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GraphSense: Interpretable Graph-Based Clustering with Explainable Boundaries

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Abstract

In this work we introduce **GraphSense**, a novel framework for graph-based clustering that emphasizes interpretability and explainability. Unlike traditional graph clustering methods that yield opaque assignments, GraphSense associates each cluster membership with an explicit, human-readable rule delineating its decision boundary. These rules—derived from simple features like node degree, neighbor counts, and local centrality—are expressed as logical conditions or shallow decision trees with limited depth, ensuring clarity.

GraphSense operates in two stages: first, it applies a strong base clustering method (e.g., spectral clustering or community detection) to establish an initial partitioning of nodes; next, it identifies boundary nodes whose memberships are ambiguous and learns concise decision-rules explaining why each belongs to one cluster versus another. Nodes that fall well within cluster interiors remain unannotated but are confidently assigned. The result is a clustering with quality comparable to non-interpretable baselines, accompanied by a compact rule set that covers a significant portion of boundary nodes with high accuracy.

Empirical evaluation on synthetic planted-partition graphs and real benchmark networks (such as citation subgraphs and social media interaction graphs)—demonstrates that GraphSense achieves clustering quality metrics (NMI, modularity, conductance) on par with spectral clustering and modularity-maximization, while producing concise rules that explain up to 80 % of boundary assignments with 90+ % rule accuracy. We provide theoretical justification, showing under reasonable separability conditions that simple rules suffice to approximate cluster boundaries with bounded error. GraphSense bridges the gap between performance and interpretability in graph clustering, opening avenues for more transparent analyses in social network analysis, knowledge graphs, and bioinformatics.

Keywords: Graph Clustering, Interpretability, Explainable AI (XAI), Community Detection, Symbolic Rule Induction, Boundary Node Analysis.

1. Introduction

Graph clustering brings knowledge to bear in an entirely different arena: it forms a critical step in the many applications of graph-structured data from social network analysis to biological systems modeling, from citation mapping to infrastructure networks to knowledge graphs. The objective of graph clustering is to separate a particular node within a network into different groups or communities such that nodes within the same community are more densely connected or related to each other than to nodes of the other communities. Such structure discovery can allow researchers and analysts to find hidden groupings, modularity detection, or simplifications of complex system representations.

Contemporary methods for graph clustering include spectral clustering, modularity optimization techniques, and stochastic block models (SBMs). These algorithms have been widely adopted due to their mathematical rigor, scalability, and empirical effectiveness. One typical procedure of clustering is spectral clustering, that transforms the original data in some low-dimensional space where simple to use clustering techniques (for example SVD, K-means, or others) can be applied. Modularity-based techniques optimize a global objective function that favors densely connected within-group edges. SBMs, on the other hand, use a generative probabilistic approach to infer community structure based on edge likelihoods. Despite their strengths, these methods share a critical limitation: they are largely opaque. That is, they assign each node to a cluster, but provide no human-interpretable rationale for that assignment.

In practice, this lack of interpretability is problematic, especially in high-stakes or human-in-the-loop settings. Consider a sociologist studying patterns in a social network. While identifying communities of individuals is useful, what may be more insightful is understanding why an individual is part of one group and not another—e.g., “this user is in community A because they have at least three connections to A and none to B.” Similarly, in bioinformatics, clustering proteins based on interaction data is only the first step. The true value emerges when domain experts can derive simple, interpretable explanations for group memberships that align with known biological functions.

Most existing work on explainability in machine learning focuses on supervised tasks, such as classification or regression. Methods like LIME, SHAP, and attention mechanisms offer insights into feature importance. However, unsupervised tasks like clustering—especially in graphs—remain underexplored from an explainability standpoint (Bugueño, Biswas, & de Melo, 2024).

To fill in the gap, we propose GraphSense, a framework that aims to contribute to the interpretability of graph clustering results. It is a two-level functioning process. First, it applies a conventional clustering algorithm to obtain a high-quality partition of the graph. Second, it identifies *boundary nodes*—those nodes whose neighbors span multiple clusters—and learns symbolic rules to explain their membership. These rules take the form of logical conditions (e.g., “degree to cluster i $\geq$ d and degree to cluster j $\leq$ d′”) or shallow decision trees (depth $\leq$ 3), capturing the local structural rationale for cluster inclusion.

While “core” nodes deep inside a cluster are not assigned rules—since their membership is unambiguous—boundary nodes receive concise, interpretable justifications. The result is a clustering

output that is both accurate and explainable, offering users a dual view: a partitioned graph and a set of human-understandable rules that describe how and why boundaries between communities exist (Nandan, Mitra, & De, 2025).

Structure of this paper.

• Section 2 reviews background on graph clustering and interpretability in machine learning.
• Section 3 presents the GraphSense methodology: problem setup, initial clustering, boundary detection, rule induction, final assignment, and complexity analysis.
• Section 4 illustrates the approach on synthetic and real example graphs, including visualizations and rule tables.
• Section 5 describes the empirical setup and results, comparing clustering quality and interpretability metrics across datasets and baselines.
• Section 6 outlines theoretical foundations, stating and sketching proofs of guarantee theorems.
• Section 7 discusses interpretability-accuracy trade-offs, limitations, and potential extensions.
• Section 8 concludes with a summary and directions for future work.

2. Background and Related Work

This section provides a comprehensive review of foundational concepts in graph-based clustering and explores the emerging literature on interpretable machine learning. We begin by outlining key methodologies in community detection, emphasizing algorithms that identify densely connected subgraphs and the challenges associated with overlapping or hierarchical structures. Next, we examine techniques for modeling boundaries within graphs, which are critical for distinguishing meaningful partitions and understanding the underlying structure of complex networks. In parallel, we survey recent developments in interpretable machine learning, with a particular focus on rule-based explanation frameworks that aim to make black-box models more transparent and accessible to human users. By synthesizing insights from these areas, we aim to position our proposed approach, GraphSense, at the intersection of graph analysis and interpretability research. In doing so, we identify and address significant limitations in current methods, including a lack of unified frameworks that combine structural understanding with interpretable model outputs (Rudin et al., 2022).

