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# Level Dependent Perishable Inventory System in Supply Chain Environment 

Dr. Mohammad Ekramol Islam, Professor of Mathematics, Department of Business Administration, Northern University Bangladesh.

Rupen Barua, Assistant professor of Mathematics, Directorate of Secondary and Higher Education, Bangladesh, Dhaka.

Dr. Ganesh Chandra Ray, Professor, Department of Mathematics, Chittagong University, Chittagong, Bangladesh.


This paper analyzes an ( $s, S$ ) Inventory system in supply chain environment. In this paper, we consider a level dependent perishable inventory system, where raw materials arrive from two warehouses which are situated nearby the central processing unit. Arrival of demands follows Poisson process with rate $\lambda$. Production takes place when at least one component of each category is available in both the warehouses. Replenishment for the warehouses occurs in negligible time once the component amounts reaches to zero unit. It is assumed that the initially inventory level is in $S$ and system is in OFF mode. Inventory level decreases due to demands and perishability. When the inventory level reaches to $s$ then the system converted ON mode from OFF mode. The production follows exponentially distributed with parameter $\mu$. Perishability follows exponentially distributed with parameter $\theta$. Perishability will be level dependent that is rate of perishability will depend on the amount of inventory available in the stock. Steady State analysis is made and some system characteristics are evaluated by numerical illustration.
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## Introduction:

The analysis of inventory systems is primarily focused on the tactical question of which inventory control policies to use and the operational questions of when and how much inventory to order. By and large, these are the main questions for managing the inventory of perishable items as well. A lot of work has been done in inventory modeling with remarkable consideration of parishability of the items. Huge literature can be found in Nahmias S. [1] and later by the same author in [2]. Karaesmen et al. [3] also mentioned the inventory problem in future directions. But recently inventory system with supply chain management addressed by few researchers. Datta and Pal [4] extended the model to the case. in which the demand rate of an item is dependent on the instantaneous inventory level until a given inventory level is achieved, after which the demand rate becomes constant. They assumed that at the end of each cycle, the inventory level is zero. Hwang and Hahn [5] dealt with an optimal procurement policy of perishable item with stock dependent demand rate and FIFO issuing policy. Since the stock-dependent demand rate implicitly implies that all items in inventory are displayed for sale, the customers enforce the issuing policy and last-in-first-out (LIFO) issuing is a natural choice with prudent customers who are always looking for the freshest ones among the displayed items. On the other hand, most retailers arrange their displayed goods from the oldest up or front to the new goods down or back hoping that customers may pick the oldest ones first, which results in first-in-first-out (FIFO) issuing. Consequently, among displayed goods some are sold by LIFO principle while others by FIFO principle, which we call mixed issuing policy.

Blackburn and Scudder [6] discussed their paper, the challenge for companies in managing the supply chain of perishable foods is that the value of the product deteriorates significantly over time at rates that are highly dependent on the environment .
Leat [7] discussed that in the future food system will have to joint four major characteristics: resilience, sustainability, competitiveness, and ability to manage and meet customer expectations. Mohammad Ekramol Islam [8], discussed a perishable ( $s, S$ ) inventory system with postponed demands. They assume that customers arrive to the system according to a Poisson process with rate $\lambda>0$. When inventory level depletes to $s$ due to demands or decay or service to a pooled customer, an order for replenishment is placed. The lead time is exponentially distributed with parameter $\gamma$. When inventory level reaches zero, the incoming customers are sent to a pool of capacity $M$. Any demand that takes place when the pool is full and inventory level is zero, is assumed to be lost. After replenishment, as long as the inventory level is greater than $s$, the pooled customers are selected according to an exponentially distributed time lag, with rate depending on the number in the pool. Earlier we developed perishable inventory system in supply chain environment[9]. This paper is extension of our previous paper mentioned earlier.

In this model we consider, a level dependent perishable inventory system, where raw materials arrive from two warehouses which are situated nearby the central processing unit. Production takes place when at least one component of each category is available in both the warehouses. Replenishment for the warehouses occurs in negligible time once the component amounts reaches to zero unit. It is assumed that the initial level is in $S$ and system is in OFF mode. Inventory level decreases due to demands and perish ability. When the inventory level reaches to $s$ then the system converted ON mode from OFF mode. The production follows exponentially distributed with parameter $\mu$. Perishability follows exponentially distributed with parameter $\theta$. Perishability will be level dependent. When inventory reaches to order level $S$ system converted to ON to OFF mode.
The paper is arranged in the following ways. In section 2.1: assumptions, 2.2: notations, section 3: model \& analysis, section 4: steady state analysis, section 5: system characteristics, section 6: cost function of the system, section 7: numerical illustrations, section 8: graphical presentation of the system, section 9: conclusion, section 10 : references, section 11: appendix ( $1 \& 2$ ).

