_2 + \mid -\alpha \ \rangle_1 \mid -\beta \ \rangle_2)]} \tag{11}$$

Finally, the expectation value for $\langle \hat{a}_1\hat{a}_2\rangle$ is also determine in similar ways as $\langle \hat{a}_1\hat{a}_2\rangle = \langle \Psi_f \mid \hat{a}_1\hat{a}_2 \mid \Psi_f\rangle$, after some step of calculation, we find

$$\langle \hat{a}_1\hat{a}_2\rangle = N^2 \left((\langle \alpha \mid \langle \beta \mid -\langle -\alpha \mid \langle -\beta \mid)\hat{a}_1\hat{a}_2(\mid \alpha \ \rangle_1 \mid \beta \ \rangle_2 - \mid -\alpha \ \rangle_1 \mid -\beta \ \rangle_2)\right), \tag{12}$$

by substituting Eqs. (10), (11) and (12) into the Eq. (8) the quantum correlation measurement can be determine as,

$$\delta' =| \alpha\beta || 1 - A^2(\alpha\beta) |, \tag{13}$$

where A is a function of α and β, which give by

$$A = \frac{[(\langle \alpha \mid \langle \beta \mid -\langle -\alpha \mid \langle -\beta \mid) \ (\mid \alpha \ \rangle_1 \mid \beta \ \rangle_2 + \mid -\alpha \ \rangle_1 \mid -\beta \ \rangle_2)]}{[(\langle \alpha \mid \langle \beta \mid -\langle -\alpha \mid \langle -\beta \mid) \ (\mid \alpha \ \rangle_1 \mid \beta \ \rangle_2 + \mid -\alpha \ \rangle_1 \mid -\beta \ \rangle_2)]} \tag{14}$$

Moreover, if we consider the two mode of entangled coherent fields within their frequency Ω_1 and Ω_2 are given as complex form

$$\mid \Psi_f\rangle = N\left\{\mid \alpha\rangle_1 \mid -\beta\rangle_2 + e^{-i\Phi} \mid -\alpha\rangle_1 \mid \beta\rangle_2\right\}. \tag{15}$$

Then, the QC between the two-mode complex number form of coherent states are expressed by using the same procedures as in Eq. (13), and we obtained

$$\delta' =\mid \alpha\beta\left\{1-\left[\frac{-\langle -\alpha \mid\mid \alpha\rangle_1\langle \beta \mid\mid -\beta\rangle_2 \sin\Phi}{1+\langle -\alpha \mid\mid \alpha\rangle_1\langle \beta \mid\mid -\beta\rangle_2 \cos\Phi}\right]^2\right\}\mid. \tag{16}$$

This can be written as compact form

$$\delta' =\mid \alpha\beta\left\{1-\left[\frac{-e^{-2|\alpha|^2}e^{-2|\beta|^2}\sin\Phi}{1+e^{-2|\alpha|^2}e^{-2|\beta|^2}\cos\Phi}\right]^2\right\}\mid \tag{17}$$

Therefore, we have confirmed that QC between two mode of entangled coherent fields in a coupled cavities before interacting with the two level atoms as shown in Eq. (13) and Eq. (16) or Eq. (17), is a non zero value. Since, the QC measurement (δ') determines only a real value QC for all equations , weather the two-mode entangled coherent fields are complex or real form.

A. Atoms Initially in Ground States

Here, we consider the case, when the cavity fields are initially entangled while the two atoms are in the separable ground state. Initially the two atoms-cavity have no QC between them. i.e., the atoms-cavity joint state is the product state of the two-level atoms initially prepared in the separable ground state of: $\mid \Psi_g\rangle =\mid g\rangle_1 \mid g\rangle_2$ and the field in the coherent states associated with two modes of entangled coherent field is given in Eq. (9). The initial state of the two level atoms at the ground state and two mode of entangled coherent fields are evolve in the following. That is, if the two separable atoms at the ground state interacts with two mode of entangled coherent fields, then the initial state of the two level atoms and two mode of entangled coherent fields (the initial state $\mid \Psi_{in}\rangle$ of the combined atom-field interaction) of the system can be written as:

$$| \Psi_{in}\rangle =| \Psi_g\rangle \mid \Psi_f\rangle. \tag{18}$$

Thus, the two mode of entangled state as the initial condition for the bipartite atomic system with n and m Fock states in cavities. The system is in the dynamics then the stimulated emission process involves during atom-field interaction and both atoms are demoted from the excited state to ground state in the presence of n and m photons in the some resonant mode. Then the atoms are in the ground state and the two entangled coherent fields are left with $n+1$ and $m+1$ photons[20]. Therefore, the initial state of atom-field interaction described with the creation of one photon of fields are

$$| \Psi_{in}\rangle =\sum_{nm} A_n B_m \mid g, n+1\rangle_1 \mid g, m+1\rangle_2 - \sum_{nm} A'_n B'_m \mid g, n+1\rangle_1 \mid g, m+1\rangle_2 \tag{19}$$