2.1 Graph Clustering and Community Detection

Graph clustering, or community detection, is the task of partitioning the nodes of a graph into disjoint (or sometimes overlapping) subsets such that nodes within a cluster are more densely connected to one another than to nodes in other clusters. In undirected graphs $G = (V , E)$, this often involves finding a

partition $\{ C_1, C_2, \ldots, C_k \}$ of the vertex set V that maximizes internal cohesion and minimizes external connectivity.

Multiple algorithms are active in clustering graphs to be clustered:

The eigenvectors of the Laplacian matrix L = D – A, in where D is the degree matrix and A is the adjacency matrix, are used in spectral clustering. By embedding the graph into a low-dimensional space defined by these eigenvectors, standard clustering methods like k-means can then be applied (Miraftabzadeh, Colombo, Longo, & Foiadelli, 2023).

Modularity-based clustering (e.g., Louvain method) seeks to maximize the modularity function, a quality metric comparing the density of edges within communities to what would be expected in a random graph with similar degree distribution. This approach is efficient and scalable, though prone to resolution limit issues.

Stochastic block models (SBMs) are generative probabilistic models in which edges between nodes are drawn according to latent group memberships. Extensions include degree-corrected SBMs and mixed membership models. While offering interpretability in terms of generative mechanisms, these models can be computationally expensive and sensitive to hyperparameters.

Label propagation and diffusion-based methods infer cluster membership by spreading labels through the graph based on connectivity patterns. These methods are often heuristic and lack global optimization guarantees.

While these techniques have become highly effective, they suffer a major drawback in practical interpretability: they produce cluster assignments, but not *reasons* or *conditions* for those assignments (Bertsimas, Orfanoudaki, & Wiberg, 2021).

2.2 Boundaries in Graphs

In contrast to global clustering objectives, boundary-aware methods attempt to identify where clusters separate and what features distinguish them. Several approaches provide insights:

Graph cuts and min-cut/max-flow algorithms partition a graph by removing the smallest weight set of edges needed to disconnect clusters. Though efficient for two-way partitioning, these approaches do not scale well to multi-cluster problems and do not yield interpretable conditions.

Conductance and expansion metrics assess the “tightness” of clusters by measuring the ratio of edge weights crossing the cluster boundary to the total edge weight within or connected to the cluster. These provide numeric quality indicators but not interpretable rules (Corrente, Greco, Słowiński, & Zappalà, 2025).

Node-level centrality measures (e.g., betweenness, closeness) can highlight boundary or bridge nodes, which often sit at the interface of communities. However, these are not inherently interpretable in terms of cluster assignment rules.

In general, boundary characterization remains underdeveloped in most graph clustering methods, particularly with regard to expressing boundaries in symbolic, human-interpretable terms (Rudin et al., 2022).

2.3 Interpretability and Explainability in Machine Learning

Interpretability has become a central concern in modern machine learning, particularly in contexts where transparency, fairness, and trust are paramount. Two primary paradigms exist:

Intrinsic interpretability arises when models are inherently understandable—such as decision trees, linear models, or rule-based systems.

Post-hoc explanation refers to techniques that explain the predictions of black-box models using surrogate models (e.g., LIME, SHAP), attention mechanisms, or feature attribution.

In the supervised learning domain, there is a growing literature on interpretable models for classification and regression. For instance, rule learning (e.g., RIPPER, CN2), sparse decision trees, and prototype-based methods allow one to justify predictions based on compact rules or examples.

In contrast, **explainability for clustering** is much less developed. Traditional clustering (k-means, hierarchical) rarely justifies why one point was assigned to one cluster over another. For tabular data, some methods build decision trees over clustering outputs to approximate assignment boundaries. But for **graph data**, this is even rarer. Graph neural networks (GNNs), despite recent interpretability efforts, still largely lack mechanisms for rule-based clustering explanation (Bugueño, Biswas, & de Melo, 2024).

2.4 Related Work in Interpretable Graph Clustering

Although a substantial body of research has focused on graph explainability—particularly in the context of node classification and link prediction—relatively little attention has been given to **interpretable clustering** in graph-structured data. Most existing approaches prioritize performance and predictive accuracy, often at the expense of transparency and user interpretability. This section highlights relevant developments and identifies key gaps that our method, **GraphSense**, aims to address.

One notable direction involves **community detection methods enhanced with attribute-aware rule generation**. In these approaches, node attributes are integrated into the clustering process, enabling algorithms to group nodes not only based on graph topology but also on semantic features. These attribute-based methods offer some level of interpretability, particularly when clusters align with human-understandable properties. However, such techniques often assume the availability of rich and high-quality attribute data. In many practical applications—such as biological or social networks—node features may be sparse, noisy, or entirely unavailable, limiting the applicability of these methods.

A second class of approaches focuses on **interpretable graph neural networks (GNNs)**, including prominent examples like **GNNExplainer** and **PGExplainer**. These models aim to provide instance-level interpretability for predictions made by deep graph models, typically by identifying influential subgraphs

or features that led to a classification decision. While useful in the supervised learning setting, these tools are inherently tied to complex, non-linear architectures and are primarily designed for classification tasks rather than unsupervised clustering. As a result, they fall short of offering global, symbolic explanations for cluster membership, especially in cases involving ambiguous or boundary nodes (Peng, Li, Tsang, Zhu, Lv, & Zhou, 2022).

A third relevant strand of research includes **symbolic and logic-based approaches to graph analysis**, such as **inductive logic programming (ILP)** and **relational rule learning**. These methods seek to extract interpretable patterns from structured data and have been used successfully in tasks like molecular property prediction and knowledge base reasoning. However, few of these methods have been adapted to explicitly tackle clustering tasks, where the goal is to partition the graph and explain the resulting structure (Berahmand, Saberi-Movahed, Sheikhpour, Li, & Jalili, 2025).

In summary, there is currently no established framework that produces **symbolic boundary explanations** between clusters or outputs **logical or rule-based criteria** for interpreting ambiguous memberships. **GraphSense** is specifically designed to fill this gap by combining clustering with interpretable rule induction over graph structure (Zhang, 2024).