### 2.1. Assumption:

i) Initially the inventory level is $S$
ii) Demands arrive according to Poisson process with rate $\lambda$
iii) Raw materials arrive from two warehouses, situated nearby the central processing unit,
iv) Production occurred when at least one component of each category is available in both the warehouses,
v) Replenishment for the warehouses instantaneous.
vi) When inventory level reaches to re-order level $s$ then the system converted to OFF mode to ON mode and production starts,
vii) When the inventory level reaches to zero, the arriving demands are lost forever.
viii) Production will be ON until the inventory level reaches to order level $S$. Production follow exponentially distributed with parameter $\mu$.
ix) Perishiability follows exponentially distributed with parameter $\theta$
x) Perishiability will be level dependent i.e, rate of Perishiability will depend the amount of inventory available in the stock

### 2.2 Notations:

a) $S \quad \rightarrow \quad$ Maximum Inventory Level (Order level)
b) $s \quad \rightarrow \quad$ Re-order level
c) $\lambda \quad \rightarrow \quad$ Demand rate
d) $\mathrm{Q}_{1} \rightarrow$ Amount of first warehouse component
e) $\mathrm{Q}_{2} \rightarrow$ Amount of second warehouse component
f) $\mu \rightarrow \quad$ Production rate
g) $I(t) \rightarrow \quad$ Inventory Level at time $t$
h) $X(t) \rightarrow\left\{\begin{array}{l}1 \text { if production is in ON mode } \\ 0 \text { if production is in OFF mode }\end{array}\right\}$
i) $w_{1}(\mathrm{t}) \rightarrow \quad$ Warehouse - 1
j) $w_{2}(\mathrm{t}) \rightarrow$ Warehouse - 2
k) $E \rightarrow E_{1} \times E_{2} \times E_{3} \times E_{4} \rightarrow$ the state space of the process
$\mathrm{E}_{1}=\{0,1,-----S\}$
$E_{2}=\left\{0,1,-\cdots-Q_{1}\right\}$
$\mathrm{E}_{3}=\left\{0,1,-\cdots--\mathrm{Q}_{2}\right\}$
$\mathrm{E}_{4}=\{0,1\}$
i) e is the component column vector of I's.
j) $i \theta \rightarrow$ level dependent inventory perishable rate.

## 3. Model and Analysis:

In this model, the inventory system starts with order level $S$ units of the item on stock. Demands arrive according to Poisson process with rate $\lambda$. We consider a level dependent perishable inventory system, where raw materials arrive from two warehouses, situated nearby the central processing unit. Production occurred when at least one component of each category is available in both the warehouses. Replenishment for the warehouses occurs in negligible time once the component amounts reaches to zero. When inventory level reaches to re-order level $s$ then the system converted to OFF mode to ON mode and production starts. When the inventory level reaches to zero then arriving demand is lost for ever. Production will be ON until the inventory level reaches to order level $S$. Production Upto the inventory level $s+1$ the system is act as a death process as the system is in OFF mode. But from $s$ onwards, the system is act as a birth \& death process as the system is in ON Mode. Production follows exponentially distributed with parameter $\mu$. Perishability follows exponentially distributed with parameter $\theta$.


Figure: Level Dependent Perishable Inventory System in Supply Chain Environment

## The Infinitesimal generator $\widetilde{\boldsymbol{A}}$ of the four dimensional Markov Process:

$\left[\mathrm{I}(\mathrm{t}), \mathrm{W}_{1}(\mathrm{t}), \mathrm{W}_{2}(\mathrm{t}), \mathrm{X}(\mathrm{t}) ; \mathrm{t} \geq 0\right.$ ] can be defined
$\tilde{A}=(a(i, j, k, l: u, v, w, y)):(i, j, k, l),(u, v, w, y) \in E$ Where

## For ON Mode:

$A^{*}=(a(i, j, k, l: u, v, w, y)=$


## For OFF Mode:



Now, the infinitesimal generator $\tilde{A}$ can be conveniently express as a partition matrix $\tilde{A}=\left(A_{i, j, k, l}\right)$ Where the submatrices are

$$
\begin{aligned}
& A_{1}=a(i, j, k, l)_{\left[\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}, A_{2}=a(i, j, k, l)_{\left[\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}, \\
& A_{3}=a(i, j, k, l)_{\left[\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}, \quad A_{4}=a(i, j, k, l)_{\left[2\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times 2\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}, \\
& A_{5}=a(i, j, k, l)_{\left[2\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times 2\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}, A_{6}=a(i, j, k, l)_{\left[\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}, \\
& A_{7}=a(i, j, k, l)_{\left[2\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}, \\
& A_{8}=a(i, j, k, l)_{\left[\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}, \\
& A_{9}=a(i, j, k, l)_{\left[\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}, \\
& A_{10}=a(i, j, k, l)_{\left[\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}, \\
& A_{11}=a(i, j, k, l)_{\left[\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]},
\end{aligned} A_{12}=a(i, j, k, l)_{\left[\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]} .
$$