According to the general information of the system of atom field interaction. Thus, the initial state of the two non entangled atoms and two mode of entangled coherent fields interaction at the ground state is

$$\begin{aligned} | \Psi_{in}\rangle =&\sum_{nm} A_n B_m[(\sin\theta_n \mid +, n\rangle_1 + \cos\theta_n \mid -, n\rangle_1) \times (\sin\theta_m \mid +, m\rangle_2 + \cos\theta_2 \mid -, m\rangle_2)] \\ &-\sum_{nm} A'_n B'_m \ [(\sin\theta_n \mid +, n\rangle_1 + \cos\theta_n \mid -, n\rangle_1) \times (\sin\theta_m \mid +, m\rangle_2 + \cos\theta_2 \mid -, m\rangle_2)] \end{aligned} \tag{20}$$

The wave function of the system) $| \Psi(t)\rangle$ at $t \geq 0$ can be expressed as

$$| \Psi\,(t)\rangle = U\,(t) \mid \Psi_{in}\rangle. \tag{21}$$

So, the time dependent wave function of system is given in Eq. (21) and it can obtained from Eq. (20) as simplified form

$$
\begin{aligned}
\mid \Psi(t)\rangle =& \sum_{nm} [\alpha_{nm}\delta_{nm} \mid e,n\rangle_1 \mid e,m\rangle_2 + \beta_{nm}\gamma_{nm} \mid g,n+1\rangle_1 \mid g,m+1\rangle_2 \\
&+\alpha_{nm}\gamma_{nm} \mid e,n\rangle_1 \mid g,m+1\rangle_2 + \beta_{nm}\delta_{nm} \mid g,n+1\rangle_1 \mid g,m\rangle_2] \\
&-\sum_{nm} [\alpha'_{nm}\delta'_{nm} \mid e,n\rangle_1 \mid e,m\rangle_2 + \beta'_{nm}\gamma'_{nm} \mid g,n+1\rangle_1 \mid g,m+1\rangle_2 \\
&+\alpha'_{nm}\gamma'_{nm} \mid e,n\rangle_1 \mid g,m+1\rangle_2 + \beta'_{nm}\delta'_{nm} \mid g,n+1\rangle_1 \mid g,m\rangle_2]
\end{aligned} \tag{22}
$$

The density matrix of the system of Eq. (22) tells us all information about the system that can be defined as

$$\rho(t) = \mid \Psi(t)\rangle\langle\Psi(t) \mid \tag{23}$$

Taking the partial traces for computing reduced density matrices, or related functions, is a ubiquitous procedure in the quantum mechanics of composite systems. Therefore, we can use partial trace (reduced density matrix) to obtained one of them from the system. This describes the subsystem of information about atoms or fields. Accordingly, the reduced density matrix ρ_f of two mode of coherent fields can be obtained by tracing over the variable of the atomic part of the system

$$\rho_f(t) = tr_{atom}(u(t) \mid \Psi(t)\rangle\langle\Psi(t) \mid u(t)^{\dagger}). \tag{24}$$

This becomes

$$\rho_f\ (t) = \langle g \mid_1 \langle g \mid_2 \mid \Psi(t)\rangle\langle\Psi(t) \mid\mid g\rangle_1 \mid g\rangle_2 \tag{25}$$