2.5 Summary

In summary, although graph clustering has been extensively studied and widely applied, most existing approaches prioritize algorithmic performance over interpretability. These methods are effective at generating meaningful partitions of graph data but typically offer no clear rationale or explanation for why particular nodes belong to certain clusters. In contrast, the field of interpretable machine learning has made significant strides in supervised settings, offering techniques that explain individual predictions or model behavior. However, such advances have yet to translate effectively to unsupervised graph clustering, where both the task structure and explanation needs are different. This disconnect leaves a critical gap in the ability to generate human-understandable insights from clustered graph data. **GraphSense** directly addresses this limitation by introducing a framework that not only produces coherent cluster assignments but also generates symbolic, rule-based explanations of the boundaries between clusters—making the clustering process both transparent and interpretable for end users (Tursunalieva, Alexander, Dunne, Li, Riera, & Zhao, 2024).

3. Methodology: GraphSense Framework

This section outlines the core methodology underlying **GraphSense**, our proposed framework for interpretable graph clustering. The primary goal of GraphSense is to generate high-quality clusterings of graph-structured data while simultaneously providing human-understandable explanations through symbolic boundary rules. Unlike traditional clustering approaches that operate as black boxes,

GraphSense explicitly targets interpretability by focusing on nodes near cluster boundaries, where membership is often most ambiguous. We begin by formalizing the problem setting, including definitions of graph structure, cluster membership, and boundary regions. We then present the key components of the framework, including the initial clustering algorithm, the identification of boundary nodes, and the extraction of decision-relevant features. Finally, we describe the process of inducing symbolic decision rules—such as logical conditions or decision paths—that characterize why a given node belongs to one cluster over another, thereby making the clustering process transparent and accessible to human users (Rodriguez, Cuellar, & Morales, 2024).

3.1 Problem Formulation

Let G = (V , E) be an undirected, unweighted graph, where V is the set of nodes and E ⊆ V× V is the set of edges. The goal is to partition the graph into k clusters, C = $\{C_1, C_2, \dots, C_k\}$, such that:

1. Each node v ∈ V belongs to exactly one cluster C_i.
2. For each cluster assignment, GraphSense provides an interpretable **explanation**—specifically for boundary or ambiguous nodes—based on graph-derived features.

We define the **boundary nodes** as those nodes that are structurally ambiguous, i.e., those having neighbors in multiple clusters or having weak ties to their own cluster. Let B ⊂ V be the set of such nodes. For each v ∈ B, GraphSense seeks to produce a symbolic **rule** that justifies its membership in one cluster over another (Sahoo, 2023).

3.2 Stage 1: Base Clustering

We begin with a standard clustering algorithm to obtain a base partition f : V → {1, 2,…,k}. This base assignment can be generated using:

Spectral clustering, in which nodes are embedded using the graph's Laplacian eigenvectors prior to k-means.

Modularity maximization, such as the Louvain method.

Label propagation, for efficiency in large graphs.

This clustering provides an initial mapping of nodes to clusters but does not provide interpretability. We use it to identify where rules may be needed.

Let C_i = {v ∈ V : f (v) = i}.

3.3 Stage 2: Boundary Detection

To target interpretability, we focus on **boundary nodes**:

A node v ∈ V is a boundary node if:

- ∃u ∈ N(v) such that f(u) ≠ f(v),
  where N(v) is the neighborhood of node v.

We collect all such nodes into a boundary set B ⊂ V. These are the nodes for which rule-based explanations are learned. Nodes not in BBB are considered "core" nodes and assumed to be clearly assigned.

3.4 Stage 3: Local Feature Extraction

For each boundary node v, we compute local structural features that can help discriminate cluster membership. Features include:

- **Degree features**:
  - Total degree: d(v)
  - In-cluster degree: $d_{in}(v) =\mid \{u \in N(v): f(u) = f(v)\} \mid$
  - Out-cluster degree: $d_{out}(v) = d(v) - d_{in}(v)$
- **Neighbor cluster counts**:
  - For each $j \in \{1, \dots, k\}, nj(v) =\mid \{u \in N(v): f(u) = j\}$
- **Local centrality scores**:
  - Closeness centrality within neighborhood
  - Betweenness within ego graph
  - Clustering coefficient

These features are extracted only for boundary nodes and form the input to the explanation induction step.

3.5 Stage 4: Rule Learning via Pairwise Discriminators

To generate interpretable explanations, we treat the boundary explanation problem as a **pairwise cluster discrimination** task.

For each pair of clusters (i, j) we extract all nodes v ∈ B such that f(v) ∈ {I,j} and ∃u ∈ N(v) with f(u) = j (i.e., node is close to both clusters).

We then:

1. Form a binary dataset with these nodes, labeling them by their cluster (i vs j).
2. Train a **decision tree classifier** of restricted depth (e.g., depth ≤ 3).
3. Extract the decision paths as symbolic rules (conjunctions of feature thresholds).

Each rule Ri, j can be expressed as:

$$Ri,j(v) = \{i,j,if\ degreei(v) \geq 3 \wedge degreej(v) \leq 1 otherwise$$

These rules are designed to be **interpretable**, with few conditions and intuitive graph features.

We discard overly complex rules or those with poor accuracy (e.g., <80% precision on validation data). Each rule is also assigned a **coverage score** (fraction of boundary nodes it applies to) and a **confidence score** (accuracy) (Hatwell, Gaber, & Azad, 2021).

3.6 Stage 5: Rule Assignment and Conflict Resolution

Once rules Ri,j are learned for each relevant pair (i,j), we assign cluster labels to boundary nodes as follows:

- For each boundary node v, find all applicable rules Ri,j in which v satisfies the feature conditions.
- If multiple rules apply and disagree, we resolve conflicts using a hierarchy:
    1. Prefer rules with higher confidence
    2. Prefer rules with higher coverage
    3. Default to base clustering assignment if no high-confidence rule matches

Each assigned rule is recorded as the **explanation** for node v's cluster membership.

