Global Scientific JOURNALS

GSJ: Volume 11, Issue 10, October 2023, Online: ISSN 2320-9186

www.globalscientificjournal.com

In Search of Series to Calculate the Number Pi(π) and its Most Exact Value and With the Largest Number of Decimal Places.

Welken Charlois Gonçalves

Information:

http://lattes.cnpq.br/7210808817928708

Welken Charlois Gonçalves, Email: welkengoncalvesc@gmail.com; Brazil, São Paulo, Bauru, University Unesp and Anhanguera , Physics and Pedagogy.

Abstract: This article is a Quantitative analysis of the various possible ways of calculating the number π , based on the deduction of Gottfried Wilhelm Leibniz and his 1686 formula through the generation of a Fortran program, an algorithm that creates the possibilities of calculation, as Leibniz defined $\arctan(1) = \frac{\pi}{4}$ and others, such as Riemann Zeta functions $\xi(2) = \frac{\pi^2}{6}$, and several starting possibilities, finally what remains is the analysis of numbers that are congruent to certain calculations of the conditions of the number π , a relevant fact that promotes the search for a study of number and various ramifications of calculations in conjectures, lemmas, axioms and mathematical theorems, as well as its use in Physics and other sciences used to describe nature. Furthermore, it is shown that calculations and manipulation possibilities that took years, decades and centuries have their outcome in Fortran or Matlab programming in hours or minutes.

Keywords: Number π , the biggest number π , Series and Numbers Pi(π).

Index:

1. Introduction:

The number Pi(π), is a numerical proportion defined by the relationship between the perimeter of a circle and its diameter; if the circle has perimeter p and diameter d, in the number π in equal w/d. Propellant William Jones in 1706, adopted it from the Greek word περίμετρος and was later popularized by Leonard Euler. Other names are circular constants or Ludolph number.

It is known that the number π belongs to rational numbers, most of them approximate to 3.14, coincidentally with Albert Einstein's birthday, which is the special day of Americans 3/14, on interplanetary routes NASA uses 3.141592653589792 with 15 decimal places, where calculates a circle with a radius of 46 billion light years around the observable universe, an approximation of π with 40 decimal places to guarantee precision to 1 Hydrogen atom.

Calculating the number π appeared for calculating the area of the circle. Among the Egyptians, in the papyrus of Ahmmes, the value attributed was $\left(\frac{4}{3}\right)^4$, however, there is also the value $3\frac{1}{6}$, the ancient Jewish people found the value of the number π, in 1 Kings 7:23, and the value used among the Babylonians was the number 3, the approximation of $3\frac{1}{8}$.

1-1 Calculation Methods:

There are many ways to calculate the number π which are approximations of numerical methods, the number π irrational and transcendent, in which calculations always involve approximations, successive approximations and/or infinite series of additions, multiplications and divisions.

The first rigorous attempt to find π was proposed by Archimedes, through the construction of inscribed and circumscribed polygons with 96 sides, the value found was between 223/71 and 22/7 (3.1408 and 3.1429), called the classical method.

Ptolemy of Alexandria in the 3rd century AD, took as a basis a polygon with 720 sides inscribed in a circle of 60 radius units, the approximate value was 3.1416. Where to search for π arrived in China, where Liu Hui, obtained the value of 3.14159 with a polygon of 3072 sides, and in the 5th century the mathematician Tsu Chung Chih, approximated between 3.1415926 and 3.1415927. At the same time Arybhata wrote in his book "Add 4 to 100, multiply by 8 and add 62,000. The result is approximately a circle of diameter 20,000". The above equation is $c=\pi.d$, analyzing, we have:

$$(4+100).8+62000 \approx \pi.20000$$
$$104.8+62000 \approx \pi.20000$$
$$832+62000 \approx \pi.20000$$
$$62832 \approx \pi.20000$$
$$\frac{62832}{20000} \approx \pi$$

Therefore there is $\pi=3{,}1416$,, Ghiyath al-Kashi in the 15th century calculated 3.1415926535897932, the Dutch Ludoph van Ceulen , in the 16th century, calculated π to 35 decimal places, starting with a 15-sided polygon, doubling the number of sides 37 times.

Archimedes' method

For a polygon with any number of sides, Archimedes formulated a mathematical representation for calculating π.

By the laws of cosines and considering a polygon with n sides and radius 1

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

We have formed an isosceles triangle, with base mand sides r :

$$r^2 = m^2 + m^2 - 2m^2\cos\alpha$$
$$r^2 = 1^2 + 1^2 - 2\cos\alpha$$
$$r^2 = 2 - 2\cos\alpha$$
$$r = \sqrt{2 - 2\cos\alpha}$$

The angle of the isosceles triangle at the center of the polygon is expressed as 360º divided by the number of sidesn, therefore:

$$l = \sqrt{2 - 2\cos\left(\frac{360}{n}\right)}$$

Therefore, perimeter will be:

$$p = n\sqrt{2 - 2\cos\left(\frac{360p}{n}\right)}$$

Being π is represented by the perimeter of the polygon divided by its diameter, we have:

$$\pi = \frac{n\sqrt{2 - 2\cos\left(\frac{360p}{n}\right)}}{2}$$

Applying trigonometric transformations, we have:

$$\pi = n\cos\left(\frac{180p}{n}\right)$$

Statistical methods:

Calculating the number π , is also calculated using Monte Carlo, using statistics. Where points are randomly drawn in a square between the coordinates.

$O=(0,0)$ e $B=(1,1)$. The distance between the drawn points is then calculated. $c_n=(x_n,y_{\tilde{n}})$ to the origin $O=(0,0)$ It is π can be approximated through points inscribed on the circumference of radius 1 in relation to the total number of points drawn in the square with side 1.

For example:

$$\pi \cong \frac{4.386}{500} = 3,088$$

There is also Buffon's needle, proposed by the French in the 18th century by the naturalist Georges de Buffon.

The most interesting and worked on in the article is the infinite series method, in particular the Leibniz series, we have the series by the Frenchman François Viete, who studying Archimedes' method, developed a calculation of the number π in 1593:

$$\frac{\sqrt{2}}{2}.\frac{\sqrt{2+\sqrt{2}}}{2}.\frac{\sqrt{2+\sqrt{2+\sqrt{2+}}}}{2}...$$

Just as the method of mathematician John Wallis, developed in 1655, an infinite series:

$$\frac{2}{1}.\frac{2}{3}.\frac{4}{3}.\frac{2}{5}.\frac{6}{5}.\frac{6}{7}.\frac{8}{7}.\frac{8}{9}...$$

In 1770 Johann Henrich Lambert published a series of infinite divisions:

$$\frac{4}{\pi} = 1 + \cfrac{1^2}{3 + \cfrac{2^2}{5 + \cfrac{3^2}{7 + \cfrac{4^2}{9 + \cfrac{5^2}{11 + \cfrac{6^2}{...}}}}}}...$$

Finally, Leibniz's infinite series:

In mathematics, Leibniz's formula for π, establishes that:

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-...=\frac{\pi}{4}$$

Using summation notation:

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}=\frac{\pi}{4}$$

The infinite series above is called the Leibniz series. It is also called the Gregory-Leibniz series, recognizing the work of James Gregory. The formula was discovered by Madhava of Sangamagrama and is thus called the Madhava–Leibniz series.

Proof:

$$\frac{\pi}{4}=\arctan(1)=\int_0^1\frac{1}{1+x^2}dx=$$

$$=\int_0^1\left(\sum_{k=0}^{n}(-1)^k x^{2k}+\frac{(-1)^{n+1}x^{2n+2}}{1+x^2}\right)dx=$$

$$=\sum_{k=0}^{n}\frac{(-1)^k}{2k+1}+(-1)^{n+1}\int_0^1\frac{x^{2n+2}}{1+x^2}dx$$

Considering only the integral in the last line:

$$0<\int_0^1\frac{x^{2n+2}}{2k+1}<\int_0^1 x^{2n+2}dx=\frac{1}{2n+3}\to 0 \text{ with } n\to\infty$$

Therefore, with $n\to\infty$ we obtain the Leibniz series:

$$\frac{\pi}{4}=\sum_{k=0}^{\infty}\frac{(-1)^k}{2k+1}$$

Formula inefficiency:

The formula converges slowly. To calculate π to 10 correct decimal digits using direct sum requires approximately 5 billion terms because $\frac{1}{2k+1}<10^{-10} \quad para \quad k>\frac{10^{-10}-1}{2n+1}$ .12

However, Leibniz's formula can be used to calculate π with great precision (hundreds of digits or more) using various convergence acceleration techniques. For example, the Shanks transformation, binomial transformation or Van Wijngaarden transformation, which are general methods for alternating series, can be applied to the partial sums of the Leibniz series. Additionally, combining terms in pairs gives the non-alternating series

$$\frac{\pi}{4}=\sum_{n=0}^{\infty}\left(\frac{1}{4n+1}-\frac{1}{4n+3}\right)=\sum_{n=0}^{\infty}\left(\frac{2}{(4n+1)(4n+3)}\right)$$

which can be evaluated with great precision with a small number of terms, using Richardson extrapolation or the Euler–Maclaurin formula.**This series can also be transformed into an integral using the Abel–Plana formula and evaluated using numerical integration techniques.**

Isolated calculation method for decimals

In 1995, David Harold Bailey, in collaboration with Peter Borwein and Simon Plouffe, discovered a formula for calculating π , an infinite sum (often called the BBP formula), here is just a quote from the best optimization of the number π in infinite series, formulated in 1996: David-Bailey, Peter Borwein and Simon Plouff 1996

$$\pi = \sum_{n=0}^{\infty}\left(\frac{4}{8n+1}-\frac{2}{8n+3}-\frac{1}{8n+5}-\frac{1}{8n+6}\right)$$

Or:

$$\pi = \frac{1}{64}\sum_{n=0}^{\infty}\frac{(-1)^n}{2^{10n}}\left(\begin{array}{l} -\frac{32}{4n+1}-\frac{2}{4n+1}+\frac{256}{10n+1}-\frac{64}{10n+3} \\ \frac{4}{10n+5}-\frac{4}{10n+7}+\frac{1}{10n+9} \end{array}\right)$$

Allowing you to easily calculate the nth binary or hexadecimal decimal of π **without having to calculate the preceding decimals.**Bailey's website contains its derivation and implementation in several programming languages.