The reduced density matrix of the two mode of entangled coherent fields from the system can be obtained and through some step of calculations and by substituting Eq. (22). In the field-field Fock states of bases $\mid n\rangle_1 \mid m\rangle_2, \mid n+1\rangle_1 \mid m+1\rangle_2, \mid n\rangle_1 \mid m+1\rangle_2$ and $\mid n+1\rangle_1 \mid m\rangle_2$ the reduced density matrix of the sub system composed of the two mode of entangled coherent fields are

$$\rho_f(t) = \sum_{nm} [| \alpha_{nm} |^2 | \delta_{nm} |^2 | n\rangle_1 | m\rangle_2 \langle n |_1 \langle m |_2 + | \beta_{nm} |^2 | \gamma_{nm} |^2 | n+1\rangle_1 | m+1\rangle_2 \langle n+1 |_1 \langle m+1 |_2$$
$$+ | \alpha_{nm} |^2 | \gamma_{nm} |^2 | n\rangle_1 | m+1\rangle_2 \langle n |_1 \langle m+1 |_2 + | \beta_{nm} |^2 | \delta_{nm} |^2 | n+1\rangle_1 | m\rangle_2 \langle n+1 |_1 \langle m |_2]$$
$$- \sum_{nm} [| \alpha'_{nm} |^2 | \delta'_{nm} |^2 | n\rangle_1 | m\rangle_2 \langle n |_1 \langle m |_2 + | \beta'_{nm} |^2 | \gamma'_{nm} |^2 | n+1\rangle_1 | m+1\rangle_2 \langle n+1 |_1 \langle m+1 |_2$$
$$+ | \alpha'_{nm} |^2 | \gamma'_{nm} |^2 | n\rangle_1 | m+1\rangle_2 \langle n |_1 \langle m+1 |_2 + | \beta'_{nm} |^2 | \delta'_{nm} |^2 | n+1\rangle_1 | m\rangle_2 \langle n+1 |_1 \langle m |_2], \quad (26)$$

where α_{nm},β_{nm},δ_{nm},γ_{nm},α'_{nm}, β'_{nm},δ'_{nm},γ'_{nm} are the probability amplitudes of the transition system can be obtained as

$$\begin{aligned}
\alpha_{nm} &= M[e^{-i\omega_{+n}t} - e^{-i\omega_{-n}t}] \cos\theta_n \sin\theta_n, \\
\beta_{nm} &= M[e^{-i\omega_{+n}t} \sin^2\theta_n - e^{-i\omega_{-n}t} \cos^2\theta_n], \\
\delta_{nm} &= M[e^{-i\omega_{+m}t} - e^{-i\omega_{-m}t}] \cos\theta_m \sin\theta_m, \\
\gamma_{nm} &= M[e^{-i\omega_{+m}t} \sin^2\theta_m - e^{-i\omega_{-m}t} \cos^2\theta_m], \\
\alpha'_{nm} &= M'[e^{-i\omega_{+n}t} - e^{-i\omega_{-n}t}] \cos\theta_n \sin\theta_n, \\
\beta'_{nm} &= M'[e^{-i\omega_{+n}t} \sin^2\theta_n - e^{-i\omega_{-n}t} \cos^2\theta_n], \\
\delta'_{nm} &= M'[e^{-i\omega_{+m}t} - e^{-i\omega_{-m}t}] \cos\theta_m \sin\theta_m, \\
\gamma'_{nm} &= M'[e^{-i\omega_{+m}t} \sin^2\theta_m - e^{-i\omega_{-m}t} \cos^2\theta_m],
\end{aligned} \quad (27)$$

where $A_n B_m = M$ and $A'_n B'_n = M'$ represents for the normalization constant of the two entangled coherent fields, which are found as

$$\begin{aligned}
A_n B_m = M &= e^{\frac{-|\alpha|^2}{2}} \frac{\alpha^n}{\sqrt{n!}} e^{\frac{-|\beta|^2}{2}} \frac{\beta^m}{\sqrt{m!}}, \\
A'_n B'_m = M' &= e^{\frac{-|\alpha|^2}{2}} \frac{(-\alpha)^n}{\sqrt{n!}} e^{\frac{-|\beta|^2}{2}} \frac{(-\beta)^m}{\sqrt{m!}}.
\end{aligned} \quad (28)$$

Moreover, the QC measurement at the ground (QC of the two mode entangled coherent fields interacts with separable two atoms at the ground state) can be obtained by using Eqs. (8) and (26) and numerical simulations with illustrate in Fig. (2).