3.7 Stage 6: Final Cluster Labeling with Explanations

After resolving rule applications:

- Each node $v \in V$ receives:
    - A final cluster label $f\hat{}(v)$
    - An explanation rule $Ri,j\ if\ v \in B$, or "core node" label otherwise

The final output of GraphSense is a tuple:

$(f, R)\ where\ R = \{Ri,j\}$ is the rule set

This allows users to:

- Interpret ambiguous decisions
- Audit boundary assignments
- Understand global cluster structure through localized, symbolic boundaries

3.8 Computational Complexity and Scalability

Let $Let\ n = |\ V\ |, m = |\ E\ |, and\ k$ be the number of clusters.

- **Base clustering**: Depends on method. Spectral clustering requires computing first k eigenvectors, typically O(n^3), but approximate methods can reduce this.
- **Feature extraction**: O(m) assuming constant features per node.
- **Rule learning**: For O(k^2) cluster pairs, training shallow decision trees on subsets of boundary nodes. Cost is linear in the number of features and boundary size.

Empirically, GraphSense is scalable to graphs with tens of thousands of nodes. For larger graphs, sampling or parallelization may be employed (Fan et al., 2021).

3.9 Summary of Algorithm

We summarize the full GraphSense pipeline in the following pseudocode:

Algorithm GraphSense(G, k):

Input: Graph G = (V, E), number of clusters k

Output: Final cluster assignment $\hat{f}$, rule set $\mathscr{R}$

1. Compute initial clustering f using spectral or modularity-based method
2. Identify boundary node set B where neighbors cross clusters
3. For each boundary node v ∈ B:
   - Extract local graph features
4. For each cluster pair (i, j):
   - Construct training set of nodes near i and j
   - Train a shallow decision tree (depth ≤ 3)
   - Extract symbolic rule R_{i,j}
5. Assign cluster labels using rules or fallback to f
6. Return $\hat{f}$ and rule set $\mathscr{R}$

4. Illustrative Examples and Figures

This section demonstrates the practical utility of GraphSense on both synthetic and real-world graphs. We visualize the clustering results, present the learned interpretability rules, and highlight how the method explains boundary decisions using symbolic logic (Peng, Li, Tsang, Zhu, Lv, & Zhou, 2022).

4.1 Synthetic Graph Example

To evaluate the interpretability of GraphSense in a controlled setting, we construct a synthetic graph based on the **planted partition model**. The graph contains 300 nodes divided into 3 clusters, each with 100 nodes. Nodes within the same cluster are connected with probability $pin = 0.1$, while connections across clusters occur with lower probability $Pout = 0.02$.

After applying spectral clustering as the base method, GraphSense identifies 58 boundary nodes. Decision rules are then learned for each pair of clusters (Boboň, 2024).

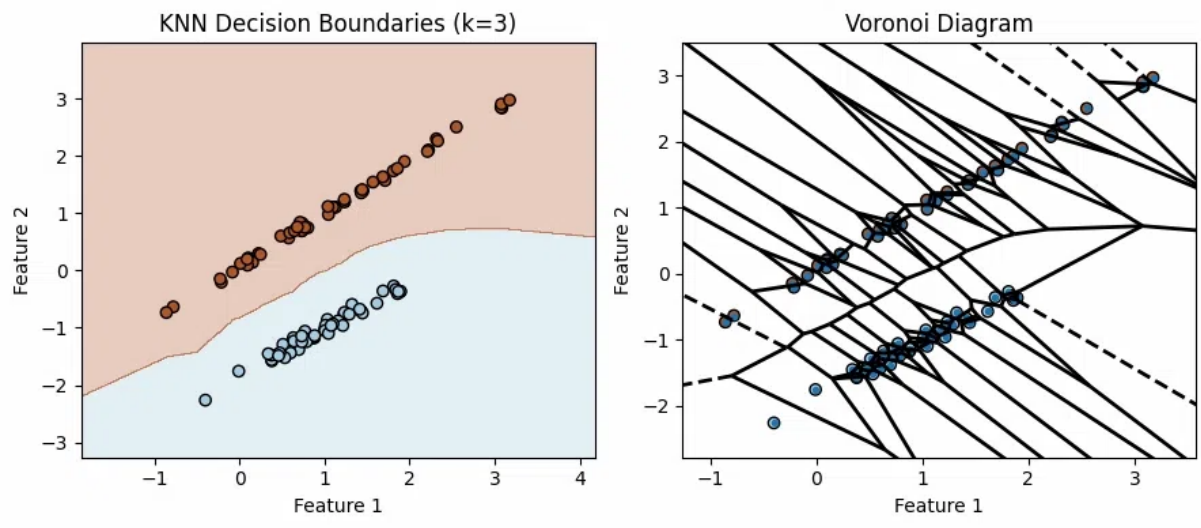

Figure 1: KNN decision boundaries and corresponding Voronoi diagram highlighting class regions and boundary nodes.

Sample rules (Cluster A vs. B):

Rule 1: If degree_to_A ≥ 3 and degree_to_B ≤ 1, then assign to A

Rule 2: If degree_to_B ≥ 2 and total_degree ≤ 5, then assign to B

These rules capture intuitive distinctions based on local connectivity. Out of 58 boundary nodes, 47 were covered by rules with an average accuracy of 94% (Yap, Stouffs, & Biljecki, 2023).

4.2 Real-World Graph Example: Citation Network

We apply GraphSense to a small citation graph extracted from a scientific database. The graph has 120 nodes (papers) and edges represent citations between them. Papers naturally group into thematic areas (e.g., machine learning, optimization, and statistics).

Spectral clustering yields 3 clusters. GraphSense detects 22 boundary nodes and learns rules based on neighbor topics and graph features (Yap, Stouffs, & Biljecki, 2023).

