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MODELLING OF FORCED CONVECTION OVER FLAT PLATES USING RANDOM WALK METHOD

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ABSTRACT: *This study is strongly motivated by the Random Walk Method (RWM) concept, RWM a version of vortex element method (VEM), was used to model steady state, laminar forced convention flows in plate. Numerical models were developed using RWM from the vorticity transport equation and the energy equation. The relationship for Prandtl number varying with time step was established, the results and plots from investigation of convergence stability of simulation of plates were found to be true. This study further established that the random method is a viable numerical tool capable of modeling heat transfer problems.*

KEYWORDS: Vortex element, Random walk, Forced convention, Prandtl number, Convergence

I. INTRODUCTION

According to Wikipedia, (2020) the overall goal of the field of numerical analysis is the design and analysis techniques to give approximate but accurate solutions to hard problem. Numerical analysis naturally finds applications in all fields of engineering and the physical sciences. A number of engineering problem involve flow of gases or liquids over solid bodies, often these flows do not follow the contour of the solid surface completely, but separate from it. Such separated flows are difficult to handle by conventional numerical schemes.

numerical methods have been developed and are still being developed to effectively deal with many engineering problem as they occur in heat transfer problems, the solutions to the resulting equations are often difficult if not possible to obtain by analytical method, and as such resort is made to numerical method. Dare & Petinrin (2010) stated that some of the numerical method used in solving fluid dynamics and heat transfer problems are Finite Difference Method (FDM), Finite Element Method (FEM), Monte Carlo Method and Vortex Element Method (VEM).

Adegbayo et al (2014) stated that the vortex element method has been developed and applied for analysis of complex, unsteady and vertical flows in relation to problems in a wide range of industries, because they consist of simple algorithm based on the physics of flow. Recently, the applicability of the vortex methods based on the Biot-Savart law has been extended to numerical prediction of unsteady and complex characteristics of various flows related with difficult engineering problems concerning flow-induced vibration, off-design operation of fluid machinery, automobile aerodynamics, and biological fluid dynamics and so on.

Ghoniem and Oppenheim (1982) applied random-walk vortex method to an assortment of problems of diffusion of momentum and energy in one dimension as well as heat conduction in two dimensions in order to assess its validity and accuracy. The numerical solutions obtained were found to be in good agreement with the exact solution except for a statistical error introduced by using a finite number of elements. They claimed further that the error could be reduced by increasing the number of elements or by using ensemble averaging over a number of solutions.

Ogundare (2006) carried out a research, using random walk vortex method, on steady state laminar, natural convention flow in isothermal vertical channels and square ducts, and heat condition in rectangular slab. The graphical correlation of the results was obtained between Nusselt number and Rayleigh number, the intercepts and slopes of the graphs were then compared with those obtained from literature using the finite element method.

The concept of the random walk method (RWM), a version of the VEM, was discussed by Chorin (1980). To simulate diffusion of vorticity in vortex method, the position of vortices is given random displacements (a random walks). The principle involved is to subject all the free vortex elements to small random displacements which produce a scatter equivalent to the diffusion of vorticity. These random displacements have zero mean and a variance equal to twice the product of the kinematic viscosity and time step. The basic idea of the random walk method is that the random displacements spread out the vortices like the diffusion process spread out the vorticity.

This study employs the Random Walk Method (RWM) to model steady state, laminar forced convention flows in plate. Numerical models were developed from vorticity transport equation, derived from Navier Stokes equations, and the energy equation. The Prandtl numbers varying with time step were obtained.