Thanks to a formula derived from the BBP formula, the 4 000 000 000 000 000th digit of in base 2 was obtained in 2001.

With the discovery of the BBP formulaof base 16 and related formulas, similar formulas in other bases have been investigated. Borwein, Bailey and Girgensohn in 2004 recently showed that There is no Machin-type BBP arctangent formula that is non-binary, although this does not preclude a completely different scheme fordigit extraction algorithmson other bases. S. Plouffe developed an algorithm for calculating the tenthdigitin any basis in phases. A number of additional identities due to Ramanujan, Catalan and Newton are provided by Castellanos (1988ab, pp. 86-88), including several involving sums ofFibonacci numbers. Ramanujan found [2].

$$\sum_{k=0}^{\infty}\frac{(-1)^k\,(4\,k+1)\,[(2\,k-1)\,!!]^3}{[(2\,k)\,!!]^3} = \sum_{k=0}^{\infty}\frac{(-1)^k\,(4\,k+1)\left[\Gamma\left(k+\frac{1}{2}\right)\right]^3}{\pi^{3/2}\,[\Gamma\,(k+1)]^3} = \frac{2}{\pi}$$

A complete list of Ramanujan series found in his second and third notebooks is provided by Berndt (1994, pp. 352-354)[2],

$$\frac{4}{\pi} = \sum_{n=0}^{\infty}\frac{(6n+1)\left(\frac{1}{2}\right)_n^3}{4^n\,(n!)^3}$$

$$\frac{16}{\pi}=\sum_{n=0}^{\infty}\frac{(42n+5)\left(\frac{1}{2}\right)_n^3}{64^n\,(n!)^3}$$

$$\frac{32}{\pi}=\sum_{n=0}^{\infty}\frac{\left(42\sqrt{5}n+5\sqrt{5}+30n-1\right)\left(\frac{1}{2}\right)_n^3}{64^n\,(n!)^3}\left(\frac{\left(\sqrt{5}-1\right)}{2}\right)^{8n}$$

$$\frac{27}{4\pi}=\sum_{n=0}^{\infty}\frac{(15n+2)\left(\frac{1}{2}\right)_n\left(\frac{1}{3}\right)_n\left(\frac{2}{3}\right)_n}{64^n\,(n!)^3}\left(\frac{2}{27}\right)^n$$

$$\frac{5\sqrt{5}}{2\pi\sqrt{3}}=\sum_{n=0}^{\infty}\frac{(11n+4)\left(\frac{1}{2}\right)_n\left(\frac{1}{6}\right)_n\left(\frac{5}{6}\right)_n}{(n!)^3}\left(\frac{4}{125}\right)^n$$

$$\frac{15\sqrt{3}}{2\pi}=\sum_{n=0}^{\infty}\frac{(33n+4)\left(\frac{1}{2}\right)_n\left(\frac{1}{3}\right)_n\left(\frac{2}{3}\right)_n}{(n!)^3}\left(\frac{4}{125}\right)^n$$

$$\frac{85\sqrt{85}}{18\pi\sqrt{3}}=\sum_{n=0}^{\infty}\frac{(133n+8)\left(\frac{1}{2}\right)_n\left(\frac{1}{6}\right)_n\left(\frac{5}{6}\right)_n}{(n!)^3}\left(\frac{4}{85}\right)^n$$

$$\frac{4}{\pi}=\sum_{n=0}^{\infty}\frac{(-1)^n(20n+3)\left(\frac{1}{2}\right)_n\left(\frac{1}{4}\right)_n\left(\frac{3}{4}\right)_n}{(n!)^3\,2^{2n+1}}$$

$$\frac{4}{\pi\sqrt{3}}=\sum_{n=0}^{\infty}\frac{(-1)^n(28n+3)\left(\frac{1}{2}\right)_n\left(\frac{1}{4}\right)_n\left(\frac{3}{4}\right)_n}{(n!)^3\,3^n4^{2n+1}}$$

$$\frac{4}{\pi}=\sum_{n=0}^{\infty}\frac{(-1)^n(260n+23)\left(\frac{1}{2}\right)_n\left(\frac{1}{4}\right)_n\left(\frac{3}{4}\right)_n}{(n!)^3\,(18)^{2n+1}}$$

$$\frac{4}{\pi\sqrt{5}}=\sum_{n=0}^{\infty}\frac{(-1)^n(644n+41)\left(\frac{1}{2}\right)_n\left(\frac{1}{4}\right)_n\left(\frac{3}{4}\right)_n}{(n!)^3\,5^n\,(72)^{2n+1}}$$

$$\frac{4}{\pi}=\sum_{n=0}^{\infty}\frac{(-1)^n(21460n+1123)\left(\frac{1}{2}\right)_n\left(\frac{1}{4}\right)_n\left(\frac{3}{4}\right)_n}{(n!)^3(882)^{2n+1}}$$

$$\frac{2\sqrt{3}}{\pi}=\sum_{n=0}^{\infty}\frac{(8n+1)\left(\frac{1}{2}\right)_n\left(\frac{1}{4}\right)_n\left(\frac{3}{4}\right)_n}{(n!)^3 9^n}$$

$$\frac{1}{2\pi\sqrt{2}}=\sum_{n=0}^{\infty}\frac{(-1)^n(10n+1)\left(\frac{1}{2}\right)_n\left(\frac{1}{4}\right)_n\left(\frac{3}{4}\right)_n}{(n!)^3 9^{2n+1}}$$

$$\frac{1}{3\pi\sqrt{3}}=\sum_{n=0}^{\infty}\frac{(40n+3)\left(\frac{1}{2}\right)_n\left(\frac{1}{4}\right)_n\left(\frac{3}{4}\right)_n}{(n!)^3(49)^{2n+1}}$$

$$\frac{2}{\pi\sqrt{11}}=\sum_{n=0}^{\infty}\frac{(280n+19)\left(\frac{1}{2}\right)_n\left(\frac{1}{4}\right)_n\left(\frac{3}{4}\right)_n}{(n!)^3(99)^{2n+1}}$$

$$\frac{1}{2\pi\sqrt{2}}=\sum_{n=0}^{\infty}\frac{(26390n+1103)\left(\frac{1}{2}\right)_n\left(\frac{1}{4}\right)_n\left(\frac{3}{4}\right)_n}{(n!)^3(99)^{4n+2}}$$

These equations were first proven by Borwein and Borwein (1987a, pp. 177-187).**Borwein and Borwein (1987b, 1988, 1993) proved other equations of this type, and Chudnovsky and Chudnovsky (1987) found similar equations for other transcendental constants (Bailey et al. 2007, pp. 44-45).**A complete list of known independent equations of this type is given by**[2]**

$$\frac{4}{\pi}=\sum_{n=0}^{\infty}\frac{(6n+1)\left(\frac{1}{2}\right)_n^3}{4^n(n!)^3}$$

$$\frac{16}{\pi}=\sum_{n=0}^{\infty}\frac{(42n+5)\left(\frac{1}{2}\right)_n^3}{64^n(n!)^3}$$

$$\frac{32}{\pi}=\sum_{n=0}^{\infty}\frac{\left(42\sqrt{5}n+5\sqrt{5}+30n-2\right)\left(\frac{1}{2}\right)_n^3}{64^n(n!)^3}\left(\frac{\sqrt{5}-1}{2}\right)^{8n}$$

$$\frac{5^{1/4}}{\pi}=\sum_{n=0}^{\infty}\frac{\left(540\sqrt{5}n-1200n-525+235\sqrt{5}\right)\left(\frac{1}{2}\right)_n^3\left(\sqrt{5}-2\right)^{8n}}{\left(n!\right)^3}$$

$$\frac{12^{1/4}}{\pi}=\sum_{n=0}^{\infty}\frac{\left(24\sqrt{3}n-36n-15+9\sqrt{3}\right)\left(\frac{1}{2}\right)_n^3\left(2-\sqrt{3}\right)^{4n}}{\left(n!\right)^3}$$

