

Difficult High-Level Math Questions

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Abstract

In this document, I describe fifteen high level math problems. I explain how to break apart the questions and fully solve each problem. This concept of practicing with complex problems helps us learn how to solve problems in a more intelligent manner, rather than brute forcing.

Contents

1. Question One	1122
2. Question Two	1124
3. Question Three	1125
4. Question Four	1126
5. Question Five	1127
6. Question Six	1128
7. Question Seven	1133
8. Question Eight	1135
9. Question Nine	1139
10. Question Ten	1140
11. Question Eleven	1145
12. Question Twelve	1148
13. Question Thirteen	1154
14. Question Fourteen	1157
15. Question Fifteen	1159

Introduction

Equation 1

If we look at the following equation, assuming ψ is $\psi(t)$

$$\frac{\left[\left(\psi^\psi \right)' + \int i^i \right]}{\int_{\tan(x)\psi}^{-\sigma^{2^j}} \left[\int \int \int x \right]}$$

We can solve for an approximate solution, but first we must focus on finding the derivative of $\psi(t)^\psi(t)$. Doing this we can find the answer to be:

$$\left[\psi(x)^\psi(x) \right]' = \left(\ln(\psi(x))\psi'(x) + \psi'(x) \right) \psi(x)^\psi(x)$$

When we plug this result back into the original equation, and simplify some expressions a little bit, we get the following.

$$\frac{\left(\ln(\psi(x))\psi'(x) + \psi'(x) \right) \psi(x)^\psi(x) + \int i^i}{\int_{\tan(x)\psi(x)}^{-\sigma^{2^j}} \int \int \int x dx dx dx dx}$$

If we also simplify the indefinite integrals in the equation we get the following.

$$\frac{\left(\ln(\psi(x))\psi'(x) + \psi'(x) \right) \psi(x)^\psi(x) + i^i x + c}{\int_{\tan(x)\psi(x)}^{-\sigma^{2^j}} \frac{x^4}{24} + \frac{cx^2}{2} + cx + c dx}$$

Lastly, we must deconstruct the definite integral.

$$\frac{\left(\ln(\psi(x))\psi'(x) + \psi'(x) \right) \psi(x)^\psi(x) + i^i x + c}{\frac{1}{120} \left(-\sigma^{5 \cdot 2^j} - \tan^5(x)\psi(x)^5 \right) + \frac{c}{6} \left(-\sigma^{3 \cdot 2^j} - \tan^3(x)\psi(x)^3 \right) + c \left(\frac{\sigma^{2^{j+1}}}{2} - \frac{1}{2} \tan^2(x)\psi(x)^2 \right) - c\sigma^{2^j} - c \tan(x)\psi(x)}$$

Now simplify to yield the final result:

$$\frac{120 \left(\left(\ln \psi(x) \right) \psi'(x) + \psi'(x) \right) \psi(x)^\psi(x) + i^i x + c}{-\sigma^{5 \cdot 2^j} - \tan^5(x)\psi(x)^5 + 20c \left(-\sigma^{3 \cdot 2^j} - \tan^3(x)\psi(x)^3 \right) + 60c \left(\sigma^{2^{j+1}} - \tan^2(x)\psi(x)^2 \right) - 120c\sigma^{2^j} - 120c \tan(x)\psi(x)}$$

Conclusion: The initial equation may seem complex at first; however, when we break it down into the fundamentals, we see that the equation becomes more approachable. The final equation is a simplified version of the initial equation.

Equation #2

If we look at the following equation

$$\frac{(\alpha - \beta)^{4 - \tan(x)\delta(-x + \cot(x))}}{\frac{-i\sqrt{3i}}{i}} + \left(x^{i^\infty} \left(\infty^{-i \cdot \infty} \cdot i \left(i^{-i+i} \left((i) \right) x \right) \left(\sqrt{\infty} \right)^{-i} \right) x^i \right) 30^\circ$$

Breaking the equation up into two separate fractions gets us the following equation.

$$-\frac{(\alpha - \beta)^{4 - \delta \tan(x) (-x + \cot(x))}}{\sqrt{3} \sqrt{i}} + \frac{180^\circ i x^{i+i^\infty} + 1 \infty^{-\frac{i}{2} \infty} - \infty \cdot i^2}{6}$$

Simplify the following a little more.

$$-\frac{\sqrt{3} \sqrt{i} (\alpha - \beta)^{-\delta \tan(x) (-x + \cot(x))} + 4}{3i} + \frac{180^\circ i x^{i+i^\infty} + 1 \infty^{-\frac{i}{2} \infty} - \infty \cdot i^2}{6}$$

Simplify and expand anything needed to be expanded.

$$\left(\frac{(\alpha - \beta)^{\delta \tan(x) - \delta \tan(x) \cot(x)} + 4}{3^{\frac{1}{2}} i^{\frac{1}{2}}} + \frac{3^{\frac{1}{2}} 180^\circ i^{\frac{3}{2}} x^{i+i^\infty} + 1 \infty^{-\frac{i}{2} \infty} - \infty - i^2}{6 \cdot 3^{\frac{1}{2}} i^{\frac{1}{2}}} \right)$$

Conclusion: We can deduce that the equation above is simplified, through the use of some basic trigonometric concepts and a little rearrangement.