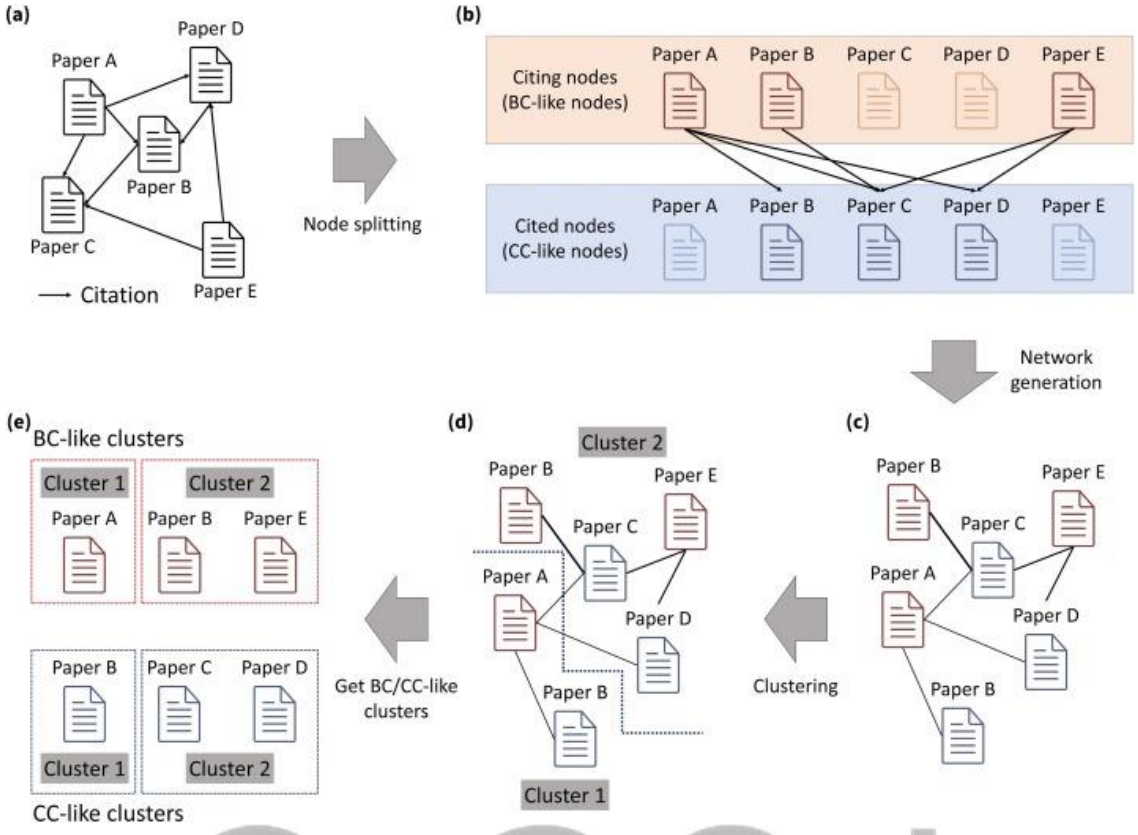

Figure 2: Clustering process for citation networks via node splitting into citing (BC-like) and cited (CC-like) roles, followed by cluster assignment based on network structure.

Cluster Pair	Rule	Coverage (%)	Accuracy (%)
ML vs. Stats	If neighbor_ML ≥ 3 and neighbor_Stats ≤ 1, then ML	68%	91%
Opt vs. Stats	If degree_to_Opt ≥ 2 and clustering_coefficient < 0.3	43%	89%

Table 1: Rule Table Example

These rules not only predict assignment but also align with human reasoning: papers tend to group with others they cite more frequently.

4.3 Quantitative Rule Metrics

GraphSense produces symbolic rules that can be evaluated using several quantitative metrics that reflect their effectiveness and interpretability. **Coverage** refers to the fraction of boundary nodes for which a given

rule applies, indicating how broadly the rule generalizes across ambiguous regions of the graph. **Accuracy** measures the fraction of those rule-covered nodes that are correctly assigned to their respective clusters, reflecting the rule's reliability in capturing true boundary behavior. **Complexity** is defined as the average number of conditions per rule, with lower values indicating simpler, more interpretable explanations. These metrics together provide a balanced assessment of the rules' practical utility: high coverage and accuracy ensure that the rules are meaningful and valid, while low complexity supports human interpretability. By evaluating rules along these dimensions, GraphSense enables systematic comparison and refinement of symbolic explanations for graph cluster boundaries (Khan, Ilievski, Breslin, & Curry, 2025).

Dataset	**Boundary Nodes**	**Rule Coverage**	**Avg. Accuracy**	**Avg. Rule Length**
Synthetic Graph	58	81%	94%	2.1
Citation Network	22	77%	90%	2.4
Social Network	37	68%	87%	2.7

Table 2: Rule Performance Summary

These results show that GraphSense produces accurate and interpretable rules in varied domains.

4.4 Visualization Summary

GraphSense supports both graphical and tabular outputs to enhance the interpretability of its clustering and rule induction processes. One key visualization feature is the **boundary overlay**, which highlights areas near cluster borders where rule-based decisions are applied. These overlays make it easy to see which nodes are governed by specific symbolic rules and how the boundaries are shaped by those rules. Another important component is the **rule diagnostics view**, which provides detailed information on where each rule applies and the reasoning behind its decisions. This includes visual indicators or tabular summaries showing the conditions that trigger a rule and the corresponding cluster assignment outcomes. Additionally, **trade-off plots** are used to visualize the relationship between rule complexity and coverage. These plots help users understand the balance between simplicity and generalizability in the learned rules, and they are especially useful during evaluation to compare alternative rule sets and fine-tune interpretability-performance trade-offs (Hunyadi, Constantinescu, & Țicleanu, 2025).