B. Atoms Initially in Excited States

In second case, we consider both the cavity fields and two atoms are initially in the entangled states. Thus, the two-level atoms initially prepared in the non-separable states (entangled states) can be described by $\mid \Psi_e\rangle = \mid e\rangle_1 \mid e\rangle_2$ and the field in the coherent states associated with two modes of entangled coherent field given in Eq. (9). The initial state of the the two level atoms at excited state and the two mode of entangled coherent fields interaction. Therefore, when the two entangled atoms at excited state interacts with two modes of entangled coherent fields, and the initial state of the two level atoms and two mode of entangled coherent fields. For such the initial state $\mid \Psi_{in}\rangle$ of the combined atom-field system can be obtained as:

$$\mid \Psi_e\rangle = \mid e\rangle_1 \mid e\rangle_2\rangle N\{\mid \alpha\rangle \mid \beta\rangle - \mid -\alpha\rangle \mid -\beta\rangle\} \tag{29}$$

Then after, some step of calculation by using the general information of the atom field interaction of the dressed state, the inial state of the atom field interaction system at the excited state is

$$\begin{aligned} \mid \Psi_{in}\rangle = & \sum_{nm} A_n B_m \left[\cos\theta_n \mid +, n\rangle_1 - \sin\theta_n \mid -, n\rangle_1\right] \times \left[\cos\theta_m \mid +, m\rangle_2 - \sin\theta_m \mid -, m\rangle_2\right] \\ & - \sum_{nm} A'_n B'_m \left[\cos\theta_n \mid +, n\rangle_1 - \sin\theta_n \mid -, n\rangle_1\right] \times \left[\cos\theta_m \mid +, m\rangle_2 - \sin\theta_m \mid -, m\rangle_2\right] \end{aligned} \tag{30}$$

Furthermore, the time dependent wave function of system is given in Eq. (21). By using the same procedures, the reduced density matrix (partial trace) ρ_f of two mode of coherent fields can be obtained by tracing out the atomic part as

$$\rho_f(t) = \langle e \mid_1 \langle e \mid_2 \mid \Psi(t)\rangle\langle\Psi(t) \mid\mid e\rangle_1 \mid e\rangle_2 \tag{31}$$

the reduced density matrix ρ_f of two mode of entangled coherent fields can be obtained after some step of calculation as,

$$\begin{aligned}
\rho_f(t) =&\sum_{nm}[\mid a_{nm}(t) \mid^2 \mid c_{nm}(t) \mid^2 \mid n\rangle_1 \mid m\rangle_2 \langle n \mid_1 \langle m \mid_2 \\
&+ \mid b_{nm}(t) \mid^2 \mid d_{nm}(t) \mid^2 \mid n+1\rangle_1 \mid m+1\rangle_2 \langle n+1 \mid_1 \langle m+1 \mid_2 \\
&+ \mid a_{nm}(t) \mid^2 \mid d_{nm}(t) \mid^2 \mid n\rangle_1 \mid m+1\rangle_2 \langle n \mid_1 \langle m+1 \mid_2 \\
&+ \mid b_{nm}(t) \mid^2 \mid c_{nm}(t) \mid^2 \mid n+1\rangle_1 \mid m\rangle_2 \langle n+1 \mid_1 \langle m \mid_2] \\
&-\sum_{nm}[\mid a'_{nm}(t) \mid^2 \mid c'_{nm}(t) \mid^2 \mid n\rangle_1 \mid m\rangle_2 \langle n \mid_1 \langle m \mid_2 \\
&+ \mid b'_{nm}(t) \mid^2 \mid d'_{nm}(t) \mid^2 \mid n+1\rangle_1 \mid m+1\rangle_2 \langle n+1 \mid_1 \langle m+1 \mid_2 \\
&+ \mid a'_{nm}(t) \mid^2 \mid d'_{nm}(t) \mid^2 \mid n\rangle_1 \mid m+1\rangle_2 \langle n \mid_1 \langle m+1 \mid_2 \\
&+ \mid b'_{nm}(t) \mid^2 \mid c'_{nm}(t) \mid^2 \mid n+1\rangle_1 \mid m\rangle_2 \langle n+1 \mid_1 \langle m \mid_2],
\end{aligned} \tag{32}$$

where the $a_{nm}(t)$, $b_{nm}(t)$, $c_{nm}(t)$, $d_{nm}(t)$, $a'_{nm}(t)$,$b'_{nm}(t)$, $c'_{nm}(t)$ and $d'_{nm}(t)$ are the probability amplitudes of the transition system, and given in