1125

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Equation #3

If we view the following equation

$$\frac{x}{x^2} \div \frac{x^3}{x^4} \div \frac{x^5}{x^6} \div \frac{x^{7i + \pi}}{x^8} \div \frac{9\tan(x)}{x^{-i+10}} \div \frac{x^{11}\sqrt{i}}{x^{12}} \div x^{13i}$$

After dividing all possible values, we find a fairly simple fraction. That we can simply further.

$$\frac{x}{\frac{\sqrt{i}x^{9\tan(x) + 61 - i}(x^{7i + \pi})}{x^{13i}}}$$

Next, we apply the fraction rule, giving us the following.

$$\frac{x}{(x^{7i + \pi})\sqrt{i}x^{9\tan(x) + 61 - i}x^{13i}}$$

Simplify.

$$\frac{x^{-9\tan(x) + i - 60}}{(x^{7i + \pi})\sqrt{i}x^{13i}}$$

Simplify.

$$\frac{x^{-9\tan(x) - 12i - 60}}{(x^{7i + \pi})\sqrt{i}}$$

Finally, we find the fully simplified answer to be:

$$\frac{\sqrt{i}x^{-9\tan(x) - 12i - 60}}{i(x^{7i + \pi})}$$

Conclusion: This question inevitably seems quite daunting because of its size; however, in reality the question is conceptually quite simple.

1126

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Equation #4

If we view the following equation

$$\frac{|\beta^2| \left[x^i \sqrt[3]{i + \sqrt{ix + \cot(x)}} \right] \phi \left[x \sqrt[3]{i - 3i} + \frac{-3x}{\psi} \right]}{\left[|\beta^2| \left[x^i \sqrt[3]{i + \sqrt{ix + \cot(x)}} \right] \phi \left[x \sqrt[3]{i - 3i} + \frac{-3x}{\psi} \right] \right]^2}$$

Rearrange and simplify.

$$\frac{\beta^2 x^i \sqrt[3]{i + \sqrt{ix + \cot(x)}} \phi^{x \frac{-3x}{\psi} - \sqrt[3]{2i}}}{\left(x^i \sqrt[3]{i + \sqrt{ix + \cot(x)}} \phi^{x \frac{-3x}{\psi} + \sqrt[3]{i - 3i}} |\beta^2| \right)^2}$$

Simplify the absolute value of beta in the denominator.

$$\frac{\beta^2 x^i \sqrt[3]{i + \sqrt{ix + \cot(x)}} \phi^{x \frac{-3x}{\psi} - \sqrt[3]{2i}}}{\left(\beta^2 x^i \sqrt[3]{i + \sqrt{ix + \cot(x)}} \phi^{x \frac{-3x}{\psi} - \sqrt[3]{2i}} \right)^2}$$

Keep in mind that.

$$\phi^{x \frac{-3x}{\psi} - \sqrt[3]{2i}} = \phi^{x \frac{-3x - \sqrt[3]{2} \sqrt[3]{i} \psi}{\psi}}$$

Knowing this, we can apply it to our problem.

$$\frac{1}{\beta^2 x^i \sqrt[3]{i + \sqrt{ix + \cot(x)}} \phi^{x \frac{-3x}{\psi} - \sqrt[3]{2i}}}$$

Dividing everything by psi, we can simplify the power into one fraction.

$$\frac{1}{\beta^2 x^i \sqrt[3]{i + \sqrt{ix + \cot(x)}} \phi^{x \frac{-3x - \sqrt[3]{2} \sqrt[3]{i} \psi}{\psi}}}$$

1127

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Equation #5

If we view the following equation

$$\sqrt{\frac{\sqrt[3]{i^{-1}}}{\sqrt[3]{\int \tan(x)}} + \frac{\sqrt[3]{i} [\tan(x)\cot(x)]}{\frac{-3i+2}{\int \tan(x)}}$$

Start by solving the integrals.

