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IMPLEMENTATION OF WAVELET REPRESENTATION.

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ABSTRACT: *Wavelets and wavelet transforms are a relatively new topic in signal processing. Their development and, in particular, their application remains an active area of research. The paper concentrates on the application of the wavelet transform. Specially, the implementation of Wavelet Representation.*

KEYWORDS: Signal processing, Wavelet transform, Wavelets, Orthonormal.

INTRODUCTION: This size defines a resolution of reference for measuring the local variations of the image. Generally, the structures we want to recognize have very different sizes. Hence, it is not possible to define a priori an optimal resolution for analyzing images. Several researchers have developed pattern matching algorithms which process the image at different resolutions. For this purpose, one can reorganize the image information into a set of details appearing at different resolutions. Given a sequence of increasing resolutions then the details of an image at the resolution are defined as the difference of information between its approximation at the resolution 'i and its approximation at the lower resolution. The wavelet analysis procedure is to adopt a wavelet prototype function, called an analyzing wavelet or mother wavelet. Temporal analysis is performed with a contracted, high-frequency version of the prototype wavelet, while frequency analysis is performed with a dilated, low-frequency version of the same wavelet. Because the original signal or function can be represented in terms of a wavelet expansion (using coefficients in a linear combination of the wavelet functions), data operations can be performed using just the corresponding wavelet coefficients. Wavelet analysis is an exciting new method for solving difficult problems in mathematics, physics and engineering with modern applications as diverse as wave propagation, data compression, signal and image processing, pattern recognition, computer graphic, the detection of aircraft and submarines and other medical image technology. Wavelet allow complex information such as music. Other applied field that are making use of wavelet including astronomy, acoustics, nuclear engineering, sub-band coding.

Implementation of Wavelet Representation:

Firstly a pyramidal algorithm to compute the wavelet representation will be described. We show that $D_{2j}f$ can be calculated by convolving $\boldsymbol{A^d_{2j+1}f}$ with a discrete filter G whose form will be characterize. For any n belongs to Z, the function $\boldsymbol{\varphi_{2j}(x-2^{-j}n)}$ is a member of $\boldsymbol{O_2 \subset V_{2^{j+1}}}$. In the same manner this function can be expanded in an orthonormal basis of $\boldsymbol{V_{2^{j+1}}}$.

$$\boldsymbol{\varphi_{2j}(x-2^{-j}n) = 2^{-j-1}\sum_{k=\propto}^{\propto} < \varphi_{2j}(\ u-2^{-j}n), \omega_{2j+1}(u-2^{-j-1}k) >. \omega_{2j+1}(x-2^{-j-1}k)}$$

Hence, by computing the inner product of f(x) with the functions of both sides we obtain,

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$$< f(\boldsymbol{u}), \boldsymbol{\varphi_{2j}}(\boldsymbol{u} - \boldsymbol{2^{-j}n}) > = \sum_{\boldsymbol{k}=\propto}^{\propto} < \boldsymbol{\varphi_{2j}}(\ \boldsymbol{u}), \boldsymbol{\omega}(\boldsymbol{u} - (\boldsymbol{k} - \boldsymbol{n}) >< f(\boldsymbol{u}), \boldsymbol{\omega_{j+1}}(\boldsymbol{x} - \boldsymbol{2^{-j-1}k})$$

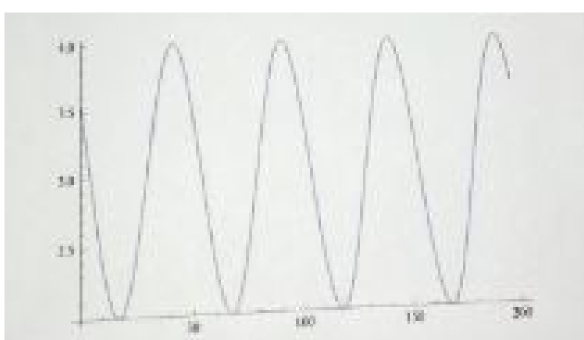
Let G be the discrete filter with impulse response,

g(n) = $< \varphi_{2^{-1}}(u), \omega(u-n) >$

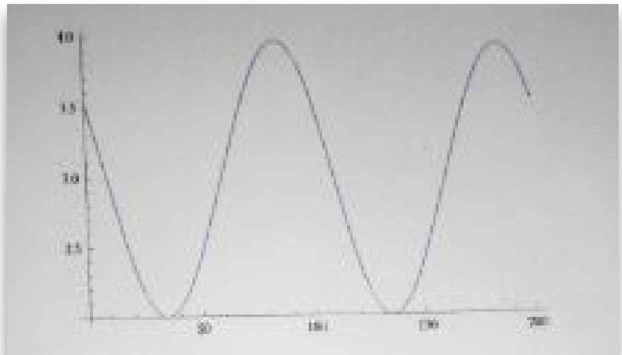
and G be the symmetricfilter with impulse response g(n) = g(-n), the transfer function of this filter is the function G(α). So we get,

$$< f(u), \varphi_{2j}\left(u - 2^{-j}n\right) > = \sum_{k=\propto}^{\propto} < g(\ 2n-k), < f(u), \omega_{j+1}(x - 2^{-j-1}k)$$