Which are given below:

$$
\begin{array}{ll}
A_{1}=a(i, j, k, l)_{\left[\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]} & \\
(i, j, k, l) \rightarrow\left(i, j, k+Q_{2}, l\right) \text { is } Q_{2} & : \forall, i=S ; j=0 \ldots . Q_{1} ; k=0 ; l=0 \\
(i, j, k, l) \rightarrow\left(i, j+Q_{1}, k, l\right) \text { is } Q_{1} & : \forall, i=S ; j=0 ; k=0, \ldots Q_{2} ; l=0 \\
(i, j, k, l) \rightarrow(i, j, k, l) \text { is }-\lambda & : \forall, i=S ; j=1, \ldots . Q_{1} ; k=1, \ldots . Q_{2} ; l=0 \\
(i, j, k, l) \rightarrow(i, j, k, l) \text { is }-\left(\lambda+Q_{2}\right) & : \forall, i=S ; j=1, \ldots . Q_{1} ; k=0 ; l=0 \\
(i, j, k, l) \rightarrow(i, j, k, l) \text { is }-\left(\lambda+Q_{1}\right) & : \forall, i=S ; j=0 ; k=1, \ldots . Q_{2} ; l=0 \\
(i, j, k, l) \rightarrow(i, j, k, l) \text { is }-\left(\lambda+Q_{1}+Q_{2}\right) & : \forall, i=S ; j=0 ; k=0 ; l=0
\end{array}
$$

## : Otherwise

$$
A_{2}=a(i, j, k, l)_{\left[\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}=
$$

$$
\left[\begin{array}{l}
(i, j, k, l) \rightarrow(i-1, j, k, l) \text { is } \lambda \\
0
\end{array}\right.
$$

$$
: \forall, i=S ; j=0 \ldots Q_{1} ; k=0 \ldots Q_{2} ; l=0
$$

$$
A_{3}=a(i, j, k, l)_{\left[\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}=
$$

$$
\left[(i, j, k, l) \rightarrow(i+1, j, k, l) \text { is } \mu \quad: \forall, i=S-1 ; j=0 \ldots Q_{1} ; k=0 \ldots Q_{2} ; l=1\right.
$$

$$
A_{4}=a(i, j, k, l)_{\left[2\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times 2\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}=
$$

$\begin{cases}(i, j, k, l) \rightarrow\left(i, j, k+Q_{2}, l\right) \text { is } Q_{2} & : \forall, i=s+1, \ldots S \\ (i, j, k, l) \rightarrow\left(i, j, k+Q_{2}, l\right) \text { is } Q_{2} & : \forall, i=s+1, \ldots S \\ (i, j, k, l) \rightarrow\left(i, j+Q_{1}, k, l\right) \text { is } Q_{1} & : \forall, i=s+1, \ldots S \\ (i, j, k, l) \rightarrow\left(i, j+Q_{1}, k, l\right) \text { is } Q_{1} & : \forall, i=s+1, \ldots S \\ (i, j, k, l) \rightarrow(i, j, k, l) \text { is }-\lambda & : \forall, i=s+1, \ldots S \\ (i, j, k, l) \rightarrow(i, j, k, l) \text { is }-\left(\lambda+Q_{2}\right) & : \forall, i=s+1, \ldots S \\ (i, j, k, l) \rightarrow(i, j, k, l) \text { is }-\left(\lambda+Q_{2}\right) & : \forall, i=s+1, \ldots S \\ (i, j, k, l) \rightarrow(i, j, k, l) \text { is }-\left(\lambda+Q_{1}\right) & : \forall, i=s+1, \ldots S \\ (i, j, k, l) \rightarrow(i, j, k, l) \text { is }-\left(\lambda+Q_{1}\right) & : \forall, i=s+1, \ldots S \\ (i, j, k, l) \rightarrow(i, j, k, l) \text { is }-(\lambda+\mu) & : \forall, i=s+1, \ldots S \\ (i, j, k, l) \rightarrow(i, j, k, l) \text { is }-\left(\lambda+Q_{1}+Q_{2}\right) & : \forall, i=s+1, \ldots S-1 \\ (i, j, k, l) \rightarrow(i, j, k, l) \text { is }-\left(\lambda+Q_{1}+Q_{2}\right) & : \forall, i=s+1, \ldots S \\ 0 & : \text { Otherwise }\end{cases}$

