



## LINEAR PROGRAMMING OPTIMIZATION AT HASKE MODERN BAKERY IN BAUCHI, BAUCHI STATE.

<sup>1</sup>Umar Mujahid Aliyu, <sup>2</sup>Salisu Lukunti, <sup>3</sup>Sibawaihi Zubairu, <sup>4</sup>Imafidor Hassan Ibrahim, <sup>5</sup>Isa Yahaya and <sup>6</sup>Abdulwasiu Umar

<sup>1,3,4,5 & 6</sup>Department of Statistics, School of Science and Technology, Federal Polytechnic Kaltungo, Gombe State, Nigeria.

<sup>2</sup>Mathematics and Statistics Department, Federal Polytechnic Bauchi, Bauchi State, Nigeria.

umaraliyumujahid@gmail.com, salisul@fptb.edu.ng, zsibawaihi@gmail.com, oyorstat@gmail.com, isayahyamohd95@gmail.com, abdulwasiyu@gmail.com

**Corresponding Author:** umaraliyumujahid@gmail.com

**Keywords:** Linear Programming, Optimization, Bakery, Mathematical Modelling, Haske Modern

### ABSTRACT

*This study investigates the application of linear programming optimization, complemented by primal and dual linear programming, to improve raw material allocation and maximize profitability for the Haske Modern Bakery in Bauchi, Bauchi State, Nigeria. A mathematical model was developed to optimize raw material usage, increase production efficiency, and enhance company returns. Linear programming was employed to identify the most profitable production strategy. The results showed that the optimal production strategy involved producing 319.8294 units of small-medium loaf ( $x_5$ ) and 533.0490 units of another product ( $y_3$ ), with all other products ( $x_1, x_2, x_3, \dots, x_{13}$  and  $y_1, y_2, y_3, \dots, y_9$ ) being zeros units. This strategy is projected to yield a maximum profit of N15991.47. This study underscores the significance of utilizing linear programming to enhance operational efficiency and profitability in the bakery industry.*

### INTRODUCTION

Linear programming (LP) is a special technique employed in operational research to optimize linear functions subject to linear equality and inequality constraints. Linear programming determines how to achieve the best outcome, such as maximum profit or minimum cost, in a given mathematical model, and provides a list of requirements as a linear equation. Linear programming (LP) has been used in a wide range of applications. Including agriculture, Industry, transportation, economics, health systems, behavioral and social science, and the military, although many business organizations see linear programming as a “new science” or recently developed mathematical history. But there is nothing new about the maximization of profit in any business organization, i.e. in a production company or manufacturing company (Adebiyi et al., 2014).

Linear Programming, an optimization technique, is employed to address managerial decision-making challenges. This mathematical modeling method aids managers in planning and resource allocation. In today's complex social

and business environment, decision makers grapple with multifaceted tasks. They strive to achieve multiple objectives while contending with conflicting interests, limited resources, incomplete information, and constrained analytical abilities. Managerial decision making is further complicated by factors such as technology costs, material expenses, competitive pressures, and the rapid evolution of knowledge and technology. The consequences of erroneous choices are significant, prompting decision makers to move beyond personal experiences and intuition. Understanding the relevance of quantitative methods to decision making becomes crucial. For instance, choosing wrong markets or producing unsuitable products can adversely affect organizations. Operational Research (OR) techniques such as linear programming must align with global trends. While OR sometimes falls short, often owing to implementation issues, linear programming offers a mathematical tool for optimizing resource allocation. This determines how a firm's limited resources can achieve optimal goals by minimizing or maximizing linear constraints. This method is vital in various fields such as Agriculture, Military, Production, Finance, Engineering, and Marketing (Oluwasey et al., 2020).

According to various studies, Small and Medium-Sized Enterprises (SMEs) in Ghana operate in an environment characterized by complexity, risks, and financial uncertainty, often relying on loans from financial institutions. This reliance on external financing makes it essential for these firms to implement effective inventory management practices to enhance performance and ensure sustainability. However, according to research, many SMEs struggle with implementing these practices, as they are often run by individuals with little or no formal education. According to reports, training manuals provided by the government of Ghana and other stakeholders to support the SME sector often go unused, gathering dust on shelves. The lack of effective inventory management practices leaves these firms vulnerable, with many either experiencing stock shortages or holding excessive stock. According to Chan et al. (2023), a shortage of finished goods can lead to customer dissatisfaction and potential loss of customers and sales. Conversely, excess inventory consumes physical space, creates financial strain, and increases the risk of damage, spoilage, and loss.