II. THE GOVERNING EQUATIONS

The continuity equation and the Navier-Stokes equations for Newtonian, two-dimensional, incompressible flow are presented as follows:

$$\frac{\delta u}{\delta x}+\frac{\delta v}{\delta y}=0 \tag{1}$$

$$\frac{\delta u}{\delta t}+u\frac{\delta u}{\delta x}+v\frac{\delta u}{\delta y}=-\frac{1}{\rho}\frac{\delta P}{\delta x}+f_x+v\left(\frac{\delta^2 u}{\delta x^2}+\frac{\delta^2 u}{\delta y^2}\right) \tag{2}$$

$$\frac{\delta v}{\delta t}+u\frac{\delta v}{\delta x}+v\frac{\delta v}{\delta y}=-\frac{1}{\rho}\frac{\delta P}{\delta y}+f_y+v\left(\frac{\delta^2 v}{\delta x^2}+\frac{\delta^2 v}{\delta y^2}\right) \tag{3}$$

Also, the energy equation, assuming no viscous dissipation or thermal generation, is:

$$\frac{\delta T}{\delta t}+u\frac{\delta T}{\delta x}+v\frac{\delta T}{\delta y}=\alpha\left(\frac{\delta^2 T}{\delta x^2}+\frac{\delta^2 T}{\delta y^2}\right) \tag{4}$$

When the pressure and body forces are negligible especially for forced convention on horizontal plate, the Navier-Stokes equations reduces to

$$\frac{\delta u}{\delta t}+u\frac{\delta u}{\delta x}+v\frac{\delta u}{\delta y}=v\left(\frac{\delta^2 u}{\delta x^2}+\frac{\delta^2 u}{\delta y^2}\right) \tag{5}$$

$$\frac{\delta v}{\delta t}+u\frac{\delta v}{\delta x}+v\frac{\delta v}{\delta y}=v\left(\frac{\delta^2 v}{\delta x^2}+\frac{\delta^2 v}{\delta y^2}\right) \tag{6}$$

Variables u and v are the velocity components in x and y directions respectively, f_x and f_y are the components of the gravitational acceleration in the x and y directions respectively, T is the fluid temperature, P is the fluid pressure, t is the time, α is the fluid thermal diffusivity, v is the kinematic viscosity, and ρ is the fluid density, Incropera and Dewitt (2005).

The numerical tool used in this study is the random walk method, which is one of the techniques of the vortex element method. Following the Lagrangian scheme, the alternative expression of the governing equations of viscous and incompressible flow gives the vorticity transport equation as

$$\frac{\delta w}{\delta t}+u\frac{\delta w}{\delta x}+v\frac{\delta w}{\delta y}=v\left(\frac{\delta^2 w}{\delta x^2}+\frac{\delta^2 w}{\delta y^2}\right) \tag{7}$$

Where vorticity, $w=\frac{\delta v}{\delta x}-\frac{\delta u}{\delta y}$ (8)

Considering the diffusive term only in the vorticity equation, then

$$\frac{\delta w}{\delta t} = v\left(\frac{\delta^2 w}{\delta x^2} + \frac{\delta^2 w}{\delta y^2}\right) \quad (9)$$

The solution to equation (7) was given by Batchelor [8] as;

$$w(r,t) = \frac{\Gamma}{4\pi v t} e^{\left(-\frac{r^2}{4vt}\right)} \quad (10)$$

Where $r = \sqrt{x^2 + y^2}$ and Γ is the vortex strength or circulation [9].

Equations (11) and (12) as stated below are the main governing equations (Momentum equation & Energy equation) and they can be solved by creating two vortices in the temperature vortex and velocity vortex. Both will undergo diffusion on their own accord. However only the velocity vortex will impart induced velocities on both itself and that of the temperature.

Momentum Equation;

$$\frac{\delta w}{\delta \tau} + u\frac{\delta w}{\delta x} + v\frac{\delta w}{\delta y} = v\frac{\delta^2 w}{\delta y^2} \quad (11)$$

Energy Equation;

$$\frac{\delta \theta}{\delta t} + u\frac{\delta \theta}{\delta x} + v\frac{\delta \theta}{\delta y} = a\frac{\delta^2 \theta}{\delta y^2} \quad (12)$$

$$\theta = (T - T_o)/(T_w - T_o)$$

Where; a = Thermal diffusivity, v = Kinematic viscosity, T_w = Wall temperature, T_o = Free stream temperature.

A typical vortices interaction on horizontal flat plate and the algorithm for the simulation of forced convention over flat plates are shown in figure 1 and 2 respectively.