$$
\begin{aligned}
& A_{5}=a(i, j, k, l)_{\left[2\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times 2\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}= \\
& \begin{cases}(i, j, k, l) \rightarrow(i-1, j, k, l) \text { is } \lambda & : \forall, i=s+2 ; j=0 \ldots . Q_{1} ; k=0 \ldots Q_{2} ; l=0 \\
(i, j, k, l) \rightarrow(i-1, j, k, l) \text { is } \lambda & : \forall, i=s+2 ; j=0 \ldots . Q_{1} ; k=0 \ldots . Q_{2} ; l=1\end{cases} \\
& \mathrm{C}_{0} \quad: \text { Otherwise } \\
& A_{6}=a(i, j, k, l)_{\left[\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}= \\
& {\left[\begin{array}{ll}
(i, j, k, l) \rightarrow(i+1, j-1, k-1, l) \text { is } \mu & : \forall, i=s+1 ; j=0 \ldots . Q_{1} ; k=0 \ldots . Q_{2} ; l=1 \\
0 & : \text { Otherwise }
\end{array}\right.} \\
& A_{7}=a(i, j, k, l)_{\left[2\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}= \\
& {\left[\begin{array}{ll}
(i, j, k, 0) \rightarrow(i-1, j, k, 1) \text { is } \lambda & : \forall, i=s+1 ; j=0 \ldots . Q_{1} ; k=0 \ldots . Q_{2} \\
0 & : \text { Otherwise }
\end{array}\right.} \\
& A_{8}=a(i, j, k, l)_{\left[\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}= \\
& {\left[(i, j, k, l) \rightarrow(i+1, j-1, k-1, l) \text { is } \mu \quad: \forall, i=s ; j=0 \ldots . Q_{1} ; k=0 \ldots . Q_{2} ; l=1\right.} \\
& 0 \quad: \text { Otherwise } \\
& A_{9}=a(i, j, k, l)_{\left[\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}= \\
& (i, j, k, l) \rightarrow\left(i, j, k+Q_{2}, l\right) \text { is } Q_{2} \\
& : \forall, i, \ldots \ldots . . s ; j=0 \ldots . Q_{1} ; k=0 ; l=1 \\
& (i, j, k, l) \rightarrow\left(i, j+Q_{1}, k, l\right) \text { is } Q_{1} \quad: \forall, i=i, \ldots \ldots \ldots . . . s ; j=0 ; k=0, \ldots . Q_{2} ; l=1 \\
& (i, j, k, l) \rightarrow(i, j, k, l) \text { is }-\left(\lambda+Q_{2}\right) \quad: \forall, i=i, \ldots \ldots \ldots . . . s ; j=1, \ldots . Q_{1} ; k=0 ; l=1 \\
& (i, j, k, l) \rightarrow(i, j, k, l) \text { is }-\left(\lambda+Q_{1}\right) \quad: \forall, i=i, \ldots \ldots \ldots . . . s ; j=0 ; k=1, \ldots . Q_{2} ; l=1 \\
& (i, j, k, l) \rightarrow(i, j, k, l) \text { is }-(\lambda+\mu) \quad: \forall, i=i, \ldots \ldots \ldots . . . s ; j=1, \ldots . Q_{1} ; k=1, \ldots . Q_{2} ; l=1 \\
& (i, j, k, l) \rightarrow(i, j, k, l) \text { is }-\left(\lambda+Q_{1}+Q_{2}\right): \forall, i=i, \ldots \ldots . . . . . s ; j=0 ; k=0 ; l=1 \\
& \text { : Otherwise } \\
& A_{10}=a(i, j, k, l)_{\left[\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}= \\
& : \forall, i=1 \text {, } \\
& s ; j=0 \ldots Q_{1} ; k=0, \ldots Q_{2} ; l=1 \\
& \text { : Otherwise } \\
& A_{11}=a(i, j, k, l)_{\left[\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]}= \\
& {\left[\begin{array}{ll}
(i, j, k, l) \rightarrow(i+1, j-1, k-1, l) \text { is } \mu & : \forall, i=0, \ldots \ldots . s-1 ; j=0 \ldots . Q_{1} ; k=0, \ldots . Q_{2} ; l=1 \\
0 & : \text { Otherwise }
\end{array}\right.}
\end{aligned}
$$

$$
\begin{aligned}
& A_{12}=a(i, j, k, l)_{\left[\left(Q_{1}+1\right)\left(Q_{2}+1\right) \times\left(Q_{1}+1\right)\left(Q_{2}+1\right)\right]} \\
& \left(\begin{array}{ll}
(i, j, k, l) \rightarrow\left(i, j, k+Q_{2}, l\right) \text { is } Q_{2} & : \forall, i=0 ; j=0 \ldots Q_{1} ; k=0 ; l=1 \\
(i, j, k, l) \rightarrow\left(i, j+Q_{1}, k, l\right) \text { is } Q_{1} & : \forall, i=0 ; j=0 ; k=0, \ldots Q_{2} ; l=1 \\
(i, j, k, l) \rightarrow(i, j, k, l) \text { is }-Q_{2} & : \forall, i=0 ; j=1, \ldots Q_{1} ; k=0 ; l=1 \\
(i, j, k, l) \rightarrow(i, j, k, l) \text { is }-Q_{1} & : \forall, i=0 ; j=0 ; k=1, \ldots Q_{2} ; l=1 \\
(i, j, k, l) \rightarrow(i, j, k, l) \text { is }-\mu & : \forall, i=0 ; j=1, \ldots Q_{1} ; k=1, \ldots Q_{2} ; l=1 \\
(i, j, k, l) \rightarrow(i, j, k, l) \text { is }-\left(Q_{1}+Q_{2}\right) & : \forall, i=0 ; j=0 ; k=0 ; l=1 \\
0 & : \text { Otherwise }
\end{array}\right.
\end{aligned}
$$

So we can write the partitioned matrix as follows:


## 4. Steady State Analysis:

It can be seen from the structure of matrix $\tilde{A}$ that the state space $E$ is irreducible. Let the limiting distribution be denoted by $x^{(i, j, k, l)}$

$$
\begin{aligned}
& x^{(i, j, k, l)}=\lim _{x \rightarrow \infty} \operatorname{Pr}\left[I(t), w_{1}(t), w_{2}(t), X(t)(i, j, k, l) \mid I(0), w_{1}(0), w_{2}(0), X(0)\right. \\
& =(S, 0,0,0)],(i, j, k, l) \in E .
\end{aligned}
$$

