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Graphing exercises for linear functions in A covariance-based algebra classroom [

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Introduction

Graphing is a fundamental topic in algebra that is notoriously difficult for students. Much of the past research has focused on conceptions and misconceptions. This study extends past research by looking at the mathematical practices of a practitioner, specifically one instructor of a function-based covariation-focused algebra class in the linear functions unit. Considering practices in addition to conception adds dramatically to our understanding of mathematical activity because it leads to explicit descriptions of normative purposes that are connected to particular situations or problems and also specifies how tools and symbols are coordinated to achieve these purposes. The results of this study are three levels of empirically proven practices associated with the conception of one advanced level of covariational reasoning, chunky continuous covariation. This study not only describes how practices may be described at different levels of complexity, but also demonstrates how smaller practices may be combined to form larger, more complex practices.

RATIONALE

Graphing on the Cartesian coordinate plane is a fundamental component of high school mathematics and specifically a central theme in algebra. In the 2010 Common Core State Standards for Mathematics (CCSSM) graphs and graphing are explicitly mentioned in 4 of the 8 standards for mathematical practice (NGA Center and CCSSO, 2010). For example, the first standard for mathematical practice is, "Make sense of problems and persevere in solving them" (NGA Center and CCSSO, 2010). In the explanation of this mathematical practice, authors of the CCSSM describe a mathematically proficient student as one who "can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends" (NGA Center and CCSSO, 2010). Explanations of other mathematical practices mention the ability to model with mathematics by mapping a relationship using graphs, use appropriate tools strategically by analyzing graphs of functions created with a graphing calculator, attend to precision while labeling axes, and look for and express regularity in repeated reasoning while repeating slope calculations.

While much research has been done on graphing, this research is likely insufficient. Past research on graphing has either focused on identifying the knowledge, skills, and understandings students should acquire in mathematics classes (e.g., Friel, Curcio, & Bright, 2001; Leinhardt, Zaslavsky, & Stein, 1990; Moschkovich, Schoenfeld, & Arcavi, 1993). This work, while useful for understanding cognitive processes related to graphing, does not take into account the social nature of learning and the situated nature of knowledge, and thus is likely insufficient for informing the teaching and learning of graph creation and use.

Knowledge, skills, and understandings are embedded in the practices in which they are learned (Brown, Collins, & Duguid, 1989). The knowledge, skills, and understandings needed to engage in a practice can vary depending on the type of graph, what information is presented on a graph, and what information needs to be obtained from the graph. For example, the understanding required to find the slope of a line in an algebra class is very different from the understanding required to examine a line in a calculus class and conclude that the derivative is a constant function. As a second example, students may be able to find the slope of a line in a distance/time graph of a person walking at a constant rate by identifying two points on a line and using the slope formula. Despite this knowledge, they may be unable to answer how fast the person was walking. However, students' knowledge of slope may be situated within the practice of finding the slope of a line on a graph rather than the practice of finding the rate of change between two quantities. The two contexts are different and require the student to think about slope in very different ways. (Brown et al., 1989).

The CCSSM focuses on graphing in the context of functions on a Cartesian coordinate plane (NGA Center and CCSSO, 2010). Kaput (1999) identified the study of functions, relations, and joint variation as one of the main strands of algebra and suggested that the concept of functions as the correspondence and variation of quantities should be taught in our schools. Thompson and Carlson (2017) continued by arguing that, "emerging conceptions of continuous covariation were central to the development of the mathematical idea of a function" (p. 422) and encouraged the reform of mathematics classrooms to make the study of functions from the perspective of the covariation of quantities a core component of the curricula. Because understanding functions in algebra is so foundational, the context of a functions-based covariation-focused algebra class is a good context in which to begin a study of graphing practices. To know more about graphing practices in which students should be proficient in the context of functions-based, covariation-focused algebra class, I examined what specific graphing practices a proficient member of a function-based, covariation-focused algebra class engages in.

Cognitive Perspectives

In this section, I discuss past research that has been done in graphing. I argue that focusing on knowledge and skills has both methodological and theoretical limitations. In contrast to a cognitive perspective, other researchers have researched graphing using a sociocultural perspective (e.g., Roth & Bowen, 2001). These researchers viewed graphing as a practice that teachers apprentice their students into. This allows us to think about knowledge situated in a particular context instead of generally. research on graphing was conducted using a cognitive focus, meaning that researchers attended to the knowledge, skills, and understandings students needed in order to create and use graphs. One contribution of this review consisted of an analysis of tasks involving functions, graphs, and graphing. In their analysis, Leinhardt et al. (1990) focused on four aspects of tasks: action, situation, variable, and focus. The first aspect of tasks consists of the actions a student should be able to perform when interpreting or constructing graphs: prediction, classification, translation, and scaling. The second aspect of tasks is situation, which refers to the mathematics class in which students encounter the task, or to whether the graph is abstract or depicts a real world situation. The third category of tasks is variable. The final category of task is focus. Focus refers to what aspect of a graph students should pay attention to when performing each task. Students are said to be proficient at graphing when they are able to perform the four actions of prediction, classification, translation, and scaling in tasks involving different situations and different uses of variables while being able to focus on different aspects of a graph.