2-Discussion:

The discussion of the article is basically based on exposing an idea, how to find series that can optimize the calculation of the number π and its decimal places, basically look for the writing of the number π , the most interesting factor is the study of infinite series, starting from Leibniz's infinite series which says that:

$$\frac{\pi}{4}=\arctan\left(1\right)$$

Thus, we have the expansion of the sine and cosine, both as tangent, in exponential and in Leonard Euler's formula, given, by Maclaurin series:

$$\sin(\mathrm{x})=\mathrm{x}-\frac{1}{3!}x^3+\frac{1}{5!}x^5-\frac{1}{7!}x^7+\ldots$$

$$\cos(\mathrm{x})=1-\frac{1}{2!}x^2+\frac{1}{4!}x^4-\frac{1}{6!}x^6+\ldots$$

$$e^z=1+\frac{1}{1!}z+\frac{1}{2!}z^2+\frac{1}{3!}z^3+\frac{1}{4!}z^4\ldots$$

In particular if z=ix, we obtain:

$$e^{ix}=1+ix-\frac{1}{2!}x^2-i\frac{1}{3!}x^3+\frac{1}{4!}x^4+\frac{1}{5!}x^5\ldots$$

Leonard Euler's expression is:

$$e^{ix}=\cos\left(x\right)+i\sin\left(x\right)$$

The inverse of this equation is given by:

$$\cos\left(x\right)=\frac{e^{ix}+e^{-ix}}{2}$$

$$\sin\left(x\right)=\frac{e^{ix}-e^{-ix}}{2i}$$

Now creating the sine, cosine and tangent algorithm, in Fortran F90 language, executed by the compiled through the FORCE program, follow the codes below, sine, cosine, exponent and π/n in the APPENDIX, in the case of the article we will work with the division of infinite series sine, cosine, tangent and the Leibniz series.

CONCATENATING THE SINE AND COSINE PROGRAM, AND MAKING SOME MANIPULATIONS AN EXAMPLE AT THE END OF TWO RATIOS, SINE/COSINE AND COSINE/SINE, as an example with several possibilities, we have the first calculation hypothesis to find the desired series and subsequently formulate in a infinite sum formula, where the tangent is found, in which the source code is presented in the appendix:

Now the most essential part of programming which is the calculations of the number pi, over some number, using a loop in Fortran:

$$\frac{\pi^2}{x},\frac{\pi^3}{x},\frac{\pi^4}{x},\frac{\pi^5}{x},\frac{\pi^6}{x},\frac{\pi^7}{x} \quad e \quad \frac{\pi^8}{x}$$

Or, another possibility:

$$2\frac{\pi}{x},3\frac{\pi}{x},4\frac{\pi}{x},5\frac{\pi}{x},6\frac{\pi}{x},7\frac{\pi}{x} \quad e \quad 8\frac{\pi}{x}$$

In short, there are several possibilities for working on divisions of infinite series, for example the division of infinite series sine and cosine and its result in search of arcotg(x), you can work with a comparison of the program above, for an unlimited number of values of x, the program mentioned in this part truncates an equality of tangent, variable x and arcotg(x) resulting in and printing the exact values where the division or forms of multiplication and division with the factor π , with its respective tangent value, Leibniz defined the $\pi/4$, but there are many other ways to express the number π with certain algebraic forms.

In mathematics, Leibniz's formula for π , establishes that:

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\ldots=\frac{\pi}{4}$$

Using summation notation:

$$\frac{\pi}{4}=\sum_{k=0}^{\infty}\frac{(-1)^k}{2k+1}$$

Where:

$$\frac{\pi}{4}=\arctan(1)$$

Through the infinite series of sin(x) and cos(x), we find the Tangent:

$$\tan(x)=\frac{\sin(x)}{\cos(x)}=\frac{x-\frac{1}{3!}x^3+\frac{1}{5!}x^5-\frac{1}{7!}x^7+\ldots}{1-\frac{1}{2!}x^2+\frac{1}{4!}x^4-\frac{1}{6!}x^6+\ldots}$$

Using a method similar to long division:

$$x+\frac{x^3}{3!}+\frac{2}{15!}x^5+\ldots$$

$$\text{x}-\frac{1}{6}x^3+\frac{1}{120}x^5-\ldots$$

$$\tan(\text{x})=1-\frac{1}{2!}x^2+\frac{1}{4!}x^4-\frac{1}{6!}x^6+\ldots)\ \frac{\text{x}-\frac{1}{2}x^3+\frac{1}{24}x^5-\ldots}{\frac{1}{3}x^3+\frac{1}{30}x^5-\ldots}$$

$$\frac{\frac{1}{3}x^3+\frac{1}{6}x^5-\ldots}{\frac{2}{15}x^5+\ldots.}$$

$$\tan(\text{x})=x+\frac{x^3}{3}+\frac{2}{15}x^5+\ldots$$

Leibniz's proof:

$$\arctan(\text{x})=\int_0^x\frac{1}{1+t^2}dt=$$

$$=x-\frac{x^3}{3}+\frac{x^5}{5}-\ldots+(-1)^n\frac{x^{2n+1}}{2n+1}+(-1)^{n+1}\int_0^x\frac{t^{2n+2}}{1+t^2}dt=$$

$|x|\leq 1$ for:

$$\left|\int_0^x\frac{t^{2n+2}}{1+t^2}dt\right|\leq\left|\int_0^x t^{2n+2}dt\right|=\frac{|x|^{2n+3}}{2n+3}\to 0\quad como\quad n\to\infty$$

It is found:

$$\arctan(\text{x})=x-\frac{x^3}{3}+\frac{x^5}{5}-\ldots=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$$

This is the representation of arctan(x) for $|x|\leq 1$.

Given the equation above, now considering x=a, such that the tangent equation becomes:

$$\tan(a) = a + \frac{a^3}{3} + \frac{2}{15}a^5 + ...$$

So we have to:

$$y = arctg(x) \quad e \quad x = tg(y)$$

The definition of the Maclaurin series, table the equation, of the arctangent:

$$\arctan(x) = y = \left[x - \frac{x^3}{3} + \frac{x^5}{5} - ... = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1} \right]$$

Defining, $x = 1$ then $y = \frac{\pi}{4}$.

For the purpose of the article, the following equation below must be proven, using the definition of tangent of divisions of infinite series:

If $y = arctg(x)$, then $x = tg(y)$, then replacing the last one:

y=a

$$\tan(a) = a + \frac{a^3}{3} + \frac{2}{15}a^5 + ...$$

Some manipulations result in:

$$\tan(y) = z = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1} + \left(\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1} \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1} \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1} \right) +$$
$$+ \left(\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1} \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1} \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1} \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1} \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1} \right)$$

Where:

$$\arctan(x) = y = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$$

Rearranging the sum terms:

$$\tan(y) = z = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1} + \left(\sum_{n=0}^{\infty}\sum_{n=0}^{\infty}\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}(-1)^n \frac{x^{2n+1}}{2n+1}(-1)^n \frac{x^{2n+1}}{2n+1} \right) +$$
$$+ \left(\sum_{n=0}^{\infty}\sum_{n=0}^{\infty}\sum_{n=0}^{\infty}\sum_{n=0}^{\infty}\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}(-1)^n \frac{x^{2n+1}}{2n+1}(-1)^n \frac{x^{2n+1}}{2n+1}(-1)^n \frac{x^{2n+1}}{2n+1}(-1)^n \frac{x^{2n+1}}{2n+1} \right)$$

Now, doing the sum:

$$\tan(a)=z=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}+\left(\sum_{n=0}^{\infty}\sum_{n=0}^{\infty}\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}(-1)^n\frac{x^{2n+1}}{2n+1}(-1)^n\frac{x^{2n+1}}{2n+1}\right)+$$
$$+\left(\sum_{n=0}^{\infty}\sum_{n=0}^{\infty}\sum_{n=0}^{\infty}\sum_{n=0}^{\infty}\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}(-1)^n\frac{x^{2n+1}}{2n+1}(-1)^n\frac{x^{2n+1}}{2n+1}(-1)^n\frac{x^{2n+1}}{2n+1}(-1)^n\frac{x^{2n+1}}{2n+1}\right)$$

Which results:

$$\tan(a)=z=\left[\left(x-\frac{x^3}{3}+\frac{x^5}{5}-\ldots\right)+\frac{1}{3}\left(x-\frac{x^3}{3}+\frac{x^5}{5}-\ldots\right)^3+\frac{2}{15}\left(x-\frac{x^3}{3}+\frac{x^5}{5}-\ldots\right)^5+\ldots\right]`$$