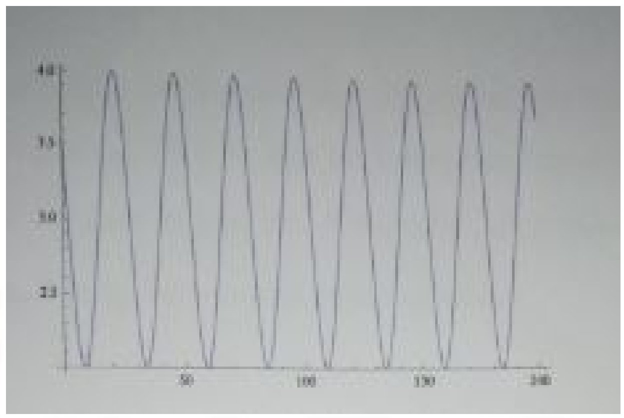
This equation shows that we can compute the detail signal $D_{2j}f$ by convolving $A^{d}_{2j+1}f$ with the filter G and retaining every other sample of the output. The orthogonal wavelet representation of a discrete signal $A^{d}_{1}f$ can therefore be computed by successively decomposing $A^{d}_{2j+1}f$ into $A^{d}_{2j}f$ and $D_{2j}f$ for $J \leq j \leq -1$. This algorithm is illustrated by the block diagram as shown as the figure. One method of handling the border problems uses a symmetry with respect to the first and the last sample as it implies that the impulse response of the filter G is related to the impulse response of the filter H by, $g(n) = (-1)^{1-n}h(1-n)$.



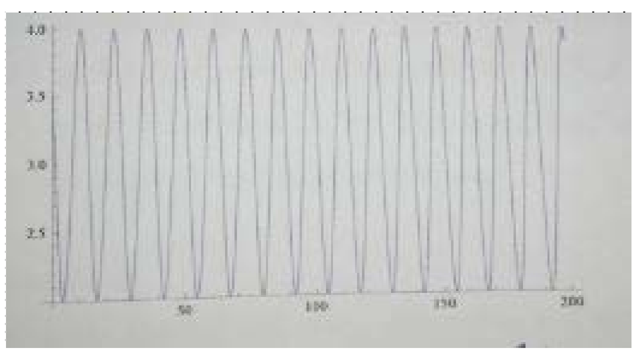
Detail Signal at resulation 1/32.



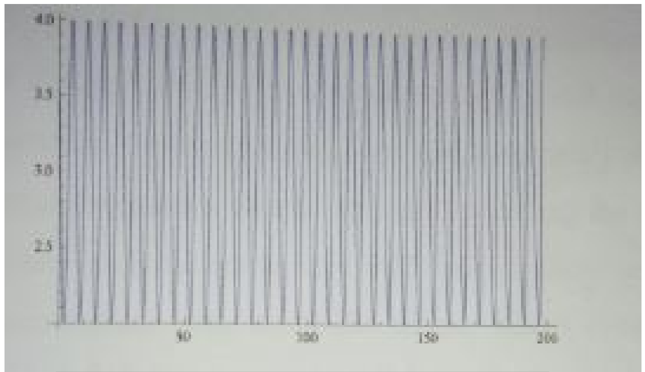
Detail Signal at resulation 1/16.



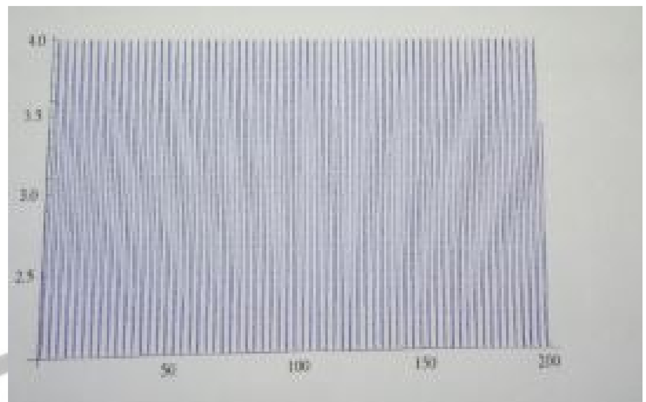
Detail Signal at resulation 1/8.



Detail Signal at resulation 1/8.



Detail Signal at resulation 1/2.



Detail Signal at resulation 1.

The above figures show the details signal as the resulation 1, ½, ¼, 1/8, 1/16, 1/32 (j = 0, -1, -2, -3, -4).

G is the mirror filter of H, and is a high-pass filter. In signal processing, g and H are called quadrature mirror filters. If the original signal has n samples, then the discrete signals $D_{2j}f$ and $\boldsymbol{A^d_{2j}f}$ have 2^jN samples each. Thus the wavelet representation

$$\boldsymbol{(A^d_{2j}f, (D_{2j}f)_{J\leq j\leq -1}}$$

has the total number of samples as the original approximation signal $A^d_1 f$**.** This occurs because the representation is orthogonal.

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