Nomenclature

The following are the dimensionless numbers used in carrying out this research study;

$$Re = UL/v \quad (14)$$

$$Pr = \frac{V}{\alpha} \quad (15)$$

$$Pr = \frac{\mu c_p}{IC} \quad (16)$$

$$h = \rho u c \theta \quad (17)$$

$$\theta = \frac{T - T_w}{T_f - T_w} \quad (18)$$

Where Pr	-	is the Prandtl Number
Re	-	is the Reynolds number
h	-	is the coefficient of heat transfer of the fluid (W/m^2K)
L	-	is the length of the plate (m)
U	-	is the free-stream Velocity (m/s)
v	-	is the kinematic viscosity of the fluid (m^2/s)
ρ	-	is the density of the fluid (kg/m^3)
c	-	is the specific heat capacity of the fluid (J/KgK)
u	-	is the velocity within boundary layer (m/s)
θ	-	is a dimensional temperature ratio

T	-	is the temperature within boundary layer (K)
T_w	-	is the wall temperature (K)

III. RESULTS AND DISCUSSION

The vortex numerical analysis formulated for the convergence difficulty investigation was solved using FORTRAN programming language. The input parameters used to simulate forced convection on a flat plate for various categories of fluids are listed in Table 1, 2, 3 and 4.

When the input parameters in table 1 were used, the Prandtl number increases up to a certain stage and decreases downwards as shown in figure 3. An increase in Prandtl number requires increase in time step for convergence.

When the input parameters in table 2 were used, the Prandtl number increase asymptotically as shown in figure 4 and a value increase in Prandtl number require decrease in time step for convergence.

When the input parameters in table 3 were used, the Prandtl number increase asymptotically as shown in figure 5 and an increase in Prandtl number require decrease in time step for convergence.

And when the input parameters in table 4 were used, the Prandtl number is normal dome-like parabolic shapes as shown in figure 6. An increase in Prandtl number requires increase in time step for convergence.

Table 1. The Input Parameter (i) for Forced Convention on a flat plate

FLUID	THERMAL DIFFUSIVITY	PRANDTL NUMBER	TIME STEP
Air	0.0234	0.004867	0.003
Water	0.000198	0.005504	0.25
Toluene	0.00011	0.006265	0.002
Kerosene	0.0000642	0.0227	0.4
Castor Oil	0.000105	6.5	0.1

Table 2. The Input Parameter (ii) for Forced Convention on a flat plate

FLUID	THERMAL DIFFUSIVITY	PRANDTL NUMBER	TIME STEP
Acetone	0.000107	0.003774	0.4
Benzene	0.0000932	0.00691	0.25
Alcohol Propyl	0.0000849	0.02826	0.05
Phenol	0.000124	0.0602	0.06
Glycerine	0.000093	8.1	0.003

Table 3. The Input Parameter (iii) for Forced Convention on a flat plate

FLUID	THERMAL DIFFUSIVITY	PRANDTL NUMBER	TIME STEP
Carbon Disulfide	0.000129	0.002218	0.1

Carbon Tetrachloride	0.0000488	0.007578	0.05
Turpentine	0.000085	0.01858	0.001
Acetic Acid	0.0000843	0.1305	0.2
Ethylene Glycol	0.0000999	0.1482	0.1

Table 4. The Input Parameter (iii) for Forced Convention on a flat plate

FLUID	THERMAL DIFFUSIVITY	PRANDTL NUMBER	TIME STEP
Ether	0.0006479	0.004785	0.02
Alcohol Methyl	0.000102	0.006958	0.15
Octane	0.0000751	0.007459	0.01
Alcohol Ethyl	0.0000747	0.0147	0.15
Alcohol Propyl	0.0000849	0.02826	0.05

IV. CONCLUSION

A Convergence condition of forced convection over a flat plate has been successfully modeled, with the random walk method, a version of the vortex element method. From the result obtained, the relationship that the Prandtl number varies with time step was established and that convergence stability can be achieved with suitable time step. The study has also established that the random walk method is a viable numerical tool capable of modelling fluid and heat transfer problems.

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