Doing:

$$a=x,\ b=-\frac{x^3}{3}\ \ e\ \ c=\frac{x^5}{5}$$

Given the tangent in its infinite series, considering:

$$(b+c)=\left(x-\frac{x^3}{3}\right)\ e\ \ a=\frac{x^5}{5}$$

We are restricted to three unknowns to create a remarkable product, as n=0,1 and 2 are considered:

$$(w+z)^3=(a+(b+c))^3,\ w=a\ \ e\ \ z=b+c$$

Becomes:

$$(a+(b+c))^3=$$

$$(w+z)^3=w^3+3w^2z+3w^2+z^3$$

$$=(b+c)^3+3a^2(b+c)+3a(b+c)^2+(b+c)^3=$$

$$\left[=b^3+3b^2c+3bc^2+c^3+3a^2b+3a^2c+3ab^2+3ac^2+6abc+a^3=\right]$$

$$a=x,\ b=-\frac{x^3}{3}\ \ e\ \ c=\frac{x^5}{5}$$

$$\left[= \left(-\frac{x^3}{3}\right)^3 + 3\left(-\frac{x^3}{3}\right)^2\left(\frac{x^5}{5}\right) + 3\left(-\frac{x^3}{3}\right)\left(\frac{x^5}{5}\right)^2 + \left(\frac{x^5}{5}\right)^3 + 3x^2\left(-\frac{x^3}{3}\right) + 3x^2\left(\frac{x^5}{5}\right) + 3x\left(-\frac{x^3}{3}\right)^2 + 3x\left(\frac{x^5}{5}\right)^2 + 6x\left(-\frac{x^3}{3}\right)\left(\frac{x^5}{5}\right) + x^3 = \right]$$

$$= -\frac{x^9}{3} + 3\frac{x^{11}}{3.5} - 3\frac{x^{13}}{3.5} - 3\frac{x^{15}}{5} - 3\frac{x^5}{3} + 3\frac{x^7}{5} - 3\frac{x^7}{3} + 3\frac{x^{12}}{5} - 6\frac{x^9}{3} + x^3 =$$

$$= x^3 - 3\frac{x^5}{3} - 6\frac{x^7}{15} - 9\frac{x^9}{3} + 3\frac{x^{11}}{15} + 3\frac{x^{12}}{5} - 3\frac{x^{13}}{15} - 3\frac{x^{15}}{5} \ldots$$

The intention is to divide sine with cosine, attempts to find other series, which express the number π , in a way, emphasizing that from these series one can find the number π more precise and exact like the references in the introduction and the Euler and Newton method.

An important observation is that, for example, finite series have a radius of convergence, when $\pi/4$ e x=1 in the Gregory-Leibniz series, there is only **convergence between [-1,1].**

Verification of the program only the insertion of arctg(x) on the tangent, it is necessary to demonstrate the calculations through notable products, the Leibniz formula is the assumption to exemplify the study and future deductions, the most interesting fact that the Leibniz formula for the number π , has its radius of convergence between [-1,1], where there are only angles between this interval, it is a necessary observation to be aware and analyze the conditions of the limits, in possible attempts, as an example cited, the analysis conditions are countless hypotheses, for example, we have that the tangent is given by, by the divisions of infinite series of sine and cosine:

$$\tan(x) = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \ldots$$

The first example deduced will be the insertion of arctg(x), on the tangent defined in infinite series between [-1,1], its radius of convergence:

$$\arctan(x) = y = \left[x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}\right]$$

The program is simple from the point of view of manipulations, since Fortran and language programs only work with defined integer and real variables and not with unknowns, which needs, the latter, to be demonstrated mathematically, working with defined variables of integers and real, there is no outline of the conjecture or theorem and the demonstration, only the verification calculation, but there are conditions for analyzing them through the algorithm of defined variables. The program provides possibilities for various manipulations and you can start from the sum example [5], page 613 to Geometric series in infinite series, chapter 11, as an example.

For, the tangent equation, below, if $a = \frac{\pi}{4}$, then $z = \tan(a) = 1$

$$\tan(a) = z = \left[a + \frac{a^3}{3} + \frac{2}{15}a^5 + \ldots\right] = 1$$

The Tangente Code is below:

Defined $y = arctg(x)$ It is $x = tg(y)$, first calculate arctg(x) we will find the variable and y. After this, calculate tan(a) and find a, for example, the possible manipulations are multiplied by factors x or 1/x or (1-x), for example, or there are several corrections to correct and find the π integer and a possible formula for π, you can also extract root integer numbers and etc. rational numbers and the like. Basically the program consists of this article. The Main Program is given below:

```
program tangente
!!!FIRST, DECLARE THE REAL AND INTEGER NECESSARY VARIABLES
Implicit none
integer :: a, exponentvalue, k, j, h, b, sum, gg, hh, t, kkb, jjb, bb, g, llb, p, pp, somatoriobb
real :: denominator, precedent, exponent ,kk ,dd ,hhh ,calculationbb ,somatn ,conv_rad , ddd
,ww ,cc , z, y, dddd, www
real :: command , sam , valor , x
real :: denominatorbb, precedentbb, valuebb, sumbb, variablebb, exponentbb, ang_degrees,
convert_rad
!!!THE DIMENSION IS IN VECTOR FORM, IT CAN BE RESOLVED IN MATRICIAL
FORM WHERE, IF NECESSARY, BETTER LATER MANIPULATIONS OF THE RESULTS
CAN BE MADE.
dimension :: valor(1000000), precedent(1000000), sam(1000000), somatn(1000000),
command(1000000)
dimension :: valuebb(1000000), precedentbb(1000000), sumbb(1000000),
calculationbb(1000000), cc(1000000)
real :: Variable
real(8), parameter :: pi=3.1415926535897932385
!!!HERE BASICALLY ARE THE OUTPUT, WITH WRITE
open(369, file='Tangente.txt', status='replace')
open(396, file='TangenteB.txt', status='replace')
open(3699, file='TangenteBB.txt', status='replace')
open(3969, file='TangenteBBB.txt', status='replace')
open(366, file='angulo.txt', status='replace')

print *, 'Tangent from Leibniz'
!!!THE MAXIMUM VALUE OF THE EXPONENT IS THE PRECISION OF THE SERIES
print *, 'Enter the Maximum Exponent Value'
read *, exponentvalue
ww=0
kkb=2
variable=0.000
do bb=1,1000
ww=ww+ 1
!!!HERE DDD HAS THE INCREMENT 0.003, IDEAL FOR PRECISION AND FINDING
THE SERIES THE MORE PRECISE THE MORE ANGLES WILL BE FOUND
ddd=ww/100+0.003
variablebb=ddd
calculationbb(1)=0.000
precedentbb(1)=0.000
!!!!DECLARE ALL VARIABLES INTEGRATE AND REAL TO THEN WORK WITH THE
NECESSARY INCREMENTS IN THE LOOP
g=0
p=1
!!!FINDING THE ANGLES THROUGH LEIBNIZ'S ARCOTG SERIES
do t = 1,51,2
```