Let $\mathrm{x}=\left(x^{(S, 0)} \ldots, x^{(S-1,0)} \ldots x^{(s+1,1)}, x^{(s, 1)} \ldots x^{(0,1)}, x^{(1,1)} \ldots x^{(S-1,1)}\right)$ with $x^{\left(\mathrm{K}, \mathrm{O}, \mathrm{Q}_{2}\right)}$,
$x\left({ }^{(\mathrm{K}, 1, \mathrm{Q}}\right)^{2} \ldots \ldots \ldots x^{\left(\mathrm{K}, \mathrm{Q}, \mathrm{Q}_{2}\right)}$ ) Where $K=((S, 0),(S-1,0) \ldots . .(S-1,1))$
$\forall, K=(S, 0), \ldots,(S-1,1),(S-2,1) \ldots(s+1,1),(s, 1) \ldots(1,1),(0,1)$
The limiting distribution exists, satisfies the following equation:
$\mathbf{x} \widetilde{A}=0$ and $\sum \sum \sum \sum x^{(i, j, k, l)}=1$.
Theorem: If $\mathbf{x}=\left\{x_{i}, i \geq 0\right\}$ is stationary distribution, them $\mathbf{x} \widetilde{A}=0$
Proof: We have from kolmogorov forward differential equation $x_{t}{ }^{\prime}=x_{t} \tilde{A}$
$x_{i j}^{\prime}(t)=-P_{i j}(t) a(j, j)+\sum_{k \neq j} x_{i k}(t) a(k, j) \ldots$
Since $\mathbf{x}$ is stationary then $t \rightarrow \infty$, if limit exists, it is independent of time parameter and hense $x_{i j}^{\prime}(t) \rightarrow 0$

From eq (a) we get
$-x_{i j}(t) a(j, j)+\sum_{k \neq j} P_{i k}(t) a(k, j)=0$
In matrix notation which can be written as $\mathbf{x} \tilde{A}=0$ and Normalizing condition hold; then $\Sigma \Sigma \Sigma \sum x^{(i, j, k, 0)}$ and $\sum \sum \sum \sum x^{(i, j, k, 1)}$ can be completely evaluated.

By using the equation (1) with normalizing condition, we calculate all the steady state probability vector by using Mathematica software can be measured (see appendix-1\&2)

## 5. System Characteristics:

a) Expected total inventory of the system:

$$
L=\sum_{i=s+1}^{s} i \sum_{j=0}^{Q_{1}} \sum_{k=0}^{Q_{2}} x^{i, j, k, 0}+\sum_{i=1}^{s-1} i \sum_{j=0}^{Q_{1}} \sum_{k=0}^{Q_{2}} x^{i, j, k, 1}
$$

b) Re-production rate of the system:
$R=\lambda \sum_{j=0}^{Q_{1}} \sum_{k=0}^{Q_{2}} x^{s+1, j, k, 0}$
c) Number of customers lost in the system:
$C L=\lambda \sum_{j=0}^{Q_{1}} \sum_{k=0}^{Q_{2}} x^{0, j, k, 1}$
d) Expected amount of inventory in warehouse-1:
$W_{1}=\sum_{j=1}^{Q_{1}} j \sum_{k=0}^{Q_{2}} \sum_{i=s+1}^{S} x^{i, j, k, 0}+\sum_{j=1}^{Q_{1}} j \sum_{k=0}^{Q_{2}} \sum_{i=1}^{S-1} x^{i, j, k, 1}$
e) Expected amount of inventory in warehouse-2
$W_{2}=\sum_{k=1}^{Q_{2}} k \sum_{j=0}^{Q_{1}} \sum_{i=s+1}^{S} x^{i, j, k, 0}+\sum_{k=1}^{Q_{2}} k \sum_{j=0}^{Q_{1}} \sum_{i=1}^{S-1} x^{i, j, k, 1}$
f) Expected amount to be perished.

$$
P=\sum_{i=s+1}^{S} i \theta \sum_{j=0}^{Q_{1}} \sum_{k=0}^{Q_{2}} x^{i, j, k, 0}+\sum_{i=1}^{S-1} i \theta \sum_{j=0}^{Q_{1}} \sum_{k=0}^{Q_{2}} x^{i, j, k, 1}
$$

## 6. Cost Function of the system

$c_{1}=$ Holding cost of the system
$c_{2}=$ Re-switching cost of the system
$c_{3}=$ Cost of customer lost in the system
$c_{4}=$ Inventory holding cost in warehouse -1
$c_{5}=$ Inventory holding cost in warehouse - 2
$c_{6}=$ Expected amount to be perished
So expected total cost of the system:
$E(T C)=C_{1} L+C_{2} R+C_{3}(C L)+C_{4} W_{1}+C_{5} W_{2}+C_{6} P$

## 7. Numerical Illustration:

By giving values to the underlying parameters we provide some numerical illustrations. Take
$\mathrm{S}=5, \mathrm{~s}=2, \mathrm{Q}_{1}=\mathrm{Q}_{2}=2, \lambda=2, \mu=2.1, \theta=0.2$
$c_{1}=1, c_{2}=2, c_{3}=3, c_{4}=1, c_{5}=2, c_{G_{5} J \odot 2018}=1$

## Then we get the measures as described in Table 7.1

| Holding cost <br> of the <br> system | Switching cost <br> of the system | Cost of <br> customer <br> lost in the <br> system | Inventory <br> holding cost <br> in warehouse <br> -1 | Inventory <br> holding cost <br> in warehouse <br> -2 | Expected <br> amount to be <br> perished | Expected <br> total cost of <br> the system |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.891969506 | 0.0030993948 | 1.0098298 | 0.456916205 | 0.479999752 | 0.4289425334 | 5.773515941 |

Table : 7.1 Numerical values of different system characteristics.