$$\sqrt{\frac{\sqrt[3]{i^{-1}}}{\sqrt[3]{-\ln|\cos(x)| + c}} + \frac{\sqrt[3]{i} \tan(x)\cot(x)}{\frac{-3i+2}{-\ln|\cos(x)| + c}}$$

Simplify the i th root.

$$\sqrt{\frac{i^{-\frac{1}{3}}}{\sqrt[3]{-\ln|\cos(x)| + c}} + \frac{\sqrt[3]{i} \tan(x)\cot(x)}{\frac{-3i+2}{-\ln|\cos(x)| + c}}$$

Remove the cube root, and apply to the numerator.

$$\sqrt{\frac{i^{-\frac{1}{3}}(-\ln|\cos(x)| + c)^{\frac{2}{3}}}{-\ln|\cos(x)| + c} + \frac{\sqrt[3]{i} \tan(x)\cot(x)}{\frac{-3i+2}{-\ln|\cos(x)| + c}}$$

Take the -3i + 2 and apply to the numerator.

$$\sqrt{\frac{i^{-\frac{1}{3}}(-\ln|\cos(x)| + c)^{\frac{2}{3}}}{-\ln|\cos(x)| + c} + \frac{\sqrt[3]{i} (2 + 3i) \tan(x)\cot(x)}{\frac{13}{-\ln|\cos(x)| + c}}$$

Simplify to only two separate fractions.

$$\sqrt{\frac{i^{-\frac{1}{3}}(-\ln|\cos(x)| + c)^{\frac{2}{3}}}{-\ln|\cos(x)| + c} + \frac{\sqrt[3]{i} \tan(x)\cot(x)(2 + 3i)}{13(-\ln|\cos(x)| + c)}$$

Add fractions together by finding a common denominator.

$$\sqrt{\frac{(2 + 3i)\left(i^{-\frac{1}{3}}(2 - 3i) + \sqrt[3]{i} \tan(x)\cot(x)(-\ln|\cos(x)| + c)\sqrt[3]{-\ln|\cos(x)| + c}\right)}{13\sqrt[3]{-\ln|\cos(x)| + c}}$$

Simplify even more.

$$\sqrt{\frac{(2 + 3i)\left(i^{-\frac{1}{3}}(2 - 3i) + \sqrt[3]{i} \tan(x)\cot(x)(-\ln|\cos(x)| + c)\sqrt[3]{-\ln|\cos(x)| + c}\right)(-\ln|\cos(x)| + c)^{\frac{2}{3}}}{-13\ln|\cos(x)| + 13c}}$$

1128

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Equation #6

You are given a 3rd degree polynomial function, with the terms listed below. You are tasked with simplifying the expression into a binomial. You then must take the partial derivative of each of the terms in the original polynomial. Once this is completed, you must plug each partial derivative into a Jacobian, then back into the original polynomial. After this is done simply solve for x.

$$ax^3 + x^2b + \frac{a^2x}{dc} + \frac{2ab}{dc} + \frac{b^2}{xdc} = 0$$

Try to find something that can multiply to give ax + b on both sides.

$$x^2(ax + b) + \frac{cd}{dc}(cx + d) = 0$$

We have ax + b meaning we can combine the outer parts.

$$x^2(ax + b) + \frac{ax + b}{dcx}(ax + b) = 0$$

This gives us:

$$\left(x^2 + \frac{ax + b}{dcx}\right)(ax + b) = 0$$

Now we take the derivative of each part within the initial equation.

$$f(x) = ax^3 \quad f'(x) = 3ax^2$$

$$f(x) = x^2b \quad f'(x) = 2bx$$

$$f(x) = \frac{a^2x}{dc} \quad f'(x) = \frac{a^2}{dc}$$

$$f(x) = \frac{2ab}{dc} \quad f'(x) = 0$$

$$f(x) = \frac{b^2}{xdc} \quad f'(x) = -\frac{b^2}{x^2dc}$$

Then, take the Jacobian.

$$J = \left[3ax^2, 2bx, \frac{a^2}{dc}, 0, -\frac{b^2}{x^2dc} \right]$$

Next set each part of the Jacobian equal to the initial equation, then solve for x.

$$3ax^2 = ax^3 + x^2b + \frac{a^2x}{dc} + \frac{2ab}{dc} + \frac{b^2}{xdc}$$

Divide everything by (3a).

$$x^2 = \frac{ax^3}{3a} + \frac{x^2b}{3a} + \frac{\frac{a^2x}{dc}}{3a} + \frac{\frac{2ab}{dc}}{3a} + \frac{\frac{b^2}{xdc}}{3a}$$

Simplify.