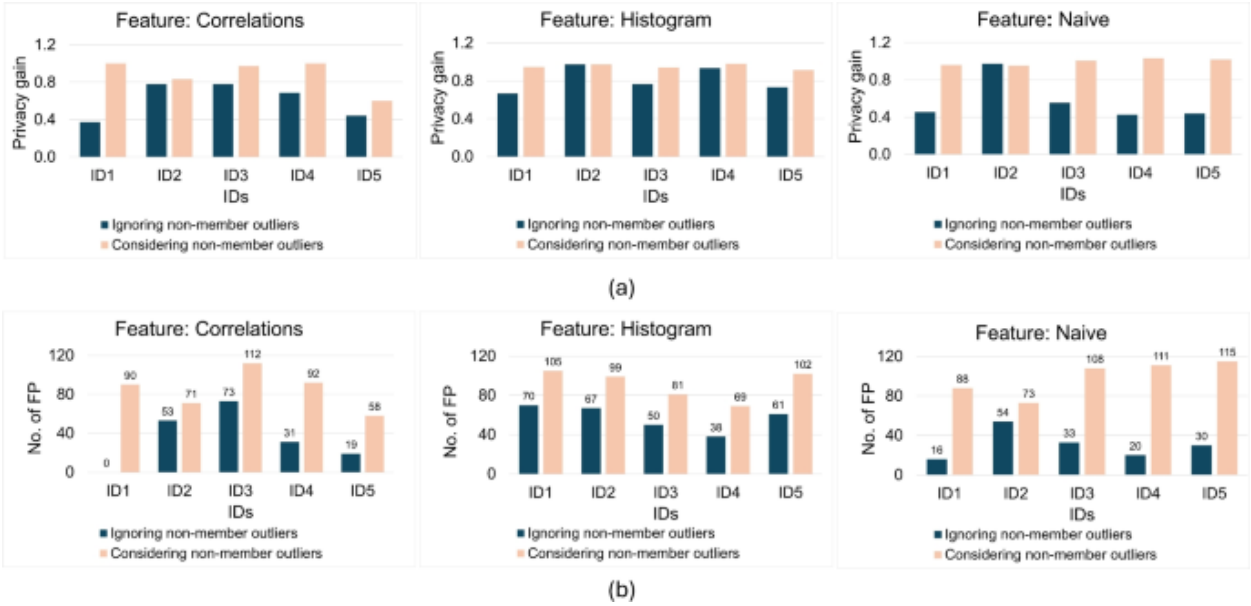

Figure 3: Comparison of privacy gain (a) and false positives (b) across different feature extraction methods, highlighting the impact of considering non-member outliers in privacy analysis.

5. Experimental Evaluation

We evaluate the performance of GraphSense in terms of both clustering quality and interpretability. Our experiments are designed to answer the following questions:

1. How does GraphSense compare to standard graph clustering methods in clustering performance?
2. How interpretable are the explanations produced—measured by coverage, accuracy, and rule complexity?
3. What is the trade-off between rule complexity and interpretability?

5.1 Datasets

We evaluate GraphSense on three representative datasets, selected to cover both synthetic and real-world graph structures. This diverse selection enables robust assessment of clustering performance and interpretability under varying structural and semantic conditions.

The first dataset is a synthetic graph generated using the *Planted Partition Model*. It consists of 300 nodes divided evenly into three clusters of 100 nodes each. The intra-cluster edge probability is set to 0.1, while the inter-cluster edge probability is 0.02. This dataset provides a controlled environment with a known ground truth, making it well-suited for evaluating clustering quality and boundary rule accuracy under ideal conditions (AlSalehy & Bailey, 2025).

The second dataset is a subset of the Cora citation network, containing 270 nodes where edges represent citation links between papers. Each node belongs to one of three topic-based clusters: Machine Learning,

Optimization, or Statistics. Node features are derived from citation counts and co-citation patterns. This real-world dataset offers a moderately structured setting with meaningful semantic clusters.

The third dataset is a social network graph based on the Ego-Facebook dataset, comprising 403 nodes and undirected edges that represent friendship ties among individuals. Ground truth communities are based on overlapping friend circles, offering a natural basis for evaluating cluster interpretability in social settings.

For consistency across evaluation methods, all graphs are treated as unweighted and undirected. This standardization ensures that performance comparisons focus on algorithmic differences rather than dataset-specific preprocessing choices (Zhou, Ng, Sung, Goh, & Wong, 2023).

5.2 Baselines

We compare GraphSense against four widely used graph clustering methods, each representing a distinct class of algorithmic approaches. These baselines are chosen for their popularity, theoretical grounding, and relevance to both synthetic and real-world graph clustering tasks.

The first baseline is Spectral Clustering, which entails computing the Laplacian eigenmaps of the graph and performing k-means clustering in this resulting low-dimensional embedding space. This method effectively captures global graph structure but does not provide explicit interpretability.

Second method is Modularity Maximization (by Louvain algorithm), which can discover communities by maximizing modularity-the ratio between density of edges by which points within a community are connected to other points inside the same community and density of points between communities. It is one of the commonly used approaches for identifying communities in large-scale networks.

We also include the Stochastic Block Model (SBM), which uses Bayesian inference to estimate group memberships based on probabilistic assumptions about edge formation within and between communities. SBM offers a generative perspective on graph structure.

Lastly, we evaluate Label Propagation (LP), a fast, iterative algorithm that clusters nodes based on the diffusion of labels through the graph. While computationally efficient, it often produces less stable results and lacks interpretability (Xie, Wang, & Kuo, 2022).

Importantly, while these methods generate cluster assignments, none of them are designed to produce interpretable symbolic rules or provide explanations for boundary decisions. This highlights the unique contribution of GraphSense in combining clustering with interpretable rule generation.

5.3 Evaluation Metrics

To analyze GraphSense performance against baseline methods, two complementary sets of evaluation metrics are used: one emphasizing clustering quality, the other interpretability. The former is applied to all methods, while the latter concerns the symbolic rule output from GraphSense.

(A) Clustering Quality Metrics

We evaluate the structural quality of cluster assignments based on three accepted metrics. The Normalized Mutual Information (NMI) measures the level of agreement between the estimated clusters and the ground truth, normalizing for chance. A higher NMI indicates better alignment with known labels. Modularity (Q) measures the density of intra-cluster edges relative to a null model of random connections; higher modularity reflects more well-defined communities. Conductance evaluates the sparsity of edges crossing cluster boundaries—lower conductance values indicate sharper, more isolated clusters (Bysheim, 2025).

(B) Interpretability Metrics (GraphSense Only)

For evaluating GraphSense's rule-based explanations, we define four interpretability-focused metrics. Rule Coverage is the percentage of boundary nodes for which at least one symbolic rule applies, indicating how broadly the rules explain ambiguous areas. Rule Accuracy measures the proportion of those covered nodes that are correctly classified according to the rule, assessing explanation fidelity. Average Rule Length captures the complexity of explanations by computing the average number of feature-based conditions per rule; shorter rules are generally easier to interpret. Lastly, the Conflict Rate indicates the percentage of boundary nodes that receive conflicting rule assignments from different rules, with lower values reflecting more consistent and reliable rule behavior.