## 8. Graphical Presentation of the System



Graph-1: Holding cost Vs Total cost of the system


Graph-2: Switching cost Vs total cost of the system



Graph-4: Inventory holding cost in warehouse-1 Vs Total cost of the system


Graph-5: Inventory holding cost in warehouse-2 Vs Total cost of the system


Graph-6: Demand rate Vs Total cost of the


Graph-7: Expected Amount to be Perished Vs Total cost of the system

## 9. Conclusion:

All cost in the present system raise the total cost. It is observed from the table tables [1-7] that for a small change of holding cost total cost increases in a remarkable amount. Hence, the holding cost is most sensitive to raise the total cost. So, we have to take care of holding cost to reduced the expected total cost of the system.


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## 11. Appendix-I

By exploiting the equation $\mathbf{x} \widetilde{A}=0$
$-\left(Q_{1}+Q_{2}\right) x_{0001}+(\lambda+\theta) x_{1001}=0$
$Q_{1} x_{0011}+(\lambda+\theta) x_{1011}=0$
$Q_{2} x_{0101}+(\lambda+\theta) x_{1101}=0$
$-\mu x_{0111}+(\lambda+\theta) x_{1111}=0$
$-Q_{1} x_{0021}+(\lambda+\theta) x_{1021}+Q_{2} x_{0001}=0$
$-\mu x_{0121}+(\lambda+\theta) x_{1121}+Q_{2} x_{0101}=0$
$-Q_{2} x_{0201}+(\lambda+\theta) x_{1201}+Q_{1} x_{0001}=0$
$-\mu x_{0211}+(\lambda+\theta) x_{1211}+Q_{1} x_{0011}=0$
$-\mu x_{0221}+(\lambda+\theta) x_{1221}+Q_{2} x_{0201}+Q_{1} x_{0021}=0$
$-\left(\lambda+\theta+Q_{1}+Q_{2}\right) x_{1001}+(\lambda+2 \theta) x_{2001}+\mu x_{0111}=0$
$-\left(\lambda+\theta+Q_{1}\right) x_{1011}+(\lambda+2 \theta) x_{2011}+\mu x_{0121}=0$
$-\left(\lambda+\theta+Q_{2}\right) x_{1101}+(\lambda+2 \theta) x_{2101}+\mu x_{0211}=0$
$-(\lambda+\theta+\mu) x_{1111}+(\lambda+2 \theta) x_{2111}+\mu x_{0221}=0$
$-\left(\lambda+\theta+Q_{1}\right) x_{1021}+(\lambda+2 \theta) x_{2021}+Q_{2} x_{1001}=0$
$-(\lambda+\theta+\mu) x_{1121}+(\lambda+2 \theta) x_{2121}+Q_{2} x_{1101}=0$
$-\left(\lambda+\theta+Q_{2}\right) x_{1201}+(\lambda+2 \theta) x_{2201}+Q_{1} x_{1001}=0$
$-(\lambda+\theta+\mu) x_{1211}+(\lambda+2 \theta) x_{2211}+Q_{1} x_{1011}=0$
$-(\lambda+\theta+\mu) x_{1221}+(\lambda+2 \theta) x_{2221}+Q_{2} x_{1201}+Q_{1} x_{1021}=0$
$-\left(\lambda+2 \theta+Q_{1}+Q_{2}\right) x_{2001}+(\lambda+3 \theta) x_{3001}+(\lambda+3 \theta) x_{3000}+\mu x_{1111}=0$
$-\left(\lambda+2 \theta+Q_{1}\right) x_{2011}+(\lambda+3 \theta) x_{3011}+(\lambda+3 \theta) x_{3010}+\mu x_{1121}=0$
$-\left(\lambda+2 \theta+Q_{2}\right) x_{2101}+(\lambda+3 \theta) x_{3101}+(\lambda+3 \theta) x_{3100}+\mu x_{1211}=0$
$-(\lambda+2 \theta+\mu) x_{2111}+(\lambda+3 \theta) x_{3111}+(\lambda+3 \theta) x_{3110}+\mu x_{1221}=0$
$-\left(\lambda+2 \theta+Q_{1}\right) x_{2021}+(\lambda+3 \theta) x_{3021}+(\lambda+3 \theta) x_{3020}+Q_{2} x_{2001}=0$
$-(\lambda+2 \theta+\mu) x_{2121}+(\lambda+3 \theta) x_{3121}+(\lambda+3 \theta) x_{3120}+Q_{2} x_{2101}=0$
$-\left(\lambda+2 \theta+Q_{2}\right) x_{2201}+(\lambda+3 \theta) x_{3201}+(\lambda+3 \theta) x_{3200}+Q_{1} x_{2001}=0$
$-(\lambda+2 \theta+\mu) x_{2211}+(\lambda+3 \theta) x_{3211}+(\lambda+3 \theta) x_{3210}+Q_{1} x_{2011}=0$
$-(\lambda+2 \theta+\mu) x_{2221}+(\lambda+3 \theta) x_{3221}+(\lambda+3 \theta) x_{3220}+Q_{1} x_{2021}+Q_{2} x_{2201}=0$
$-\left(\lambda+3 \theta+Q_{1}+Q_{2}\right) x_{3001}+(\lambda+4 \theta) x_{4001}+\mu x_{2111}=0$
$-\left(\lambda+3 \theta+Q_{1}+Q_{2}\right) x_{3000}+(\lambda+4 \theta) x_{4000}=0$
$-\left(\lambda+3 \theta+Q_{1}\right) x_{3011}+(\lambda+4 \theta) x_{4011}+\mu x_{2121}=0$
$-\left(\lambda+3 \theta+Q_{1}\right) x_{3010}+(\lambda+4 \theta) x_{4010}=0$
$-\left(\lambda+3 \theta+Q_{2}\right) x_{3101}+(\lambda+4 \theta) x_{4101}+\mu x_{2211}=0$
$-\left(\lambda+3 \theta+Q_{2}\right) x_{3100}+(\lambda+4 \theta) x_{4100}=0$
$-(\lambda+3 \theta+\mu) x_{3111}+(\lambda+4 \theta) x_{4111}+\mu x_{2221}=0$
$-(\lambda+3 \theta) x_{3110}+(\lambda+4 \theta) x_{4110}=0$
$-\left(\lambda+3 \theta+Q_{1}\right) x_{3021}+(\lambda+4 \theta) x_{4021}+Q_{2} x_{3001}=0$
$-\left(\lambda+3 \theta+Q_{1}\right) x_{3020}+(\lambda+4 \theta) x_{4020}+Q_{2} x_{3000}=0$
$-(\lambda+3 \theta+\mu) x_{3121}+(\lambda+4 \theta) x_{4121}+Q_{2} x_{3101}=0$