$$x^2 = \frac{1}{3}x^3 + \frac{1}{3a}x^2b + \frac{1}{dc}3a^3x + \frac{1}{3dc}2b + \frac{1}{3xadc}b^2$$

Square root everything to remove the square applied to the x.

$$x = \sqrt{\frac{1}{3}x^2 + \frac{1}{3a}x^2b + \frac{1}{dc}3a^3x + \frac{1}{3dc}2b + \frac{1}{3xadc}b^2}$$

Put it into one fraction.

$$x = \sqrt{\frac{ax^4dc + x^3bdc + 9a^4x^2 + 2axb + b^2}{3axdc}}$$

2nd part of the Jacobian.

$$2bx = ax^3 + x^2b + \frac{a^2x}{dc} + \frac{2ab}{dc} + \frac{b^2}{xdc}$$

Divide everything by (2b).

$$x = \frac{ax^3}{2b} + \frac{x^2b}{2b} + \frac{\frac{a^2x}{dc}}{2b} + \frac{\frac{2ab}{dc}}{2b} + \frac{\frac{b^2}{xdc}}{2b}$$

Simplify to one fraction.

$$x = \frac{ax^4dc + x^3bdc + a^2x^2 + 2axb + b^2}{2xbdc}$$

3rd part of the Jacobian.

$$\frac{a^2}{dc} = ax^3 + x^2b + \frac{a^2x}{dc} + \frac{2ab}{dc} + \frac{b^2}{xdc}$$

Multiply everything by (dc).

$$a^2 = ax^3dc + x^2bdc + a^2x + 2ab + \frac{b^2}{x}$$

Put it into one fraction.

$$a^2 = \frac{x\left(ax^2dc + x^2bdc + a^2 + \frac{2ab}{x} + b^2\right)}{x}$$

Divide (a) squared by the x in the denominator.

$$\frac{a^2}{x} = ax^2dc + x^2bdc + a^2 + \frac{2ab}{x} + b^2$$

Multiply by (a) squared on both sides.

$$x = \frac{ax^2dc + xbdc + a^2 + \frac{2ab}{x} + b^2}{a^2}$$

Solve for x.

$$x = \frac{ax^3dc + x^2bdc + a^2x + 2ab + xb^2}{a^2x}$$

4th part of the Jacobian.

$$0 = ax^3 + x^2b + \frac{a^2x}{dc} + \frac{2ab}{dc} + \frac{b^2}{xdc}$$

Combine (dc).

$$0 = ax^3 + x^2b + \frac{a^2x + 2ab}{dc} + \frac{b^2}{xdc}$$

Combine (dc) and (xdc) into one fraction.

$$0 = x^2(ax + b) + \frac{x(a^2x + 2ab) + b^2}{xdc}$$

Divide everything by ax + b.

$$\frac{-x^2(ax + b)}{ax + b} = \frac{x(a^2x + 2ab) + b^2}{xdc} \div \frac{ax + b}{1}$$

Solve for x squared.

$$x^2 = \frac{x(a^2x + 2ab) + b^2}{-adcx^2 - xdc b}$$

Take the root.

$$x = \sqrt{\frac{x(a^2x + 2ab) + b^2}{-adcx^2 - xdc b}}$$

Find the imaginary part and place it outside the root.

$$x = i\sqrt{\frac{ax + b}{xdc}}$$

5th part of the Jacobian.

$$-\frac{b^2}{x^2dc} = ax^3 + x^2b + \frac{a^2x}{dc} + \frac{2ab}{dc} + \frac{b^2}{xdc}$$

Divide everything by x squared multiplied by (dc).

$$-b^2 = ax^5dc + x^4dcb + a^2x^3 + 2abx^2 + b^2x$$

Set the equation equal to zero by adding (b) squared to the opposing side.

$$0 = ax^5dc + x^4dcb + a^2x^3 + 2abx^2 + b^2x + b^2$$

Solve for x.

$$-b^2x = ax^5dc + x^4dcb + a^2x^3 + 2abx^2 + b^2$$

Divide by negative (b) squared.

$$x = \frac{ax^5 dc}{-b^2} + \frac{x^4 dcb}{-b^2} + \frac{a^2 x^3}{-b^2} + \frac{2abx^2}{-b^2} + \frac{b^2}{-b^2}$$

Simplify down to one fraction.

$$x = -\frac{ax^5 dc + x^4 bdc + a^2 x^3 + 2ax^2 b + b^2}{b^2}$$

Use capital pi to find the product.