A second contribution of Moschkovich et al. (1993) was to consider functions in three, not necessarily comprehensive, forms: algebraic, graphical, and tabular. They noticed that teachers expected that if students could translate a function from an algebraic equation to a graph, then they could also translate a function from a graph to an algebraic equation. One important finding to come out of this study was the recognition that students think about functions differently than teachers. While teachers are able to see equations, graphs, and tables as different representations of the same function, students do not naturally make this connection.

A third trend in early cognitive research on graphing was thinking about graphing in terms of literacy, which these researchers called graph comprehension (e.g., Curcio, 1987; Wainer, 1992; Friel et al., 2001). They identified three levels of graph comprehension: basic, intermediate, and advanced shown in Figure 1. A student who has acquired basic skills is able to read data (Curcio,

1987) or, as expressed by Wainer (1992), extract data from a graph. A student who has acquired skills at an intermediate level is

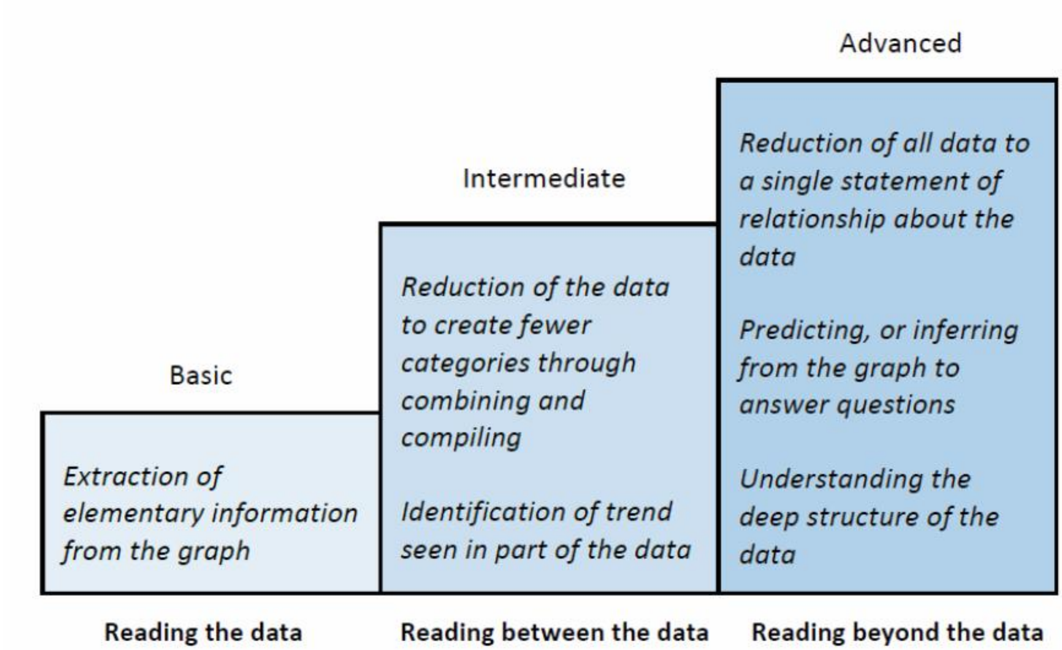

Figure 1. Three levels of graph comprehension.

able to read between the data of a graph (Curcio, 1987).

Covariation

One prominent function-based algebra curriculum is Pathways College Algebra (Carlson, 2016). This curriculum was designed from research on functions from a dynamic perspective called covariation (Carlson, Jacobs, Coe, Larsen, & Hue, 2002; Thompson & Carlson, 2017) as opposed to a static object view. A dynamic process view of functions is preferable because it requires someone to envision "quantities in their conceptualized situation as having values that varied" (Thompson & Carlson, 2017, p. 425). Conversely, having a static object view of functions is undesirable because it contributes to "[students'] inability to construct meaningful formulas to represent one quantity as a function of another" (Thompson & Carlson, 2017, p. 426). Carlson et al. In their research, Carlson et al. categorized students' ways of coordinating the change between two varying quantities into five distinct levels that emerge developmentally that they call the five mental actions of covariation, shown in Figure 2. For example, Mental Action 2 (MA2) requires coordinating the direction of change of one variable with changes in the other variable. Along with the mental actions, Carlson et al. (2002) described observable behaviors, also shown in Figure 2, that, when exhibited by students, students have the ability to reason about two varying quantities in a certain way. These observable behaviors are suggestive of some of the graphing practices participants might use as they engage in covariational reasoning.