GSJ: Volume 11, Issue 10, October 2023
ISSN 2320-9186

!!!BASICALLY THE TOTAL LOOP OF THE SERIES UP TO 51, OR THE DESIRED, THE MOST ESSENTIAL IS THE VARIABLELB, WHICH IS THE VALUE OF X DETERMINED IN THE SERIES ARCOTG(X)
denominatorbb=t
exponentbb=t
if (t>1) then
exponentbb=2*g+1
denominatorbb=2*g+1
end if
precedentbb(kkb) = ((((-1)**(p+1)))*(variablebb**exponentbb)/(denominatorbb))
calculationbb(kkb) = precedentbb(kkb)
sumbb(t) = calculationbb(kkb)
!!! HERE YOU BASICALLY WRITE THE VARIABLES
write (369,*) '***************************************'
write (369,*) 'Variable Value = ' , variablebb ,'Series Value = ', sumbb(t),ddd,'Exponent Value = ', exponentbb
write (369,*) 'Denominator Value = ' , denominatorbb, precedentbb(kkb), kkb
write (369,*) '***************************************'
!!! HERE IS THE LOGICAL CONDITION TO PRINT THE EXACT AND TOTAL VALUE OF THE SUM OF THE SERIES, HERE MAY BE THE END OF THE PROGRAM, BECAUSE YOU ALREADY HAVE THE ANGLE OF THE TOTAL SUM OF THE ARCTG(BY THE DIVISION OF THE INFINITE SERIES OF SINE AND COSINE, HOWEVER THE OTHER LOOP OF IS IS REQUIRED FOR A FUTURE CALCULATION, OF THE DETERMINED AND SUGGESTED MULTIPLICATION FACTORS, BOTH THE ARC TANGENT AND TANGENT SERIES. HERE IS THE APPROXIMATE VALUE OF THE TANGENT, IN THE AFTER LOOP THE MOST EXACT CALCULATION OF THE TANGENT SERIES.
if (t == 51) then
sam(1) = 0.000
do pp = 3,51,2
somatn(pp) = somatn(pp-2)+sumbb(pp-2)
end do
!!! HERE IS A CALCULATION OF THE TANGENT USING THE VALUE OF THE VARIABLE FOUND IN THE ARCOTG SERIES
pp=51
x = somatn(pp)+((somatn(pp))**3)/(3)+(2)*((somatn(pp))**5)/(15)
command(kkb) = x
!!! AFTER CARRYING OUT THE CALCULATIONS, THE CORRECT PRINTING OF THE VALUES
write (396,*) '***************************************'
write (396,*) 'Value Of The S,ries End', somatn(51), command(kkb)
write (396,*) 'Angle Value In Radian = ', x , somatn(1), somatn(3), somatn(5) ,'Value of the Looop Do = bb :',bb
write (396,*) '***************************************'
kkb = kkb+1
end if
p = p+1
somatoriobb = somatoriobb+1
g = g+1
Enddo
end do
!!!FINDING THE TANGENT THROUGH THE SERIES OF THE DIVISION OF INFINITE SERIES SINE AND COSINE FROM THE DEFINITION OF MACLAURIN SERIES

```
!!!HERE ONLY THE LOOP IS DEFINED WITH THE PROPER VARIABLE WITH THE INCREMENT
do b = 1, 1000
variable = pi/b
!!!www=www+1
!!!dddd=www/1000+0.1
!!!variable=dddd
sam(0) = 0.000
precedent(0) = 0.000
k = 1
hh = 1
sum = 1
conv_rad = pi/180.0
ang_degrees = b
dd = variable/convert_rad
j = 1
!!!HERE FROM THE WHILE TO THE VALUE OF THE EXPONENT CALCULATES THE TOTAL SERIES OF THE TANGENT WITH THE LOGICAL CONDITION OF THE CONSTANTS
do while (j<=51)
!!!THE EXPONENT VALUE WAS CONSIDERED = 51, WITH A VALUE = 5 OR 7 WE ALREADY HAVE THE NUMBERS AND THE ANGLE IN APPROXIMATE RADIANS, BUT THE HIGHER THE EXPONENT NUMBER THE GREATER THE ACCURACY. THE EXECUTION TIME DEPENDS ON THE CONDITION OF THE "DO" LOOP OR THE EXPONENT, THE MOST RELEVANT THE "DO" LOOP.
denominator = hh
exponent = hh
if (hh>1) then
exponent = hh
denominator = hh*((hh-2))
end if

!!! CAN DO TIMES (J-1) FOR TANGENT SERIES ACCURACY, ONLY WHEN HH>1.
precedent(k) = (((1)**(j+1)))*(variable**exponent)/(denominator)
valor(k) = precedent(k)+valor(k-1)
!!! THE EXIT OF THE SERIES VALUE
write (3699,*) '****************************************'
write (3699,*) 'Variable Value In Radians = ' , variable ,'Series Value = ', valor(k),'Exponent Value = ', exponent
write (3699,*) 'Value of b of Series and b = ',b, precedent(k), precedent(k-1) ,j,hh ,k,'Denominator Value' , denominator
write (3699,*) '****************************************'
precedent(k-1) = valor(k)
sam(sum) = valor(k)
kk = sum
gg = hh
!!!HERE IS THE CONDITION TO ONLY PRINT THE FINAL VALUE OF THE SERIES
if (kk*2-1 == gg .and. j==51) then
write (3969,*) '****************************************'
write (3969,*) 'Tangent Value = ', sam(sum), sum, variable
write (3969,*) '****************************************'
end if
```

```
!!! THE MOST IMPORTANT THING OF THE PROGRAM, TRUNCATES THE FUNCTION WITH THE PROPER REAL VARIABLE WITH AINT,, BUT BEFORE YOU CAN MULTIPLY IT BY 10, 100 OR ANY OTHER NUMBER OF PREFERENCE IN BASE 10 TO FIND THE LARGE POSSIBLE NUMBER OF ANGLES AND CONGRUENT TANGENT, A FIRST PART OF THE PROGRAM IS TANGENT BY ARCOTG(X)
!!!COMPARE THE TRUNCATED TANGENT WITH THE CALCULATED TANGENT IN THIS TANGENT SERIES BLOCK AND PRINT THE EXACT ANGLE VALUE...

do llb=1,kkb
print*, aint(command(llb)*1000),aint(sam(sum)*1000)
z = command(llb)*1000
y = sam(sum)*1000
if (aint(command(llb)*100) == aint(sam(sum)*100)) then

!!!tangent arctangent and variable when they are congruent

write (366,*) '**************************************'
write (366,*) 'Variable Value = ', variable, 'Tangent Value = ', z,y,llb
write (366,*) '**************************************'
end if
end do

k = k+1
hh = hh+2
sum = sum+1
j = j+1
Enddo
Enddo

Pause
end program
```


Due to the limit conditions imposed on the loops, it is probably necessary to split the filename.txt files. For this, it is necessary, on Windows, to use PowerShell with the following command, changing the name of the filename.txt file, to the name of the file that you will output the OPEN command of the FORTRAN program algorithm:

```
$i=0; Get-Content d:\temp\teste.txt -ReadCount 100 | %{$i++; $_ | Out-File d:\temp\out_$i.txt
```

ReadCount 100, is the generated file, for example with 100 lines, just a note, it is necessary to access the appropriate folder in Windows using the cd command, and the dir command, may be necessary to list the files and directories that are present in the corresponding folder .

Examples of some program outputs are basically below, in Radianus:

**************************************					
**************************************					
Variable Value =	**0.78539819**	**Tangent Value =**	**978.65656**	**975.15607**	**99**
**************************************					
**************************************					
Variable Value =	**0.78539819**	**Tangent Value =**	**978.65656**	**975.15826**	**99**
**************************************					
**************************************					
Variable Value =	**0.78539819**	**Tangent Value =**	**978.65656**	**975.15942**	**99**

*************************************					
*************************************					
*************************************					
Variable Value =	0.44879895	Tangent Value =	482.78043	480.26346	49
*************************************					
*************************************					
Variable Value =	0.44879895	Tangent Value =	482.78043	480.26346	49
*************************************					
*************************************					
*************************************					
*************************************					
Variable Value =	0.20943952	Tangent Value =	212.99902	212.52925	22
*************************************					
*************************************					
Variable Value =	0.20943952	Tangent Value =	212.99902	212.52925	22
*************************************					
*************************************					
Variable Value =	0.19634955	Tangent Value =	192.99951	198.89264	20
*************************************					
*************************************					
Variable Value =	0.19634955	Tangent Value =	192.99951	198.89264	20
*************************************					
Variable Value =	0.18479957	Tangent Value =	182.99965	184.79956	19
*************************************					
*************************************					
Variable Value =	0.18479957	Tangent Value =	182.99965	186.90326	19
*************************************					
*************************************					
Variable Value =	0.18479957	Tangent Value =	182.99965	186.91762	19
*************************************					
*************************************					

Note that the tangent is multiplied by 1000 for the possibility of truncation, or by doing this in the second part of the program you can optimize the increment to find an approximate value or simply just variable=pi/b, which is a multiplicative factor of Pi, as this way it is possible to construct new series and sums of series, or discover the value of decimal places for the number pi, instead of ddd=ww/1000 +0.1 , variable=dddd, that is, just optimizing the increment from 0.1 to one deviation value with more sensitivity in precision.

For example 0.28559935, from the output in radians is equal to 11.25°, which is $\frac{1}{5}.\frac{\pi}{4}$, that is, when x=(0.78539819)/5

Being:

$$\arctan(\mathrm{x}) = y = \left[x - \frac{x^3}{3} + \frac{x^5}{5} - ... = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \right]$$

The example calculation above results in:

$$\arctan(\frac{1}{5}.\mathrm{x}) = \frac{1}{5}.y = \left[\frac{1}{5}x - \frac{(1/5x)^3}{3} + \frac{(1/5x)^5}{5} - ... = \sum_{n=0}^{\infty} (-1)^n \frac{(1/5x)^{2n+1}}{2n+1} \right]$$

Where:

$$x = 1 \text{ results in } y = \frac{\pi}{4},$$

Exclusively for $\pi / 4$:

Variable Value = 0.78539819 Tangent Value = 978.65656 974.89539 99
****
****
Variable Value = 0.78539819 Tangent Value = 978.65656 975.03223 99
****
****
Variable Value = 0.78539819 Tangent Value = 978.65656 975.09674 99
****
****

Another example, for x = 0.14959966, the angle is 8.57°, tan(x) = 0.14959966,

x = 0.14959966 $= \frac{4}{21} \cdot \frac{\pi}{4}$

$$\arctan\left(\frac{4}{21}.x\right) = \frac{4}{21}.y = \left[\frac{4}{21}x - \frac{(4/21x)^3}{3} + \frac{(4/21x)^5}{5} - \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{(4/21x)^{2n+1}}{2n+1}\right]$$

Since:

$$x = 1 \text{ results in } y = \frac{\pi}{4},$$

Having,

$$\arctan(x) = y = \left[x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}\right]$$

Becomes

$$x.\arctan(x) = \left[x.y = \left[x^2 - \frac{x^4}{3} + \frac{x^6}{5} - \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+2}}{2n+1}\right]\right]$$