$$\begin{aligned}
a_{nm}(t) &= M\left[f_{+n}(t)\cos^2\theta_n + f_{-n}(t)\sin^2\theta_n\right], \\
b_{nm}(t) &= M\left[f_{+n}(t) - f_{-n}(t)\ \right]\cos\theta_n\sin\theta_n, \\
c_{nm}(t) &= M\left[f_{+m}(t)\cos^2\theta_m + f_{-m}(t)\sin^2\theta_m\right], \\
d_{nm}(t) &= M\left[f_{+m}(t) - f_{-m}(t)\ \right]\cos\theta_m\sin\theta_m\ , \\
a'_{nm}(t) &= M'\left[f_{+n}(t)\cos^2\theta_n + f_{-n}(t)\sin^2\theta_n\right], \\
b'_{nm}(t) &= M'\left[f_{+n}(t) - f_{-n}(t)\ \right]\cos\theta_n\sin\theta_n, \\
c'_{nm}(t) &= M'\left[f_{+m}(t)\cos^2\theta_m + f_{-m}(t)\sin^2\theta_m\right], \\
d'_{nm}(t) &= M'\left[f_{+m}(t) - f_{-m}(t)\ \right]\cos\theta_m\sin\theta_m,
\end{aligned} \tag{33}$$

where $f_{+n}(t) = e^{-i\omega_{+n}t}$ and $f_{-n}(t) = e^{-i\omega_{-n}t}$ represents the probability amplitudes of the transaction system at the excited states. Furthermore, the quantum correlation measurement of the two mode entangled coherent fields interacts with non separable two atoms at the excited states can be determine by using Eq. (8) and numerical simulations with illustrate in Fig. (3).

V. RESULTS AND DISCUSSION

In this section, we discuss the dynamics of field-field quantum correlations in two-mode cavities under different effects of atomic states. We have provided a very different approach to quantify quantum correlation in a bipartite quantum state. Moreover, our approach captures the essential feature of quantum correlation of field-field. Thus, by adopting realistic situation that can be realized experimentally. Parameters we use for our numerical calculations are taken from[21,22]:

Therefore, to explicitly illustrate the behaviour of the quantum correlations between the two mode of entangled coherent fields, we use the relevant physical parameters of the two mode of entangled coherent fields and the coupling strength within their constant value in all the simulations. i.e, the two mode of entangled coherent fields. For the sake of simplicity, ($\alpha = \beta = 0.01$) and the coupling strength of the two atoms with cavities are ($g_1 = g_2 = 0.1$). Using these physical parameters, we investigated the effect of the initial state on the dynamical behaviour of the QCs between the two modes of entangled coherent fields displayed in illustration figures below. These physical parameters have been reported in the generation of macroscopic entangled coherent states with large Josephson junctions[22]. Accordingly, we have investigated the dynamical behaviour of the correlations between the field-field modes under closed cavity system, which is displayed in Figs. (2)–(5).

In Fig. (2), we plot the dynamical evolution of quantum correlation of field-field as a function of time T for the two level atoms initially prepared in the non entangled ground states interacting with the two mode of the entangled coherent fields. From this Fig. (2), we observe that the evolution of quantum correlation of field-field shows periodic behavior during the time-evolution. Moreover, the evolution of the quantum correlation of field-field shows decreasing from the initial QC of the two mode of entangled coherent fields before interacting with separable atoms at the ground state. There is periodic behavior during the time-evolution as shown in Fig. (2). In this case, the maximum amount of quantum correlation of field-field is smaller than the initial value of the quantum correlation of field-field.

This implies, the initially separated atoms cannot enhance the dynamics of quantum correlations of the fields, because there is no entanglement between each initial atomic states. Therefore, the separable ground states of the atoms cannot enhance the correlations