Mental action	Description of mental action	Behaviors
Mental Action 1 (MA1)	Coordinating the value of one variable with changes in the other	• Labeling the axes with verbal indications of coordinating the two variables (e.g., *y* changes with changes in *x*)
Mental Action 2 (MA2)	Coordinating the direction of change of one variable with changes in the other variable	• Constructing an increasing straight line • Verbalizing an awareness of the direction of change of the output while considering changes in the input
Mental Action 3 (MA3)	Coordinating the amount of change of one variable with changes in the other variable	• Plotting points/constructing secant lines • Verbalizing an awareness of the amount of change of the output while considering changes in the input
Mental Action 4 (MA4)	Coordinating the average rate-of-change of the function with uniform increments of change in the input variable.	• Constructing contiguous secant lines for the domain • Verbalizing an awareness of the rate of change of the output (with respect to the input) while considering uniform increments of the input
Mental Action 5 (MA5)	Coordinating the instantaneous rate of change of the function with continuous changes in the independent variable for the entire domain of the function	• Constructing a smooth curve with clear indications of concavity changes • Verbalizing an awareness of the instantaneous changes in the rate of change for the entire domain of the function (direction of concavities and inflection points are correct)

Figure 2. Mental Actions of the Covariation Framework .

Because the behaviors that reflect particular mental actions occur on the Cartesian coordinate plane, graphing is an integral part of the curriculum developed from this framework. While these behaviors suggest graphing practices play an important role in covariational reasoning. In the updated framework, they focus on covariational reasoning independent of variational reasoning and conceptions of rates of change. In this updated framework, Thompson and Carlson continue to acknowledge that sets of behaviors accompany the different levels of their framework for covariational reasoning, but they no longer list these behaviors, leaving in question which behaviors accompany which levels of covariational reasoning. Thus, there is reason to conduct research on a teacher's graphing practices in the particular setting of a college algebra class using the function-based, covariation-focused curriculum Pathways College Algebra to study what practices are associated with advanced levels of covariational reasoning.

Practice

When analyzing the codes from the transcripts, I noticed three distinct levels of practices: micro practices, intermediate practices, and advanced practices. All three levels of practice involved an image drawn on a graph that I call the *change diagram*, shown in Figure 3. Micro practices are the smallest practices and are used to create the elements of the change diagram. Intermediate practices consist of practices that involve using one instantiation of the change diagram. Advanced practices are complex practices that are composed of micro practices and intermediate practices and involve using multiple instantiations of the change diagram. I first describe the change diagram by explaining the five micro practices in which Dr. Kemp engaged as she created one change diagram.

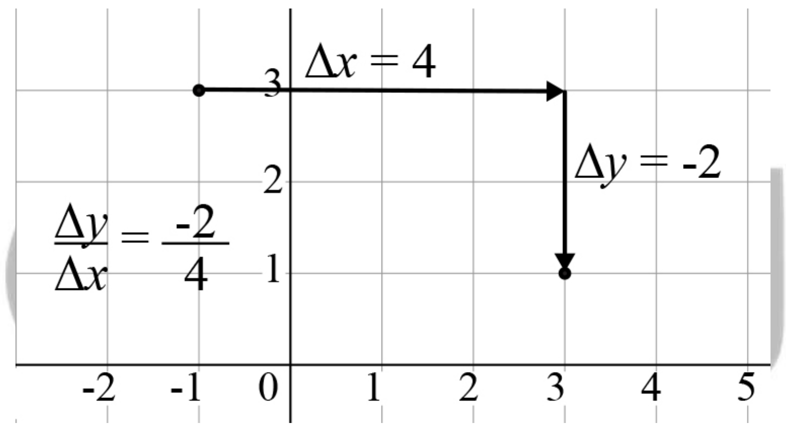

Figure 3. Change Diagram. Elements include an initial point, horizontal and vertical vectors, a final point, and a constant or average rate of change of y with respect to x.