Or divide by x

$$\frac{\arctan(x)}{x} = \left[\frac{y}{x} = \left[1 - \frac{x^2}{3} + \frac{x^4}{5} - \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{2n+1}\right]\right]$$

For example, y-xy=y(1-x)

$$\arctan(x) - x.\arctan(x) = y(1-x) = x - x^2 - \frac{x^3}{3} + \frac{x^4}{3} + \frac{x^5}{5} - \frac{x^6}{5} - \ldots =$$

Organizing in sum:

$$\left(\sum_{n=0,1,2,3,....}^{\infty}(-1)^{n+1}\frac{x^{2n+2}}{2n+1}\right)_{2n+1\equiv 0(\mathrm{mod}(2n+2))}\left(\sum_{n=0,1,2...}^{\infty}(-1)^{n}\frac{x^{2n+1}}{2n+1}\right)_{(2n+1)\equiv 0(\mathrm{mod}(2n+1))}$$

Now, for:

$$arc\tan(\mathrm{x})-\frac{\arctan(\mathrm{x})}{x}=y\left(1-\frac{1}{x}\right)=-1+x+\frac{x^2}{3}-\frac{x^3}{3}-\frac{x^4}{5}+\frac{x^5}{5}...=$$

Organizing in sum:

$$\left(\sum_{n=1,2,3,....}^{\infty}(-1)^{n+1}\frac{x^{n}}{m}\right)_{m\equiv 0(\mathrm{mod}\,n)}\left(\sum_{n=0,1,3,4...}^{\infty}(-1)^{n+1}\frac{x^{n}}{n+1}\right)_{(n+1)\equiv 1(\mathrm{mod}\,n)}$$

Or to:

$$x.\arctan(\mathrm{x})-\frac{\arctan(\mathrm{x})}{x}=\left(x.y-\frac{y}{x}\right)=y\left(x-\frac{1}{x}\right)=\left(\frac{x^2-1}{x}\right)$$

$$=\left[x.y=\left[-1+\frac{2x^2}{3}-\frac{2x^4}{15}+\frac{2x^6}{35}-...=\right]\right]=$$

Organizing the sum:

$$\left(\sum_{n=0}^{\infty}(-1)^{n+1}\frac{2x^{2n}}{(2n+1)(n+1)}\right)_{(2n\equiv 1\ \mathrm{mod}\ 2n+1)}\left(\sum_{n=0}^{\infty}(-1)^{n+1}\frac{2x^{2n}}{(2n+1)(n+2)}\right)_{(2n\equiv 1\ \mathrm{mod}\ 2n+2)}$$

For x=1,

$$y(1-x)=x-x^2-\frac{x^3}{3}+\frac{x^4}{3}+\frac{x^5}{5}-\frac{x^6}{5}-...=0$$

$$y\left(1-\frac{1}{x}\right)=-1+x+\frac{x^2}{3}-\frac{x^3}{3}-\frac{x^4}{5}+\frac{x^5}{5}...=0$$

Where for x=1 y=pi/4

Finding out whether the series diverges or converges:

For the series:

$$\arctan(\mathrm{x})=y=\left[x-\frac{x^3}{3}+\frac{x^5}{5}-...=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}\right]$$

We have:

$$\arctan(\mathrm{x}) = y = \left[x - \frac{x^3}{3} + \frac{x^5}{5} - ... = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \right]$$

$$s_0 = x$$

$$s_1 = x - \frac{x^3}{3}$$

$$s_2 = x - \frac{x^3}{3} + \frac{x^5}{5} >= x - \frac{x^5}{5} + \frac{x^5}{5} = x$$

$$s_3 = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}$$

$$s_4 = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} > s_4 = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^9}{9} + \frac{x^9}{9} = x - \frac{x^3}{3} + \frac{x^5}{5}$$

...

For

$$x.\arctan(\mathrm{x}) = \left[x.y = \left[x^2 - \frac{x^4}{3} + \frac{x^6}{5} - ... = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+2}}{2n+1} \right] \right]$$

$$s_0 = x^2$$

$$s_1 = x^2 - \frac{x^4}{3}$$

$$s_2 = x^2 - \frac{x^4}{3} + \frac{x^6}{5} > x^2 - \frac{x^6}{5} + \frac{x^6}{5} = x^2$$

$$s_3 = x^2 - \frac{x^4}{3} + \frac{x^6}{5} - \frac{x^8}{7}$$

$$s_4 = x^2 - \frac{x^4}{3} + \frac{x^6}{5} - \frac{x^8}{7} + \frac{x^{10}}{9} > x^2 - \frac{x^4}{3} + \frac{x^6}{5} - \frac{x^{10}}{9} + \frac{x^{10}}{9} = x^2 - \frac{x^4}{3} + \frac{x^6}{5},$$

Or divide by x

$$\frac{\arctan(x)}{x}=\left[\frac{y}{x}=\left[1-\frac{x^2}{3}+\frac{x^4}{5}-...=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{2n+1}\right]\right]$$

$$s_0=1$$

$$s_1=1-\frac{x^2}{3}$$

$$s_2=1-\frac{x^2}{3}+\frac{x^4}{5}>1-\frac{x^4}{5}+\frac{x^4}{5}=1$$

$$s_3=1-\frac{x^2}{3}+\frac{x^4}{5}-\frac{x^6}{7}$$

$$s_4=1-\frac{x^2}{3}+\frac{x^4}{5}-\frac{x^6}{7}+\frac{x^8}{9}>1-\frac{x^2}{3}+\frac{x^4}{5}-\frac{x^8}{9}+\frac{x^8}{9}=1-\frac{x^2}{3}+\frac{x^4}{5}$$

Therefore, the series above only converge for -1<x<1, that is, for $x\to\infty$ the series always diverge. Demonstration can be found at [5] page 715.

$$\left(\sum_{n=0,1,2,3,....}^{\infty}(-1)^{n+1}\frac{x^{2n+2}}{2n+1}\right)_{2n+1\equiv 0(\mathrm{mod}(2n+2))}\left(\sum_{n=0,1,2...}^{\infty}(-1)^{n}\frac{x^{2n+1}}{2n+1}\right)_{(2n+1)\equiv 0(\mathrm{mod}(2n+1))}$$

$$\lim_{x\to\infty}y=\lim_{x\to\infty}\left(\frac{x-x^2-\frac{x^3}{3}+\frac{x^4}{3}+\frac{x^5}{5}-\frac{x^6}{5}-...}{(1-x)}\right)$$

$$\lim_{x\to\infty}y=\lim_{x\to\infty}\frac{x}{(1-x)}-\lim_{x\to\infty}\frac{x^2}{(1-x)}-\lim_{x\to\infty}\frac{x^3}{3(1-x)}+\lim_{x\to\infty}\frac{x^4}{3(1-x)}$$
$$+\lim_{x\to\infty}\frac{x^5}{5(1-x)}-\lim_{x\to\infty}\frac{x^6}{5(1-x)}$$

$$\lim_{x\to\infty}y=\lim_{x\to\infty}\frac{x}{x\left(\frac{1}{x}-1\right)}-\lim_{x\to\infty}\sum_{n=0}^{\infty}x^{1-n}(-1)^{n+1}\frac{\left(\frac{1}{n}-\frac{x}{n}\right)}{(1-x)}$$

$$\lim_{x\to\infty} y = \lim_{x\to\infty}\sum_{n=0}^{\infty} x^{2-n}\left(-1\right)^{n+1} \lim_{x\to\infty}\sum_{n=0}^{\infty}\frac{x^2\left(\frac{1}{xn}+\frac{1}{n}\right)}{x\left(\frac{1}{x}-1\right)}$$

$$\lim_{x\to\infty} y = \lim_{x\to\infty}\sum_{n=0}^{\infty} x^{2-n}\left(-1\right)^{n+1} \lim_{x\to\infty}\sum_{n=0}^{\infty} x\frac{\left(\frac{1}{n}+\frac{x}{n}\right)}{\left(\frac{1}{x}-1\right)}$$

$$\lim_{x\to\infty}\sum_{n=0}^{\infty} x\frac{\left(\frac{1}{xn}+\frac{1}{n}\right)}{\left(\frac{1}{x}-1\right)}$$

$$\lim_{x\to\infty} x\left[\lim_{n\to\infty}\frac{1}{n}\lim_{x\to\infty}\frac{1}{\left(\frac{1}{x}-1\right)}+\lim_{x\to\infty}\frac{1}{\left(\frac{1}{x}-1\right)}\lim_{x\to\infty}\frac{1}{x}\lim_{n\to\infty}\frac{1}{n}\right]$$

$$\lim_{x\to\infty} x\left[(0)(-1)+(-1)(0)(0)\right]$$

$$\lim_{n\to\infty}\frac{1}{x^n}\to 0$$

Limit other than 0 the series diverges, divergence test, as $\infty.0=1$

Now, for:

$$\arctan(\mathrm{x}) - x.\arctan(\mathrm{x}) = y\left(1-x\right) = x - x^2 - \frac{x^3}{3} + \frac{x^4}{3} + \frac{x^5}{5} - \frac{x^6}{5} - ... =$$

$$s_0 = x$$

$$s_1 = x - x^2$$

$$s_2 = x - x^2 - \frac{x^3}{3} > x - \frac{x^3}{3} + \frac{x^3}{3}$$

$$s_3 = x - x^2 + \frac{x^3}{3} - \frac{x^5}{5}$$

$$s_4 = x - x^2 + \frac{x^3}{3} - \frac{x^5}{5} + \frac{x^6}{6} > x - x^2 + \frac{x^3}{3} - \frac{x^6}{6} + \frac{x^6}{6} = x - x^2 + \frac{x^3}{3}$$