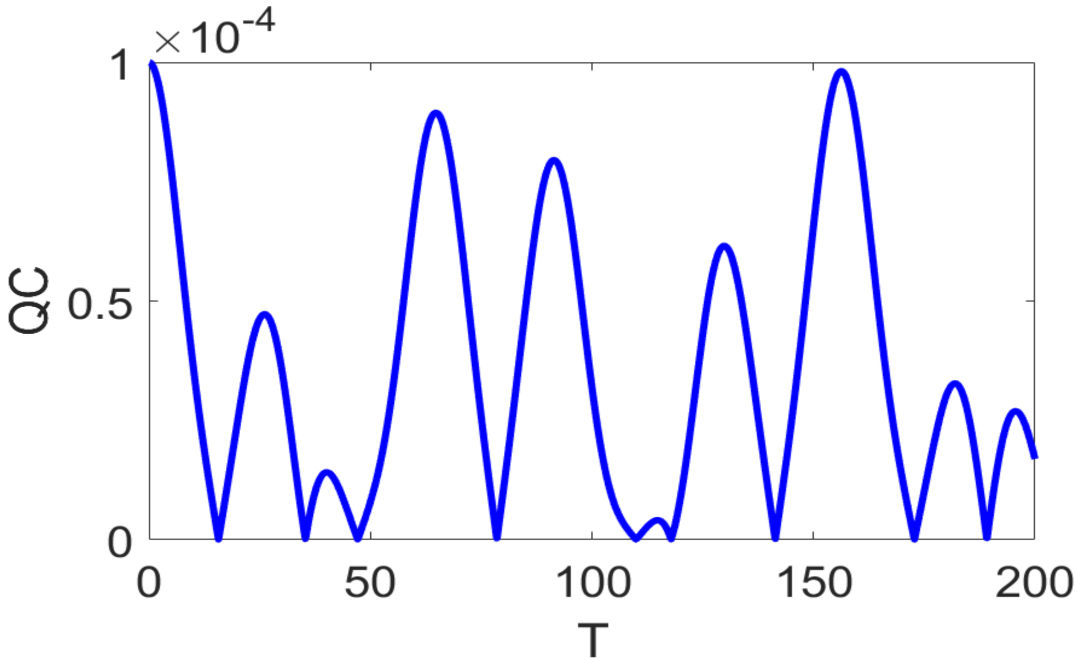

FIG. 2. The dynamical evolution of QC for the field-field as a function of time T when the two-level atoms are in the separable ground states.

between the field modes. Moreover, from the Fig. (2), we can confirm that the value of quantum correlation of field-field is not equal to zero even if the two atoms are separable. Specifically, we observe that, the evolution of the QC of field-field shows decreasing from the initial QC of the two mode of entangled coherent fields before interacting with separable atoms at the ground state. Our result have constance with some related work on quantum correlation dynamics of two separated continuous variable systems[6]

Secondly, we discuss the dynamical evolution of QC of fields-fields between the two mode of entangled coherent fields interacts with initially two level atoms at the excited states. We plot the dynamical evolution of QC of the field-field correlations as a function of time T as shown Fig. (3) for the two level atoms are in the entangled excited states interacting with the two mode of the coherent fields. This figure shows that dynamics of QC behaviour between the two entangled coherent field modes as a function of time T within their symmetric physical parameters of the two fields and coupling strength of the atoms.

Furthermore, from this figure, we observe that the evolution of the QC measurement shows gradually increasing from the initial QC of the two mode of entangled coherent fields before interacting with two non separable atoms at the excited state. It shows that periodic behavior, with similar to the ground state as shown in Fig (2) during the time-evolution.