$$
\begin{align*}
& -(\lambda+3 \theta) x_{3120}+(\lambda+4 \theta) x_{4120}+Q_{2} x_{3100}=0  \tag{39}\\
& -\left(\lambda+3 \theta+Q_{2}\right) x_{3201}+(\lambda+4 \theta) x_{4201}+Q_{1} x_{3001}=0  \tag{40}\\
& -\left(\lambda+3 \theta+Q_{2}\right) x_{3200}+(\lambda+4 \theta) x_{4200}+Q_{1} x_{3000}=0  \tag{41}\\
& -(\lambda+3 \theta+\mu) x_{3211}+(\lambda+4 \theta) x_{4211}+Q_{1} x_{3011}=0  \tag{42}\\
& -(\lambda+3 \theta) x_{3210}+(\lambda+4 \theta) x_{4210}+Q_{1} x_{3010}=0  \tag{43}\\
& -(\lambda+3 \theta+\mu) x_{3221}+(\lambda+4 \theta) x_{4221}+Q_{2} x_{3201}+Q_{1} x_{3021}=0  \tag{44}\\
& -(\lambda+3 \theta) x_{3220}+(\lambda+4 \theta) x_{42220}+Q_{1} x_{3020}+Q_{2} x_{3200}=0  \tag{45}\\
& -\left(\lambda+4 \theta+Q_{1}+Q_{2}\right) x_{4001}+\mu x_{3111}=0  \tag{46}\\
& -\left(\lambda+4 \theta+Q_{1}+Q_{2}\right) x_{4000}+(\lambda+5 \theta) x_{5000}=0  \tag{47}\\
& -\left(\lambda+4 \theta+Q_{1}\right) x_{4011}+\mu x_{3121}=0  \tag{48}\\
& -\left(\lambda+4 \theta+Q_{1}\right) x_{4010}+(\lambda+5 \theta) x_{5010}=0  \tag{49}\\
& -\left(\lambda+4 \theta+Q_{2}\right) x_{4101}+\mu x_{3211}=0  \tag{50}\\
& -\left(\lambda+4 \theta+Q_{2}\right) x_{4100}+(\lambda+5 \theta) x_{5100}=0  \tag{51}\\
& -(\lambda+4 \theta+\mu) x_{4111}+\mu x_{3221}=0  \tag{52}\\
& -(\lambda+4 \theta) x_{4110}+(\lambda+5 \theta) x_{5110}=0  \tag{53}\\
& -\left(\lambda+4 \theta+Q_{1}\right) x_{4021}+Q_{2} x_{4001}=0  \tag{54}\\
& -\left(\lambda+4 \theta+Q_{1}\right) x_{4020}+(\lambda+5 \theta) x_{5020}+Q_{2} x_{4000}=0  \tag{55}\\
& -(\lambda+4 \theta+\mu) x_{4121}+Q_{2} x_{4101}=0  \tag{56}\\
& -(\lambda+4 \theta) x_{4120}+(\lambda+5 \theta) x_{5120}+Q_{2} x_{4100}=0  \tag{57}\\
& -\left(\lambda+4 \theta+Q_{2}\right) x_{4201}+Q_{1} x_{4001}=0 \\
& -\left(\lambda+4 \theta+Q_{2}\right) x_{2200}+(\lambda+5 \theta) x_{5200}+Q_{1} x_{4000}=0 \\
& -(\lambda+4 \theta+\mu) x_{4211}+Q_{1} x_{4011}=0  \tag{60}\\
& -(\lambda+4 \theta) x_{4210}+(\lambda+5 \theta) x_{5210}+Q_{1} x_{4010}=0  \tag{61}\\
& -(\lambda+4 \theta+\mu) x_{4221}+Q_{2} x_{4201}+Q_{1} x_{4021}=0  \tag{62}\\
& -(\lambda+4 \theta) x_{4220}+(\lambda+5 \theta) x_{5220}+Q_{2} x_{4200}+Q_{1} x_{4020}=0  \tag{63}\\
& -\left(\lambda+5 \theta+Q_{1}+Q_{2}\right) x_{5000}+\mu x_{4111}=0  \tag{64}\\
& -\left(\lambda+5 \theta+Q_{1}\right) x_{5010}+\mu x_{4121}=0  \tag{65}\\
& -\left(\lambda+5 \theta+Q_{2}\right) x_{5100}+\mu x_{4211}=0  \tag{66}\\
& -(\lambda+5 \theta) x_{5110}+\mu x_{4221}=0  \tag{67}\\
& -\left(\lambda+5 \theta+Q_{1}\right) x_{5020}+Q_{2} x_{5000}=0  \tag{68}\\
& -(\lambda+5 \theta) x_{5120}+Q_{2} x_{5100}=0  \tag{69}\\
& -\left(\lambda+5 \theta+Q_{2}\right) x_{5200}+Q_{1} x_{5000}=0  \tag{70}\\
& -(\lambda+5 \theta) x_{5210}+Q_{1} x_{5010}=0  \tag{71}\\
& -(\lambda+5 \theta) x_{5220}+Q_{2} x_{5200}+Q_{1} x_{5020}=0 \tag{72}
\end{align*}
$$