Micro Practices Associated with the Construction of the Change Diagram

The change diagram is a diagram that is inscribed on a graph to invoke and support covariational reasoning (Thompson & Carlson, 2017). Elements of the change diagram include an initial point, horizontal and vertical vectors, final point, and the constant or average rate of change of *y* with respect to *x* written as the ratio of the vertical change to horizontal change. Each of these elements are constructed in a specific order. The construction of each of these elements on or near the graph is a distinct micro practice. Together the micro practices create one instantiation of the change diagram and its accompanying inscriptions. I will now illustrate these micro practices by describing instances where Dr. Kemp engaged in each of these micro practices, and I will also describe variations of the micro practices that I observed.

The prompt stated, "Illustrate the meaning of $\Delta y = -2 \cdot \Delta x$ on a graph that passes through the point (1, 7)" (Thompson & Carlson, 2017). Dr. Kemp read the prompt aloud to her students saying, "It says illustrate on the graph delta y equals negative two delta x, and it passes through the point one seven," as she wrote "$\Delta y = -2 \cdot \Delta x$" and "(1, 7)" on the board. Initially, Dr. Kemp read and wrote the prompt as it appeared in the text, but after a brief moment, she rephrased the prompt in a slow, reflective manner asking, "How are you going to show that change in x and change in y?" When she carefully rephrased the prompt using the expressions "change in x" and "change in y" instead of "delta x" and "delta y," Dr. Kemp seemed to show that she was thinking about illustrating a change in the two quantities.

After allowing students time to work, Dr. Kemp noticed that a lot of students had interpreted the prompt incorrectly by drawing lines on their graphs, so she asked the class if there was a difference between drawing a line and drawing changes. Dr. Kemp directed the students' attention to the word change by asking, "… remember this is talking about the *change* in y is always what?" She emphasized the word change by first pointing at Δy on the board, and then verbally prolonging the word while simultaneously circling Δy with her finger. Seemingly, to guide students' thinking about the changes in quantities, Dr. Kemp next wrote, "Δy is -2 times as large as the Δx," while saying, "change in y is …," nodding as students added, "negative two times," and finishing, "as large as the change in x." With this sentence, Dr. Kemp showed that she was thinking about the original equation as a specific comparison between two changes in quantity.

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To illustrate the five micro practices, I refer to an episode from class in which Dr. Kemp used a prompt from the Pathways College Algebra Student Workbook (Carlson, 2016) to introduce the change diagram to her students stated earlea

Micro Practice 1: Construct an initial point.

The change diagram begins from an ordered pair that is considered the initial point. This ordered pair is generally given, but it can be found if enough other information is given. An example of how an initial point can be found is described in the intermediate practice of finding a point. In the illustration of this micro practice, the initial point was given. Dr. Kemp began construction of a change diagram by drawing a point on her graph at the ordered pair (1, 7), shown in Figure 4a, as specified by the prompt. This point marks the beginning of the change diagram.

Micro Practice 2: Construct a horizontal vector.

From the initial point, a horizontal vector is constructed to show the distance and direction of the horizontal change (Thompson & Carlson, 2017). . The purpose of the horizontal vector is to make visible on a graph the change that is occurring with the independent variable. This purpose can be made explicit by an inscription in the form of labeling the horizontal vector using the Greek letter delta, Δ, which means “change” in mathematics, accompanied by the variable representing the horizontal quantity and a numerical evaluation of the horizontal change. The meaning of the horizontal vector can be made more explicit through

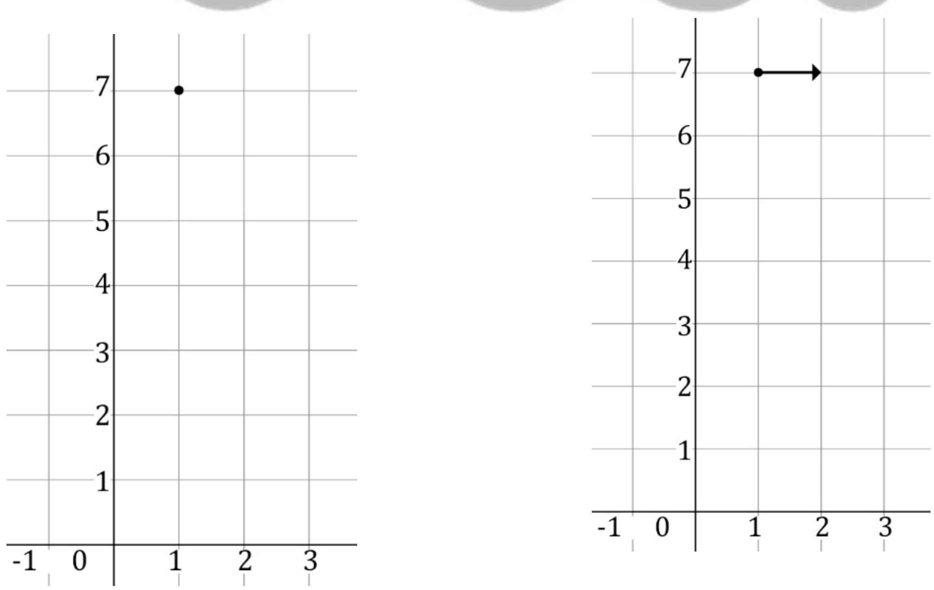

Figure 4. Micro Practices 1 and 2: a) Construct an initial point.
b) Construct a horizontal vector.