Now, for:

$$arc\tan(\mathrm{x}) - \frac{\arctan(\mathrm{x})}{x} = y\left(1-\frac{1}{x}\right) = -1 + x + \frac{x^2}{3} - \frac{x^3}{3} - \frac{x^4}{5} + \frac{x^5}{5} ... =$$

Converges or diverges:

$$arc\tan(\mathrm{x})-\frac{\arctan(\mathrm{x})}{x}=y\left(1-\frac{1}{x}\right)=-1+x+\frac{x^2}{3}-\frac{x^3}{3}-\frac{x^4}{5}+\frac{x^5}{5}...=$$

$$s_0=-1$$

$$s_1=-1+x$$

$$s_2=-1+x+\frac{x^2}{3}>-1+\frac{x^2}{3}+\frac{x^2}{3}=-1+\frac{2x^2}{3}$$

$$s_3=-1+x+\frac{x^2}{3}-\frac{x^3}{3}$$

$$s_4=-1+x+\frac{x^2}{3}-\frac{x^3}{3}-\frac{x^4}{5}>-1+x+\frac{x^2}{3}-\frac{x^4}{5}-\frac{x^4}{5}=-1+x+\frac{x^2}{3}-\frac{2x^4}{5}$$

Or to:

$$x.\arctan(x)-\frac{\arctan(x)}{x}=\left(x.y-\frac{y}{x}\right)=y\left(x-\frac{1}{x}\right)=\left(\frac{x^2-1}{x}\right)$$

$$=\left[x.y=\left[-1+\frac{2x^2}{3}-\frac{2x^4}{15}+\frac{2x^6}{35}-...=\right]\right]=$$

$$s_0=-1$$

$$s_1=-1+\frac{2x^2}{3}$$

$$s_2=-1+\frac{2x^2}{3}-\frac{2x^4}{15}>-1+\frac{2x^4}{15}-\frac{2x^4}{15}=-1$$

$$s_3=-1+\frac{2x^2}{3}-\frac{2x^4}{15}+\frac{2x^6}{35}$$

$$s_4=-1+\frac{2x^2}{3}-\frac{2x^4}{15}+\frac{2x^6}{35}-\frac{2x^8}{45}>-1+\frac{2x^2}{3}-\frac{2x^4}{15}+\frac{2x^8}{45}-\frac{2x^8}{45}=-1+\frac{2x^2}{3}-\frac{2x^4}{15}$$

Therefore, all series diverge when x>1 and only converge when -1<x<1, concluding that it is possible to find the number π written in different ways, which also makes it possible to analyze decimal places. Analyzing the formula for**Bailey-Peter-Borwein-Plouffe or Ramanujan, allows a starting precedent, or to adapt the result to greater precision in decimal places or to annotate the form of the number** π, respectively, as a reference of a starting example.

3-Conclusion:

It is then concluded that, as an example, new possibilities of writing the number π, with the exemplifications in a simpler way, the only thing to do in this case is to rewrite the algorithm adapted to these conditions, starting from this idea there are countless possibilities for manipulations, for example, arriving at a formula like Borwein and Borwein and Chudnovsky and Chudnovsky Ramanujan, Catalão and Newton, in this case using the Leibniz series as an example. Just like the Riemmann zeta function with $\xi(2)=\frac{\pi^2}{6}$, the number is also written π. The idea of the article came from the initiative of the Leibniz series, with the intention of verifying what would happen if a program algorithm was carried out, and later, for reference in the argumentation, we arrive at the examples cited in the article, noting that ideas already demonstrated, can with attempts of new ideas can expand the writing of the number π, both the Leibnizian series and others , especially the $\xi(z)$ Riemann which has several applications in the modern world and especially in Physics.

Appendix:

For SINE, we have:

```
program sin
Implicit none
integer :: a,b,j,k,c,w,d
real :: calculo,precedente,fact
real(4) :: conv_rad,ang_graus
real(8), parameter :: pi=3.14159265358979323 85
dimension :: calculo(1000),precedente(1000)
open(369, file='seno.txt', status='replace')
print *, 'Seno'
print *, 'Digite o Valor Expoente Máximo'
read *, c
print*, c
do b = 1,360
!!! CONVERSAO EM RADIANOS
conv_rad=pi/180.0
ang_graus=b
k=2
j=0
calculo(1)=0
!!!CALCULO DO SENO PELO LACO DO
do a=1,c,2

fact = 1.0
do d = 1, a
fact = fact*d
Enddo
print*, calculo(k-1)
precedente(k)=(((-1)**j)*((ang_graus*conv_rad)**a)/fact)
calculo(k)=precedente(k)+calculo(k-1)
!!!SAÍDA DO ARQUIVO SALVAR NO ARQUIVO
w=c-1
if (a == c .or. a == w) then
write (369,*) 'Angulo em Radianos e Graus = ', ang_graus*conv_rad, 'e', ang_graus , precedente(k)
write (369,*) 'Valor do Seno = ',calculo(k), b , fact, a, calculo(k-1) , j ,k,c,w
write (369,*) '********************'
Endif
calculo(k-1)=calculo(k)
j=j+1
k=k+1
enddo
enddo
pause
end program
```

TABEL 1

For COSINE, have:

```
program cosseno
Implicit none
integer :: a, b,c,d,j,k, w
real :: precedente, calculo,fact
real(4) :: conv_rad,ang_graus
real(8), parameter :: pi=3.14159265358979323 85
dimension :: calculo(1000),precedente(1000)
open(639, file='cosseno.txt', status='replace')
print *, 'Seno'
print *, 'Digite o Valor Maximo do Expoente'
read *, c
do b = 1,360
!!!CONVERSAO EM RADIANOS
conv_rad=pi/180.0
ang_graus=b
k=1
j=1
calculo(0)=1
!!!CALCULO DO COSSENO
do a=2,c, 2
print*,a,c
fact = 1.0
do d = 1, a
fact = fact*d
enddo
precedente(k)=(((-1)**j)*((ang_graus*conv_rad)**a)/fact)
calculo(k)=precedente(k)+calculo(k-1)
!!!SAIDA SALVO NO ARQUIVO
w=c-1

if (a == c .or. a == w) then
write (639,*) '***********************************************************'
write (639,*) 'Angulo em Radianos e Graus = ',ang_graus*conv_rad, 'e',ang_graus , precedente(k)
write (639,*) 'Valor do Cosseno = ',calculo(k), b , fact, a, calculo(k-1) , j ,k , c,w
write (639,*) '***********************************************************'
end if
calculo(k-1)=calculo(k)
j=j+1
k=k+1
enddo
enddo
pause
end program
```

TABEL 3

For the expoent, the program is simply:

program exponencial
Implicit none
integer :: a, b,d,j,k
real :: exponencial,exponencialpre,fact
dimension :: exponencial(1000),exponencialpre(1000)
open(396, file='expoenteAI.txt', status='replace')
print *, 'Exponencial'
print *, 'Digite o Valor M ximo'
read *, e
do b = 1,20
k=1
j=0
exponencial(0)=0
do a=0,e
fact = 1.0
do d = 1, a
fact = fact*d
Enddo
exponencialpre(k)=(((1)**j)*(b**a)/fact)
exponencial(k)=(((1)**j)*(b**a)/fact)+exponencial(k-1)
write (396,*) '********************'
write (396,*) exponencialpre(k),exponencial(k), b , fact,a,exponencial(k-1) ,j ,k
write (396,*) '********************'
exponencial(k-1)=exponencial(k)
j=j+1
k=k+1
Enddo
Enddo
Pause
end program