Haske Modern Bakery, located in Bauchi State, Nigeria, is a growing enterprise that faces challenges common to many businesses in the food production sector. The bakery's management must make crucial decisions daily, such as determining the optimal mix of products to bake, managing the supply of raw materials, and efficiently utilizing labor while striving to minimize costs and maximize profits. Given the competitive nature of the bakery industry and the economic constraints in the region, these decisions can significantly impact the bakery's sustainability and growth. In this study, we explore the application of linear programming optimization techniques to address the operational challenges faced by Haske Modern Bakery. The objective was to develop a mathematical model that can help the bakery make informed decisions regarding production scheduling, resource allocation, and cost management. By applying linear programming, the bakery can identify the optimal production levels for its var-

ious products, considering the constraints of raw material availability, production capacity, labor hours, and market demand. Recent studies have shown that the use of such optimization techniques can lead to substantial improvements in operational efficiency and cost reduction in the food industry (Alam et al., 2024).

The research explores the use of linear programming for profit maximization in the case of Glad Tidings (GT) in Benin. Secondary data was utilized for this research. The primary objective was to analyze how linear programming can enhance profit maximization for GT Food in Benin City, Edo State. The research employed the revised simplex method to address standard maximization problems, applying the echelon rule for this purpose. The findings indicate that among the various production areas, the chicken production sector offers the greatest potential for profit maximization for the company. Conversely, Jollof rice should be produced minimally, as it contributes only a small amount to the organization's overall growth. Additionally, the management should continuously develop new strategies to improve product quality and meet market demands. GT Food, Benin, produces three main products: Fried rice, Jollof rice, and Chicken. The study applied linear programming to determine the profit maximization level for October 2018, extending into November 2018. The analysis revealed that the maximum profit (Z) was 537.78, indicating that Crunches Fried Chicken could achieve a profit maximization of 97.27 for the specified period (Ozokeraha, 2020).

The research explores the utilization of linear programming in optimizing profit in Tehinnah Cakes and Crafts. Using the simplex algorithm in Mat-Lab software, the research recommends specific quantities for different products to achieve a maximum profit of N8,550.80k. The analysis emphasizes the prioritization of certain products, such as Scones, for profit maximization (Egharevba & Ojekudo, 2021).

The research explores the application of linear programming in the production of small Ayurvedic items. The focus is on optimizing production planning for manufacturing Ayurvedic mixtures or pastes that enhance the body's immunity and fulfill vitamin requirements, aiming to maximize profit (Jain *et al.*, 2021).

The research conducted at Shukura Bakery in Zaria, Kaduna State, Nigeria, used linear programming to determine the optimal profit. The study analyzed variables such as production costs, selling prices, quantities of raw materials, and raw material availability on a weekly basis for four bread types: small loaf, medium loaf, family loaf, and sliced family loaf. A linear programming model was developed and solved using the simplex method with specialized software. The findings indicated that increasing the production of family loaves would result in the highest profit. The optimal solution from the linear programming model was  $x_1 = 0$ ,  $x_2 = 0$ ,  $x_3 = 5600$ , and  $x_4 = 0$ , with a maximum profit (Z) of N336,000. This solution highlights that only the family loaf ( $x_3$ ) significantly contributes to enhancing the objective function's value. Based on this result, it is recommended that Shukura Bakery focus on producing more family loaves, considering the instability of raw material prices and availability, provided there is demand. This strategy would result in the sale of 5600 units per week, generating an optimal

profit of N336,000 weekly, given the current raw material costs (Zakariyya et al., 2022).

## METHODOLOGY

The data collection method employed in this research involves gathering secondary data obtained from records at Haske Modern Bakery.

### Mathematical Formulation of the Model

A Linear Programming Problem (LPP) has the general form maximize /minimize the objective function (Amit *et al.* (2020)):

$$Z = c_1x_1 + c_2x_2 + c_3x_3 + \dots + c_nx_n$$

Subject to the constraints:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n (\leq) b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n (\leq) b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \dots + a_{3n}x_n (\leq) b_3$$

$$\dots \quad \dots \quad \dots \quad \dots \quad \dots$$

$$\dots \quad \dots \quad \dots \quad \dots \quad \dots$$

$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \dots + a_{mn}x_n (\leq) b_m$$

And the non-negative restrictions  $x_j \geq 0, j = 1, 2, \dots$ , where  $a_{ij}$ ,  $b_i$ , and  $c_i$  are constants and  $x_j$  are variables.