gestures pointing at the vector or identifying the ends of the vector simultaneously. The end of the horizontal change is only indicated by the head of the vector. A point is not constructed to mark the end of the horizontal change because a horizontal change is generally accompanied by a vertical change. Dr. Kemp appeared to avoid graphing a point at the end of the horizontal vector to emphasize the idea of covariational reasoning that the horizontal quantity and the vertical quantity are always changing together.

Micro Practice 3: Construct a vertical vector.

Following the construction of the horizontal vector, a vertical vector is constructed. The vertical vector begins where the horizontal vector ends. The purpose of the vertical vector is to make visible on a graph the change that is occurring in the dependent variable" (Thompson & Carlson, 2017). . This construction can be made explicit by a possible inscription in the form of a label near the vertical vector using the Greek letter delta, Δ, accompanied by the variable representing the vertical quantity and a numerical evaluation of the vertical change. The meaning of the vertical vector can be made more explicit through gestures pointing at the vector or identifying the ends of the vector simultaneously. The beginning of the vertical change is indicated by the tail of the vertical vector. The end of the vertical change is indicated by the head of the vector and is additionally designated by a point, placed at the head of the vector, which will be discussed in the next micro practice.

To illustrate this practice, we return to the ongoing instructional episode. Once Dr. Kemp had clarified that a horizontal change must occur before a vertical change, she turned her attention to the vertical change by saying, "I'm going to move twice as many in the negative direction, so that would put me right there." while concurrently constructing a vertical vector from the end of the horizontal vector to a position two units down from the beginning, shown in Figure 5. After constructing the vertical vector, Dr. Kemp immediately drew a point at the end of the vertical vector, which I will discuss in the next section. Dr. Kemp continued to make the meaning of the vertical vector explicit by articulating, "This could be your change in *y*" while writing the inscription "Δy" to label the vertical vector. Dr. Kemp showed that she was thinking about the vertical change as a multiple of the horizontal change by first marking the ends of the vertical vector, , and then carefully asking her students the following question: Is my change in *y* [said while tapping her fingers marking the ends of the vertical vector on the board] negative two times [said with one more distinct tap] as large as my change in *x* [said while marking the ends of the horizontal vector,]?

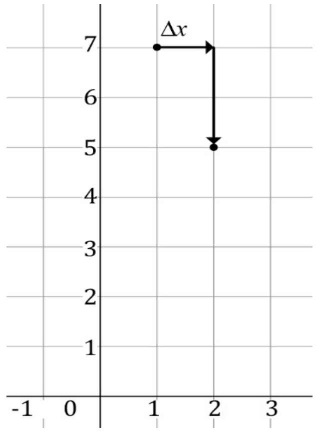

Figure 5. Micro Practice 3: Construct a vertical vector. Construct a final point.

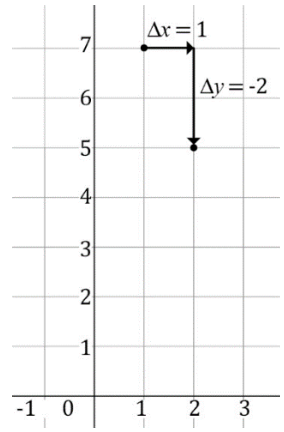

Figure 6. Micro Practice 4:

At this point, Dr. Kemp proceeded to make the comparison even more explicit by adding the numerical evaluation to the vector labels using the inscriptions "$\Delta x = 1$" and "$\Delta y = -2$," shown in Figure 6. She continued to accentuate this comparison by making a downward motion, , mimicking the vertical vector and stating that because the change is negative, the vector should be going down.

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Conclusion

Graphing is a fundamental topic in algebra and one that is notoriously difficult for students. Looking at practices may be one way to address this problem. This study looked at one teacher in one class in one unit of instruction and provided descriptions of the mathematical graphing practices of a practitioner of covariation-focused algebra curricula. These practices can be used to guide instruction to make graphing more accessible for more students. That a small study could yield such results suggests that it would be beneficial to continue studying the mathematical practices of experienced teachers.

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