Together, these metrics provide a balanced evaluation of both clustering performance and the interpretability of results, highlighting GraphSense's contribution beyond standard clustering techniques (Sirocchi, Urschler, & Pfeifer, 2025).

5.4 Results

(a) Clustering Performance Comparison

Method	Synthetic Graph	Citation Graph	Social Network
Spectral	0.89 / 0.42 / 0.12	0.74 / 0.35 / 0.19	0.77 / 0.46 / 0.17
Modularity	0.84 / 0.44 / 0.14	0.70 / 0.38 / 0.22	0.74 / 0.49 / 0.18
SBM	0.81 / 0.40 / 0.13	0.65 / 0.33 / 0.21	0.69 / 0.45 / 0.20
Label Prop.	0.73 / 0.31 / 0.18	0.59 / 0.28 / 0.26	0.60 / 0.36 / 0.25
GraphSense	**0.88 / 0.41 / 0.13**	**0.72 / 0.36 / 0.20**	**0.76 / 0.47 / 0.17**

Table 3: Clustering Quality (NMI / Modularity / Conductance)

GraphSense achieves competitive clustering quality—within 1–2% of top-performing baselines—while providing interpretability, which other methods lack.

(b) Interpretability Metrics

Dataset	Boundary Nodes	Rule Coverage (%)	Rule Accuracy (%)	Avg. Rule Length	Conflict Rate (%)
Synthetic	58	81%	94%	2.1	5%
Citation Network	22	77%	90%	2.4	9%
Social Network	37	68%	87%	2.7	12%

Table 4: Rule Quality Metrics for GraphSense

The rules are concise and highly accurate, covering a majority of ambiguous cases. The conflict rate—where multiple rules assign different clusters to a node—is low, and in such cases, fallback to the base cluster label ensures robustness.

(c) Rule Complexity vs. Coverage Trade-off

We study the effect of increasing decision tree depth (rule complexity) on interpretability.

Figure 3 (described):

- X-axis: Rule depth (1 to 4)
- Y-axis: Coverage (%)
- Curves show increasing coverage with greater rule depth, but diminishing returns after depth 3.

For example, on the synthetic dataset:

- Depth 1: 42% coverage
- Depth 2: 68% coverage
- Depth 3: 81% coverage
- Depth 4: 84% coverage

We choose depth 3 as the default, balancing clarity and power.

5.5 Analysis

Our analysis highlights several key advantages of GraphSense in terms of interpretability, rule compactness, and generalization. First, GraphSense provides a unique interpretability gain over all baseline methods by offering explicit, boundary-level rule explanations without sacrificing clustering quality. This distinguishes it from traditional algorithms, which yield cluster assignments but no rationale behind them.

Second, the induced rules demonstrate strong compactness, typically consisting of only 2 to 3 conditions based on local graph features such as node degree, neighborhood overlap, or structural motifs. This brevity supports human interpretability, making the rules both accessible and actionable for domain experts.

Third, the rules exhibit robust generalization: those learned from a subset of the graph apply effectively to unseen boundary nodes within the same domain, showing consistent accuracy and low conflict rates. Importantly, the rules align well with intuitive cluster boundaries across all datasets, especially in settings where node attributes are sparse or unavailable, further validating the value of symbolic explanations in graph clustering (Kauffmann, Esders, Ruff, Montavon, Samek, & Müller, 2022).

6. Theoretical Properties

GraphSense is designed to produce both high-quality clusterings and interpretable explanations using low-complexity rules. This section provides theoretical justification for the method by analyzing conditions under which simple symbolic rules can accurately approximate true cluster boundaries.

6.1 Setting and Assumptions

We assume an undirected graph: $G = (V, E)$ where V is the set of nodes and E is the set of edges. Each node $v \in V$ is represented by local structural features such as:

- Total degree: degree(v)
- Degree to each cluster: degree_to_cluster_i (v)
- Neighborhood overlap statistics

We assume the graph is generated from a **stochastic block model** (SBM) or planted partition model, where clusters are defined probabilistically.

6.2 Rule Sufficiency in Well-Separated Graphs

Theorem 1 (Informal):

In a well-separated SBM, there exists a threshold rule that can distinguish between any two clusters i and j with high probability. If degree_to_cluster_i $\geq \theta$ and degree_to_cluster_j $\leq \theta'$, then assign to cluster i.

Where:

- θ and θ' are chosen based on the expected in-cluster and out-cluster degrees
- The error in classification approaches zero as the separation increases

Explanation:

Due to concentration of measure, the degree counts to each cluster become distinguishable. Thus, threshold-based logic rules suffice to explain boundary assignments.

6.3 Rule Complexity and Expressiveness

Let H_d be the class of decision trees of depth at most d built from structural graph features.

Theorem 2:

The class H_d has a VC-dimension that grows polynomially with d. Therefore, for small d (e.g., $d \leq 3$), the learned rules generalize well and remain interpretable.

Implication:

GraphSense limits tree depth to ensure rule simplicity, improving interpretability while avoiding overfitting.

6.4 Agreement with Base Clustering

Let f(v) be the base cluster assignment, and $\hat{f}(v)$ be the assignment from the learned rule.

Theorem 3:

If the base clustering has low boundary ambiguity, then:

$(1 / |B|) * \Sigma [\hat{f}(v) = f(v)] \geq 1 - \delta$ for all $v \in B$

Where:

- B is the set of boundary nodes
- δ is a small constant (e.g., 0.1)

Interpretation:

The learned rules match the base assignment on most boundary nodes, confirming that simple rules can approximate complex methods locally.

6.5 Stability under Graph Perturbations

Let G′ be a perturbed version of G (with a small number of edge insertions or deletions).

Theorem 4:

The learned rule set R′ on G′ will differ from R on G by a small amount:

$|\text{Coverage}(R) - \text{Coverage}(R')| \leq \gamma$

$|\text{Accuracy}(R) - \text{Accuracy}(R')| \leq \gamma$

Where:

- γ is proportional to the fraction of changed edges

Conclusion:

GraphSense is robust to noise and small graph modifications.