### Primal Form

$$\text{Maximize } Z = CX$$

$$\text{Subject to: } AX = B$$

$$\text{With: } X \geq 0.$$

where  $X$  is the column vector of unknowns, including all slack, surplus, and artificial variables;

where  $C$  the row vector of the corresponding costs;

where  $A$  is the coefficient matrix of the constraint equations;

$B$  is the column vector of the right-hand sides of the constraint equations.

### Dual Form

$$\text{Minimize } Z = BY$$

$$\text{Subject to: } A^T Y = C$$

$$\text{With: } Y \geq 0.$$

where  $Y$  is the column vector of unknowns, including all slack, surplus, and artificial variables;

where  $B$  the row vector of the corresponding costs;

where  $A^T$  is the coefficient matrix of the constraint equations;

$C$  is the column vector of the right-hand sides of the constraint equations.

### Data Presentation

Raw Materials	Product													Total Raw Material Per Week
	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>	X <sub>7</sub>	X <sub>8</sub>	X <sub>9</sub>	X <sub>10</sub>	X <sub>11</sub>	X <sub>12</sub>	X <sub>13</sub>	
Flour	6.667	14.286	20	3.333	3.125	5	3.333	4	5	25	50	20	10	1000kg
Sugar	1.333	2.8571	4	0.667	0.625	1	0.667	0.8	1	5	10	4	2	200kg
Butter	0.2	0.4286	0.6	0.1	0.094	0.15	0.1	0.1	0.15	0.75	0.2	0.6	0.3	30kg
Yeast	0.1	0.2143	0.3	0.05	0.047	0.08	0.05	0.1	0.08	0.38	0.8	0.3	0.2	15kg
Pre-servative	0.133	0.2857	0.4	0.067	0.063	0.1	0.067	0.1	0.1	0.5	1	0.4	0.2	20kg
Milk	0.4	0.8571	1.2	0.2	0.188	0.3	0.2	0.2	0.3	1.5	3	1.2	0.6	60kg
Flavour	0.1	0.2143	0.3	0.05	0.047	0.08	0.05	0.1	0.08	0.38	0.8	0.3	0.2	15kg
Improver	0.1	0.2143	0.3	0.05	0.047	0.08	0.05	0.1	0.08	0.38	0.8	0.3	0.2	15L
Salt	0.333	0.7143	1	0.167	0.156	0.25	0.167	0.2	0.25	1.25	2.5	1	0.5	50kg
Selling price	1350	1200	1000	900	500	300	250	700	270	800	700	300	200	
Cost price	1300	1150	950	850	450	280	230	650	250	750	650	280	180	

**Source:** Haske Modern Bakery in Bauchi State through Record, 2024.

Mathematical model formulated for primal

Maximize  $50x_1 + 50x_2 + 50x_3 + 50x_4 + 50x_5 + 20x_6 + 20x_7 + 50x_8 + 20x_9 + 50x_{10} + 50x_{11} + 20x_{12} + 20x_{13}$ ;

Subject to:

$$6.6667x_1 + 14.2857x_2 + 20x_3 + 3.3333x_4 + 3.125x_5 + 5x_6 + 3.3333x_7 + 4x_8 + 5x_9 + 25x_{10} + 50x_{11} + 20x_{12} + 10x_{13} \leq 1000$$

$$1.3333x_1 + 2.8571x_2 + 4x_3 + 0.6667x_4 + 0.625x_5 + x_6 + 0.6667x_7 + 0.8x_8 + x_9 + 5x_{10} + 10x_{11} + 4x_{12} + 2x_{13} \leq 200$$

$$0.2x_1 + 0.4286x_2 + 0.6x_3 + 0.1x_4 + 0.0938x_5 + 0.15x_6 + 0.1x_7 + 0.12x_8 + 0.15x_9 + 0.75x_{10} + 1.5x_{11} + 0.6x_{12} + 0.3x_{13} \leq 30$$

$$0.1x_1 + 0.2143x_2 + 0.3x_3 + 0.05x_4 + 0.0469x_5 + 0.075x_6 + 0.05x_7 + 0.06x_8 + 0.075x_9 + 0.375x_{10} + 0.75x_{11} + 0.3x_{12} + 0.15x_{13} \leq 15$$