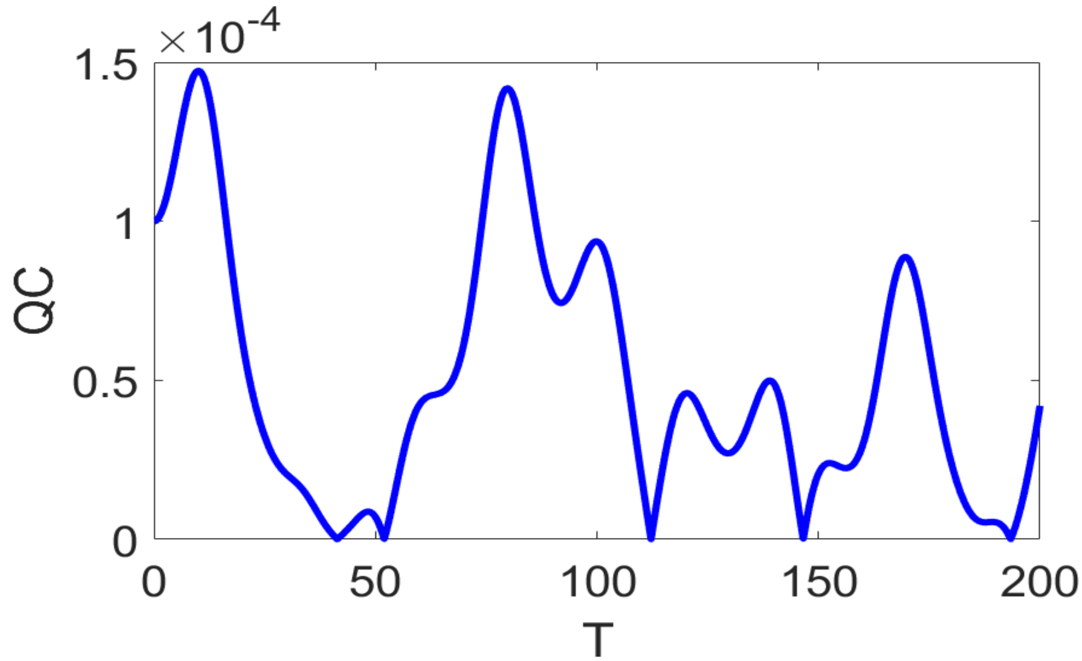

FIG. 3. The dynamical evolution of QC for the field as a function of time T when the two-level atoms are in the non separable excited state.

Moreover, from this figure we observed that the creation and annihilation of the two fcok states for two mode of entangled coherent fields during the time evolution, and it is related with the Eq. (32). The quantum correlation of the field-field at excited state is enhanced and grater than initial quantum correlation because the atoms are under entangled states interact with the second coherent states, it becomes increased. Therefore, when the atoms are in non separable states (entangle states), it becomes enhanced the quantum correlation of field-field under this conditions.

From this figure, we shows that the dynamical evolution of the QC as a function of time T and the transition frequency Ω. Accordingly, we denote the experimental parameter representing the relationship between the transition frequency Ω of atoms and the coupling strength g. Particularly, we have demonstrated that the two-level atoms are in the separable ground state, while the cavities are initially maximal entangled. Based on the intuitive observations Fig. (4), we have observe that at different initial state of atoms and transition frequency Ω the dynamics of quantum correlations of the fields is exits with different non-zero values. The maximum value of QC which is non-zero value correlations, and in some particular points there is a sudden death and sudden birth phenomena.

Such dependence significantly influence the evolution of correlation in quantum systems,

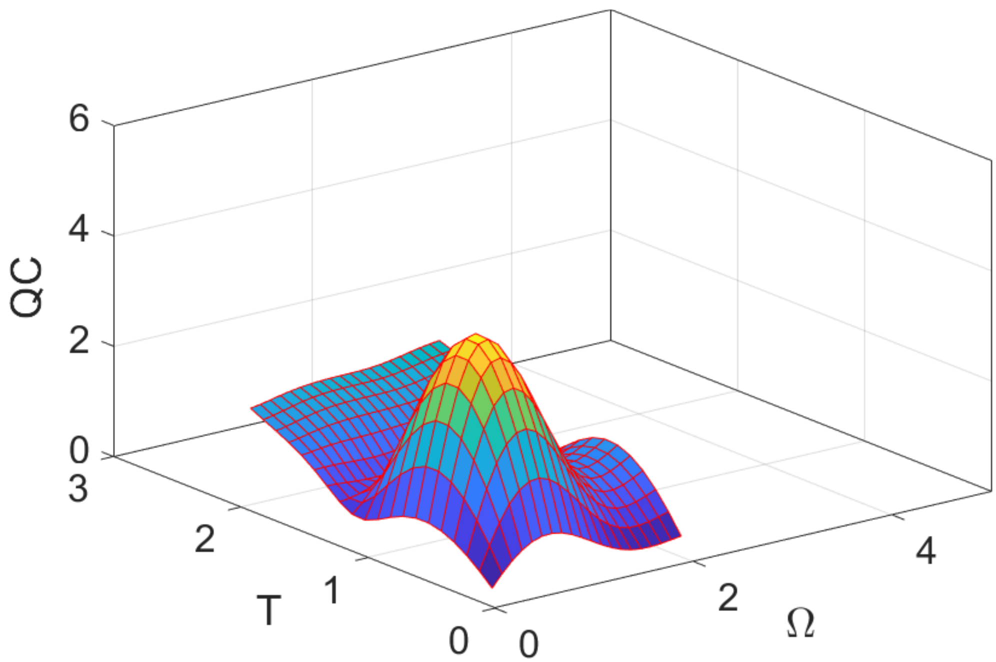

FIG. 4. The dynamical evolution of QC for the field-field as a function of time T and the transition frequency Ω. When the two-level atoms are in the non-separable state.

but still there is a correlation in the quantum system. Therefore, initially separated atoms cannot enhance the correlations of the fields larger than the initial values, because the atoms are separable atoms and no entanglement between each initial atomic states. Finally, we can say that the choice of the different initial state of atoms significantly influence the dynamics of quantum correlations behaviour between the field modes shown Fig. (4).