Appendix-II

| $\mathbf{x 0 0 0 1}$ | 0.0255588 | $\mathbf{x 0 0 1 1}$ | 0.019495 |
| :---: | :---: | :---: | :---: |
| x0101 | 0.019495 | x 0111 | 0.0977119 |
| $\mathbf{x} 0021$ | 0.061701 | x 0121 | 0.0289591 |
| x0201 | 0.061701 | x0211 | 0.0289591 |
| x 0221 | 0.0161334 | $\mathbf{x} 1001$ | 0.0464706 |
| x1011 | 0.0177227 | x1101 | 0.0177227 |
| x1111 | 0.0932704 | x1021 | 0.0328565 |
| x 1121 | 0.00992012 | x 1201 | 0.0328565 |
| x 1211 | 0.00992012 | x 1221 | 0.0418175 |
| x2001 | 0.0345511 | x2011 | 0.00567547 |
| $\times 2101$ | 0.00567547 | $\mathbf{x} 2111$ | 0.0259419 |
| $\times 2021$ | 0.0187734 | x2121 | 0.00300464 |
| x2201 | 0.0187734 | x2211 | 0.00300464 |
| $\times 2221$ | 0.0201623 | x3001 | 0.00959764 |
| x3000 | 0.00011735 | x3011 | 0.00157089 |
| x3010 | 0.0000213436 | x3101 | 0.00157089 |
| x3100 | 0.0000213436 | x3111 | 0.0102537 |
| x3110 | 0.000869945 | $\mathbf{x} 3021$ | $0.00497601$ |
| x3020 | 0.000216588 | x3121 | 0.000748043 |
| $\times 3120$ | 0.0000865472 | x3201 | 0.00497601 |
| $\times 3200$ | 0.000216588 | x3211 | 0.000748043 |
| $\times 3210$ | 0.0000865472 | x3221 | 0.00487656 |
| $\times 3220$ | 0. 00113751 | $\mathbf{x 4 0 0 1}$ | 0.0031666 |
| x4000 | 0.000276612 | x4011 | 0.000327269 |
| x4010 | 0.000350645 | x4101 | 0.000327269 |
| $\mathbf{x 4 1 0 0}$ | 0.0000350645 | x4111 | 0.00208996 |
| x 4110 | 0.000807806 | $\mathbf{x 4 0 2 1}$ | 0.00131942 |
| x4020 | 0.000272002 | x4121 | 0.000133579 |
| x 4120 | 0.0000651198 | x 4201 | 0.00131942 |
| $\mathbf{x 4 2 0 0}$ | 0.000272002 | x4211 | 0.000133579 |
| $\mathbf{x} 4210$ | 0.0000651198 | x 4221 | 0.00107707 |
| $\mathbf{x} 4220$ | 0.000746852 | x5000 | 0.000626987 |
| $\times 5010$ | 0.0000561032 | $\times 5100$ | 0.0000561032 |
| $\times 5110$ | 0.000753952 | x 5020 | 0.000250795 |
| $\times 5120$ | 0.0000374021 | $\times 5200$ | 0.000250795 |
| $\times 5210$ | 0.0000374021 | $\times 5220$ | 0.000334393 |