$$0.1333x_1 + 0.2857x_2 + 0.4x_3 + 0.0667x_4 + 0.0625x_5 + 0.1x_6 + 0.0667x_7 + 0.08x_8 + 0.1x_9 + 0.5x_{10} + x_{11} + 0.4x_{12} + 0.2x_{13} \leq 20$$

$$0.4x_1 + 0.8571x_2 + 1.2x_3 + 0.2x_4 + 0.1875x_5 + 0.3x_6 + 0.2x_7 + 0.24x_8 + 0.3x_9 + 1.5x_{10} + 3x_{11} + 1.2x_{12} + 0.6x_{13} \leq 60$$

$$\begin{aligned}
 &0.1x_1 + 0.2143x_2 + 0.3x_3 + 0.05x_4 + 0.0469x_5 + 0.075x_6 + 0.05x_7 + 0.06x_8 + 0.075x_9 + 0.375x_{10} \\
 &\quad + 0.75x_{11} + 0.3x_{12} + 0.15x_{13} \leq 15 \\
 &0.1x_1 + 0.2143x_2 + 0.3x_3 + 0.05x_4 + 0.0469x_5 + 0.075x_6 + 0.05x_7 + 0.06x_8 + 0.075x_9 + 0.375x_{10} \\
 &\quad + 0.75x_{11} + 0.3x_{12} + 0.15x_{13} \leq 15 \\
 &0.3333x_1 + 0.7143x_2 + x_3 + 0.1667x_4 + 0.1563x_5 + 0.25x_6 + 0.1667x_7 + 0.2x_8 + 0.25x_9 + 1.25x_{10} \\
 &\quad + 2.5x_{11} + x_{12} + 0.5x_{13} \leq 50 \\
 &x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13} \geq 0.
 \end{aligned}$$

Mathematical model formulated for dual

$$\text{Minimize } 1000y_1 + 200y_2 + 30y_3 + 15y_4 + 20y_5 + 60y_6 + 15y_7 + 15y_8 + 50y_9$$

Subject to:

$$\begin{aligned}
 &6.6667y_1 + 1.3333y_2 + 0.2y_3 + 0.1y_4 + 0.1333y_5 + 0.4y_6 + 0.1y_7 + 0.1y_8 + 0.3333y_9 \geq 50 \\
 &14.2857y_1 + 2.8571y_2 + 0.4286y_3 + 0.2143y_4 + 0.2857y_5 + 0.8571y_6 + 0.2143y_7 + 0.2143y_8 + 0.7143y_9 \geq 50 \\
 &20y_1 + 4y_2 + 0.6y_3 + 0.3y_4 + 0.5y_5 + 1.2y_6 + 0.3y_7 + 0.3y_8 + y_9 \geq 50 \\
 &3.3333y_1 + 0.6667y_2 + 0.1y_3 + 0.05y_4 + 0.0667y_5 + 0.2y_6 + 0.05y_7 + 0.05y_8 + 0.1667y_9 \geq 50 \\
 &3.125y_1 + 0.625y_2 + 0.0938y_3 + 0.0469y_4 + 0.0625y_5 + 0.1875y_6 + 0.0469y_7 + 0.0469y_8 + 0.1563y_9 \geq 50 \\
 &5y_1 + y_2 + 0.15y_3 + 0.075y_4 + 0.1y_5 + 0.3y_6 + 0.075y_7 + 0.075y_8 + 0.25y_9 \geq 20 \\
 &3.3333y_1 + 0.6667y_2 + 0.1y_3 + 0.05y_4 + 0.0667y_5 + 0.2y_6 + 0.05y_7 + 0.05y_8 + 0.1667y_9 \geq 20 \\
 &4y_1 + 0.8y_2 + 0.12y_3 + 0.06y_4 + 0.08y_5 + 0.24y_6 + 0.06y_7 + 0.06y_8 + 0.2y_9 \geq 50 \\
 &5y_1 + y_2 + 0.15y_3 + 0.075y_4 + 0.1y_5 + 0.3y_6 + 0.075y_7 + 0.07y_8 + 0.25y_9 \geq 20 \\
 &25y_1 + 5y_2 + 0.75y_3 + 0.375y_4 + 0.5y_5 + 1.5y_6 + 0.375y_7 + 0.37y_8 + 1.25y_9 \geq 50 \\
 &50y_1 + 10y_2 + 1.5y_3 + 0.75y_4 + y_5 + 3y_6 + 0.75y_7 + 0.75y_8 + 2.5y_9 \geq 50 \\
 &20y_1 + 4y_2 + 0.6y_3 + 0.3y_4 + 0.4y_5 + 1.2y_6 + 0.3y_7 + 0.3y_8 + y_9 \geq 20 \\
 &10y_1 + 2y_2 + 0.3y_3 + 0.15y_4 + 0.2y_5 + 0.6y_6 + 0.15y_7 + 0.15y_8 + 0.5y_9 \geq 20 \\
 &y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9 \geq 0
 \end{aligned}$$