From the above Fig. (5) shows that the excited state occupation probability versus time at resonance ($\delta = 0$). Specifically, the plot displays a quantum Rabi oscillation signal for: an atom in excited level is sent across the cavity initially in vacuum. Mainly, by considering, the states of atom-cavity interaction times, we construct the probability (Pe) to detect, and finally the atoms in excited state . Such the damping of the oscillation is due to field damping, combined with various two modes of the coherent field. Moreover, the points show qualitative behavior of spontaneous absorption, and displays as an exact Rabi treatment of two-level atom interacting with a two modes of the coherent field for which the coherent Rabi oscillation can be interrupted by applying the coherent field across the mirrors, resulting and forms Rabi pulses.

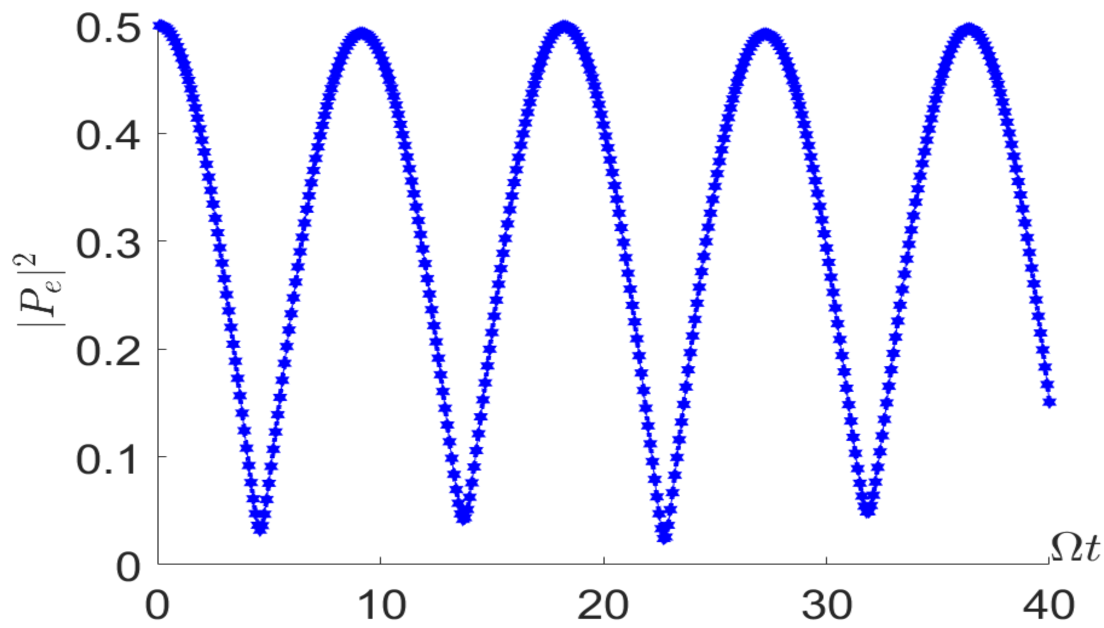

FIG. 5. The dynamical evolution of QC for the excited state occupation probability versus time at resonance.

VI. CONCLUSION

We have investigated the dynamics of quantum correlations between atoms and two mode coupled cavities using quantum correlation measurement. The system contains two non entangled atoms and two mode of entangled coherent fields. Specifically, the time evolutions of QC of field-field are investigated for both ground and excited initial states. We have shown that the two initial state of atoms can significantly affects the dynamics of quantum correlation of the two entangled coherent fields. Moreover, there is no enhancement of the QC of field-field in the ground state and it is less than the initial QC of field-field, but in the case of excited states there is an enhancement of the QC of field-field with grater than initial QC of field-field, because the atoms are non separable states. Therefore, when the atoms are in non separable states, it becomes enhanced the QC of field-field. Thus, QC measurement is flexible measurement to measure the time evolution of quantum correlation of field-field. It fits for most continuous variable systems and also a convenient measurement for quantum correlation dynamics of two-mode continuous variable systems. Therefore, we believe that our results provide a realistic route toward an optomechanical quantum correlation and a building block for the state preparation of QIP.

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