TABELA 5

The Calculation of the Tangent:

program tangente
Implicit none
integer :: a,valordoexpoente,k,j,h,b, somatorio,gg,hh ,t ,p,pp,g
real :: denominador,precedente,calculo , soma ,variavel,expoente,kk ,x,coma ,y
dimension :: calculo(1000),precedente(1000), soma(1000), somatn(1000)
real(8), parameter :: pi=3.1415926535897932385
open(369, file='cotg.txt', status='replace')
open(396, file='cotgB.txt', status='replace')
print *, 'Tangente a Partir de Leibniz'
print *, 'Digite o Valor M ximo do Expoente'
do b=1,10
variavel=b
calculo(1)=0
precedente(1)=0
k=2
g=0
p=1

```
do t = 1,51,2
denominador=t
expoente=t
if (t>1) then
expoente=2*g+1
denominador=2*g+1
end if
precedente(k)=((((-1)**(p+1)))*(variavel**expoente)/(denominador))
calculo(k)=precedente(k)
soma(t)=calculo(k)
write (369,*) '**************************************'
write (369,*)  'Valor da Variavel= ' , variavel ,'Valor  da Serie = ', soma(t),'Valor do Expoente =', expoente
write (369,*)  'Valor do Angulo Em Radiano e Graus = ',pi/b,'e'
write (369,*) 'Valor do Denominador =' , denominador,precedente(k),j,hh ,k
write (369,*) '**************************************'

if (t == 51 ) then
somatn(1)=0
do pp=3,51,2
somatn(pp)=somatn(pp-2)+soma(pp-2)
end do
pp=51
x=somatn(pp)+((somatn(pp))**3)/(3)+(2)*((somatn(pp))**5)/(15)
print*,x
write (396,*) '**************************************'
write (396,*) 'Valor do Angulo Em Radiano e Graus = ',b,'e', b,'Valor da Tangente = ', somatn(51)
write (396,*)  'Valor do Angulo Em Radiano e Graus = ',b,'e',x ,somatn(1),somatn(3),somatn(5)
write (396,*) '**************************************'
end if
k=k+1
p=p+1
somatorio=somatorio+1
g=g+1
Enddo
Enddo
Pause
end program
```

program Cálculo_Tangente
Implicit none
integer :: a,valordoexpoente,k,j,h,b, somatorio,gg,hh
real :: denominador,precedente,valor , soma ,variavel,expoente,kk
dimension :: valor(1000),precedente(1000), soma(1000)
real(8), parameter :: pi=3.1415926535897932385
open(369, file='Tangente.txt', status='replace')
open(396, file='TangenteB.txt', status='replace')
print *, 'Tangente a Partir de Leibniz'
print *, 'Digite o Valor M ximo do Expoente'
read *, valordoexpoente
do b = 1,valordoexpoente
variavel=pi/b
soma(0)=0
precedente(0)=0
k=1
hh=1
somatorio=1
j=1
do while (j<=valordoexpoente)
denominador=hh
expoente=hh
print*, denominador
if (hh>1) then
expoente=hh
denominador=hh*((hh-2))
end if
precedente(k)=(variavel**expoente)/(denominador)
valor(k)=precedente(k)+valor(k-1)
write (369,*) '***************************************'
write (369,*) 'Valor da Vari vel = ' , variavel ,'Valor da Serie = ', valor(k),'Valor do Expoente =', expoente
write (369,*) 'Valor de b de Pi/b = ',b, precedente(k), precedente(k-1) ,j,hh ,k,'Valor do Denominador' , denominador
write (369,*) '***************************************'
precedente(k-1)=valor(k)
soma(somatorio)=valor(k)
print*, somatorio,valordoexpoente
kk=somatório
if (kk==valordoexpoente) then
write (396,*) '***************************************'
write (396,*) 'Valor da Tangente = ', soma(somatorio), somatorio ,b
write (396,*) '***************************************'
end if
k=k+1
hh=hh+2

```
program tangente
Implicit none
integer :: a, b,c,d,aaa, bbb,cc,dd,e
integer :: f,g,k,i,j,x,y,w,z,h,fff,jj,kk ,qq,pp
real :: sina,ww,bb,ff,fact, tan,cosa,www   ,cot ,hhh,sin,cos
real(4) :: aa,conv_rad,ang_graus
real(8), parameter :: pi=3.1415926535897932385
dimension ::
cc(1000),ff(1000),tan(1000),cosa(1000),sin(1000),cos(1000),sina(1000),www(1000)
dimension :: ww(1000),aa(1000),bb(1000),cot(10000),valor(10000)
open(22, file='LeibnizA.txt', status='replace')
open(25, file='LeibnizB.txt', status='replace'  )
open(122, file='LeibnizC.txt', status='replace')
open(220, file='LeibnizD.txt', status='replace')
print *, 'Programa descobrindo Séries'
print *, 'Digite o Valor Máximo do expoente'
read *, e
hhh=1
ang_graus=0
do b = 1,360
!!! VARIAVEIS REAIS
!a=3.5555
aa=3.4
conv_rad=pi/180.0
ang_graus=ang_graus + 0.5
k=1
j=0
sin(0)=0
do a=1,e,2
fact = 1.0
do d = 1, a
fact = fact*d
Enddo
ww(k)=(((-1)**j)*((ang_graus*conv_rad)**a)/fact)
sin(k)=(((-1)**j)*((ang_graus*conv_rad)**a)/fact)+sin(k-1)
if ((a+1)==e) then
write (22,*)
'*****************************************************************'
write (22,*)  ww(k)    ,'Angulo =',ang_graus*conv_rad, ang_graus
write  (22,*) 'Seno =', sin(k), b , fact,a,sin(k-1) ,j ,k
write (22,*)
'*****************************************************************
*'
sina(hhh)=sin(k)
hhh=hhh  + 1
end if
```

```
Program razão_valor_das_series
Implicit none
```

integer :: a, b,c,d,aaa, bbb,cc,dd,e
integer :: f,g,k,i,j,x,y,w,z,h,fff,jj,kk
real :: valor
real(8), parameter :: pi=3.1415926535897932385
dimension ::valor(10000000)
open(137, file='LeibnizAIV.txt', status='replace')
print *, 'Leibniz'
print *, 'Digite o Valor M ximo'
do fff=1,200000
valor(fff)=pi/fff
write (137,*) '**'
write (137,*) valor(fff),pi,fff
write (137,*) '***'
end do
do fff=1,200000
valor(fff)=(pi**2)/fff
write (137,*) '********222222222********************'
write (137,*) valor(fff),pi,fff
write (137,*) '**************************************'
end do
do fff=1,200000
valor(fff)=(pi/fff)**2
write (137,*) '********2222222222222GGGGGGGGG*******************'
write (137,*) valor(fff),pi,fff
write (137,*) '**'
end do
do fff=1,200000
valor(fff)=(pi**3)/fff
write (137,*) '********33333333333333********************'
write (137,*) valor(fff),pi,fff
write (137,*) '**'
end do
do fff=1,200000
valor(fff)=(pi**4)/fff
write (137,*) '*******44444444444444*********************'
write (137,*) valor(fff),pi,fff
write (137,*) '**'
end do
do fff=1,200000
valor(fff)=(pi**5)/fff
write (137,*) '********555555555********************'
write (137,*) valor(fff),pi,fff
write (137,*) '************************************'
end do
do fff=1,200000
valor(fff)=(pi**6)/fff

write (137,*) '***6666666*************************'
write (137,*) valor(fff),pi,fff
write (137,*) '***********************************'
end do
do   fff=1,200000
valor(fff)=(pi**7)/fff
write (137,*) '***7777777777*************************'
write (137,*) valor(fff),pi,fff
write (137,*) '**************************************'
end do
do   fff=1,200000
valor(fff)=(pi**8)/fff
write (137,*) '***888888888*************************'
write (137,*) valor(fff),pi,fff
write (137,*) '*************************************'
end do
Pause
end program

[1] BOYER, C. B. *(1996). História da Matemática. Editora Edgard Blücher. [S.l.: s.n.]*

[2] https://mathworld.wolfram.com/PiFormulas.html

[3]Ian Stewart,The greatest math problems of all time– Rio de Janeiro: Zahar, 2014. (3. Puzzle of Pi, page 58)

[4] LEITHOLD, Louis. Calculus with Analytical Geometry Vol. 1. São Paulo: Harbra. 1994

[5] STEWART, J., Calculus, Vol. 2. Thomson Learning, 5th Ed., 2006. (Chapter 11, Infinite Series)

[6]https://mathworld.wolfram.com/RiemannZetaFunctionZeta2.html

[7] ROQUE, Tatiana. História da matemática: uma visão crítica, desfazendo mitos e lendas. Rio de Janeiro: Zahar, 2012.

[8] GONÇALVES, Mirian Buss; FLEMMING, Diva Marília. Cálculo B: funções de várias variáveis integrais duplas e triplas. São Paulo: Makron Books, 2005.

Note of the Article in 2024: Basically, the precision of Pi is achieved through manipulation of the resulting formula, such as that of David-Bailey, Peter Borwein and Simon Plouff 1996(To basically analyze the precision of the decimal places of PI), the article generates the idea, it is worth highlighting that the possibilities of manipulations and attempts are infinite, and each intellect can understand from its own point of view. And the possibilities generate infinite fraction factor formulas with the number pi with all types of series, such as Leibniz's, Riemann's Zeta or Aurea's law. And later coming across Fermat's last theorem as an example, proven by Wiles, in his lectures on modular forms, elliptic curves and Galois theories, in this he would have to delve into abstract algebra in the synergy of mathematical series., which resides in the Taniyama-Shimura Conjecture, that every elliptical curve is modular, but the concrete case of the written article are forms of series in which the written a better way and greater precision of the number Pi and other forms for the number PI, like Ramanujan and example of precision of number Pi like David Harold Bailey, in collaboration with Peter Borwein and Simon Plouffe.