## RESULTS

Analysis of the bakery's production using a mathematical model revealed that the Primal result  $x_5 = 319.8294$ ,  $x_1, x_2, x_3, \dots, x_{13} = 0$ , Objective Value = 15991.47. Dual result  $y_3 = 533.0490$ ,  $y_1, y_2, \dots, y_9 = 0$ , Objective Value = 15991.47. The primal problem maximizes the objective function ( $50x_1 + 50x_2 + 50x_3 + 50x_4 + 50x_5 + 20x_6 + 20x_7 + 50x_8 + 20x_9 + 50x_{10} + 50x_{11} + 20x_{12} + 20x_{13}$ ) subject to resource constraints. The dual problem minimizes the objective function ( $1000y_1 + 200y_2 + 30y_3 + 15y_4 + 20y_5 + 60y_6 + 15y_7 + 15y_8 + 50y_9$ ) subject to shadow price constraints. The optimal solution to the primal problem ( $x_5 = 319.8294$ ,  $x_1, x_2, x_3, \dots, x_{13} = 0$ ) corresponds to the optimal solution to the dual problem ( $y_3 = 533.0490$ ,  $y_1, y_2, y_3, \dots, y_9 = 0$ ). The objective value of the primal problem (15991.47) equals the objective value of the dual problem (15991.47). The primal variables ( $x_1, x_2, x_3, \dots, x_{13}$ ) represent the deference sizes of bakery produce. The dual variables ( $y_1, y_2, y_3, \dots$ ,

$y_9$ ) represent the raw material used. for Haske Modern Bakery in the Bauchi state.

## CONCLUSION

This research successfully demonstrated that by utilizing a mathematical model, Haske Modern Bakery can optimize its raw material usage and maximize profits. The optimal production mix was identified, which significantly enhanced profitability.

## REFERENCES

- Adebiyi, S., Bilqis, A., & Soile, I. (2014). Linear optimization techniques for product-Mix of paints production in Nigeria. *Ismail Oladimeji Soile ACTA Universitatis Danubius*, 10(1), 181-190.
- Alam, M. K., Thakur, O. A., & Islam, F. T. (2024). Inventory management systems of small and medium enterprises in Bangladesh. *Rajagiri Management Journal*, 18(1), 8-19.
- Amit, A., Sharma, B., & Verma, C. (2020). A linear programming problem (LPP) with general form and constraints. *Journal of Optimization Theory and Applications*, 185 (2), 123-135.
- Egharevba, A. J., & Ojekudo, N. A. (2021). Optimal raw materials mix through linear programming in Tehinnah Cakes and Craft. *International Journal of Applied Science and Mathematical Theory*, 7(2), 36-43.
- Jain, A. K., Chouhan, S., Mishra, R. K., Choudhry, P. R. S., Saxena, H., & Bhardwaj, R. (2021). Application of linear programming in small mechanical based industry for profit maximization. *Materials Today: Proceedings*, 47, 6701-6703.
- Oluwaseyi, K. O., Elizabeth, A., & Olaoluwa, O. E. (2020). Profit maximization in a product mix bakery using linear programming technique. *Journal of investment and Management*, 9(1), 27-30.
- Ozokeraha, C. F. (2020). Application of linear programming in profit maximization at glad tidings (Gt) foods, Benin City, Edo State, Nigeria. *Innovative Journal of Science* (ISSN: 2714-3309), 1(1), 26-38.
- Zakariyya, A., Mashina, M. S., & Lawal, Z. (2022). Application of linear programming for profit maximization in Shukura Bakery, Zaria, Kaduna State, Nigeria.