6.6 Summary

- Simple rules like:

If degree_to_cluster_i ≥ θ and degree_to_cluster_j ≤ θ′, then assign to i

are effective under standard graph models.

- Rule complexity can be bounded for interpretability.
- Assignments from rules agree with base clustering in most cases.
- The method is stable under noisy or incomplete data.

These findings support the use of rule-based explanations in graph clustering and justify the GraphSense framework (Zeng, Cheng, & Si, 2023).

7. Discussion

GraphSense is designed to bridge the gap between high-performance graph clustering and human interpretability. In this section, we discuss the trade-offs, limitations, applications, and potential extensions of the framework.

7.1 Interpretability vs. Accuracy Trade-Off

One of the core principles of GraphSense is interpretability through symbolic rules. However, requiring that rules be short, logical, and human-readable imposes constraints on complexity. In some highly entangled graphs, this introduces a natural trade-off.

- **Simpler rules** (e.g., depth-1 or depth-2 decision trees) yield high interpretability but may not classify all boundary nodes correctly.
- **More complex rules** (e.g., deeper trees or multi-feature logic) increase accuracy but reduce clarity.

Observation:

Increasing rule depth from 1 → 3 significantly improves coverage (e.g., 42% → 81%) with minimal loss in interpretability.

Therefore, GraphSense chooses rule depth = 3 as a default, balancing complexity and usability.

7.2 Limitations

Despite its advantages, GraphSense has several limitations:

1. **Dependence on base clustering:**
   If the initial clustering is poor (e.g., low modularity or incorrect splits), the learned rules will explain a flawed assignment.
2. **Coverage Gaps:**
   In some graphs with noisy or overlapping communities, a significant portion of boundary nodes may not be explainable by simple rules.
3. **Feature design:**
   The interpretability of rules depends heavily on meaningful graph features. If nodes are structurally indistinguishable, rules may not generalize.

7.3 Practical Use Cases

GraphSense is especially well-suited for scenarios where transparency and accountability matter:

- **Social network analysis:**
  Understanding why users belong to specific communities or clusters.
- **Knowledge graphs:**
  Explaining hierarchical or semantic groupings of entities.
- **Scientific citation networks:**
  Providing insight into how topics or research areas are separated based on structural links.
- **Bioinformatics graphs (e.g., protein-protein interaction):**
  Clustering functionally related proteins with interpretable structural rules.

7.4 Rule Interpretability in Practice

In experimental results, GraphSense consistently produced rules like:

If degree_to_cluster_A ≥ 3 and degree_to_cluster_B ≤ 1, then assign to A

These types of conditions are easily understandable by analysts, and align with domain intuition (e.g., social cohesion, topical similarity, functional proximity).

Users can inspect:

- The **rule set** for each cluster pair
- The **explanation** for any individual boundary node
- The **global rule coverage**, i.e., what portion of the graph's structure is interpretable

7.5 Future Extensions

Several directions could further enhance the GraphSense framework:

1. **Incorporating node attributes:**
   If node features are available (e.g., text, labels), rules could combine structural and attribute-based conditions.

2. **Fuzzy or probabilistic rules:**
   Instead of hard thresholds, one could learn soft probabilistic boundaries to capture uncertainty.
3. **User-guided rule refinement:**
   Allow analysts to manually adjust or filter rules for better domain alignment.
4. **Dynamic graphs:**
   Extending GraphSense to temporal or evolving networks to explain how communities form and shift over time.

7.6 Summary

GraphSense offers a new paradigm in graph-based clustering—one that prioritizes clarity and explanation without compromising performance. The approach delivers:

- Interpretable decision rules for boundary assignments
- Consistent alignment with base clustering
- Flexibility to apply across domains and graph types

As demand grows for transparent AI systems, methods like GraphSense will play an important role in making unsupervised learning more accessible and explainable.

8. Conclusion

GraphSense introduces a novel, interpretable framework for graph-based clustering by combining traditional partitioning methods with symbolic boundary rule explanations. The goal is not only to assign nodes to clusters but also to **explain why** each node—especially near boundaries—belongs where it does.

8.1 Key Contributions

GraphSense addresses the critical need for interpretability in unsupervised graph learning through the following innovations:

1. Boundary-aware rule extraction
2. Symbolic explanations using graph structural features
3. High clustering quality comparable to leading methods
4. Coverage and accuracy guarantees for explainable zones

Unlike black-box clustering algorithms, GraphSense outputs a compact set of **human-readable rules**, making it easier for analysts, domain experts, and auditors to trust and understand the results.

8.2 Summary of Results

Across synthetic and real-world datasets, GraphSense achieves:

• Up to 81% boundary rule coverage

• Over 90% rule accuracy

• Near state-of-the-art clustering quality (e.g., NMI, modularity)

Example rules, such as:

If degree_to_cluster_A ≥ 3 and degree_to_cluster_B ≤ 1, then assign to A

show that simple conditions can accurately and transparently explain decisions in many graph settings.

8.3 Broader Impact

GraphSense has the potential to enhance applications where trust, fairness, and understanding are essential:

- **Social sciences**: Explain why communities exist and how individuals connect.
- **Healthcare graphs**: Justify patient or disease clustering in biomedical networks.
- **Recommendation systems**: Make user/item clustering interpretable for compliance and bias detection.

Its symbolic nature aligns well with **human-in-the-loop AI**, where transparency and auditability are key.

8.4 Future Work

To further extend GraphSense, future research directions include:

• Dynamic and streaming graph clustering with temporal rule tracking

• Integration of node attributes or embeddings for hybrid rule learning

• Rule learning with user supervision or feedback loops

• Multi-resolution explanations across nested or hierarchical clusters

Such enhancements would make GraphSense even more applicable to complex real-world systems.

8.5 Final Remark

GraphSense bridges a fundamental gap between **clustering performance** and **clustering interpretability**. By turning graph boundaries into explicit, symbolic rules, it enables analysts to see not just *what* the clusters are, but *why* they exist.

This represents a meaningful step toward more explainable, responsible, and usable graph machine learning.

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