$$\begin{aligned} & \Pi \left[\frac{\sqrt{3} \sqrt{a} (ax^4 dc + x^3 bdc + a^2 x + 2abx + b^2)}{6abx^{\frac{3}{2}} dc^{\frac{3}{2}}} \right] \\ & + \left[\frac{\sqrt{ax^4 dc + x^3 bdc + 9a^4 x^2 + 2abx + b^2}}{6abx^{\frac{3}{2}} dc^{\frac{3}{2}}} \right] \\ & \cdot \left[\frac{ax^3 dc + x^2 bdc + a^2 x + 2ab + xb^2}{a^2 x} \right] \end{aligned}$$

Continue to break down and simplify.

$$\begin{aligned} & \Pi \left[\frac{(adcx^4 + dc bx^3 + a^2 x + 2abx + b^2)}{2 \cdot 3^{\frac{1}{2}} dba^{\frac{5}{2}} c^{\frac{3}{2}} x^{\frac{5}{2}}} \right] \\ & + \left[\frac{(adcx^3 + dc bx^2 + a^2 x + b^2 x + 2ab)}{2 \cdot 3^{\frac{1}{2}} dba^{\frac{5}{2}} c^{\frac{3}{2}} x^{\frac{5}{2}}} \right] \\ & + \left[\frac{(adcx^4 + dc bx^3 + 9a^4 x^2 + 2abx + b^2)^{\frac{1}{2}}}{2 \cdot 3^{\frac{1}{2}} dba^{\frac{5}{2}} c^{\frac{3}{2}} x^{\frac{5}{2}}} \right] \\ & \cdot \left[i \sqrt{\frac{ax + b}{xdc}} \right] \end{aligned}$$

Simplify more.

$$\begin{aligned} & \Pi \left[\frac{i(ax^4 dc + bx^3 dc + a^2 x + 2abx + b^2)}{2 \cdot 3^{\frac{1}{2}} a^{\frac{5}{2}} c^{\frac{3}{2}} x^3 dbdc^{\frac{1}{2}}} \right] \\ & + \left[\frac{(ax^3 dc + bx^2 dc + a^2 x + b^2 x + 2ab)(ax + b)^{\frac{1}{2}}}{2 \cdot 3^{\frac{1}{2}} a^{\frac{5}{2}} c^{\frac{3}{2}} x^3 dbdc^{\frac{1}{2}}} \right] \\ & + \left[\frac{(ax^4 dc + bx^3 dc + 9a^4 x^2 + 2abx + b^2)^{\frac{1}{2}}}{2 \cdot 3^{\frac{1}{2}} a^{\frac{5}{2}} c^{\frac{3}{2}} x^3 dbdc^{\frac{1}{2}}} \right] \end{aligned}$$

$$\cdot \left[-\frac{(ax^5dc + x^4bdc + a^2x^3 + 2ax^2b + b^2)}{b^2} \right]$$

Now Put it in capital pi format.

$$\Pi \left[-\frac{i(adcx^4 + dcbx^3 + a^2x + 2abx + b^2)(x)(y)(z)^{\frac{1}{2}}(h)^{\frac{1}{2}}}{2 \cdot 3^{\frac{1}{2}}d^2c^2b^3\sqrt{ax^3}} \right]$$

$$x = (adcx^3 + dcbx^2 + a^2x + b^2x + 2ab)$$

$$y = (adcx^5 + dcbx^4 + a^2x^3 + 2abx^2 + b^2)$$

$$z = (ax + b)$$

$$h = (adcx^4 + dcbx^3 + 9a^4x^2 + 2abx + b^2)$$

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1133

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Equation #7

You are given a specified polynomial that can be seen below. You must solve for delta first, then use common properties of delta to solve for x

$$\Delta x^2 h + \Delta h y + \Delta^2 x^3 + \Delta^2 x y - \Delta y + \Psi = 0$$

Find common multiples and take them out.

$$\Delta h(x^2 + y) + \Delta^2 x(x^2 + y) - \Delta y + \psi = 0$$

Combine outer parts.

$$(\Delta h + \Delta^2 x)(x^2 + y) - \Delta y + \psi = 0$$

Add the part that's not multiplied to the other side.

$$\frac{(\Delta h + \Delta^2 x)(x^2 + y)}{x^2 + y} = \frac{\Delta y - \psi}{x^2 + y}$$

Divide by x squared add y on both sides.

$$\Delta h + \Delta^2 x = \frac{\Delta y - \psi}{x^2 + y}$$

Take the delta out and divide to simplify.

$$\frac{\Delta(h + \Delta x)}{h + \Delta x} = \frac{\Delta y - \psi}{x^2 + y} \div \frac{h + \Delta x}{1}$$

Solve.

$$\Delta = \frac{\Delta y - \psi}{x^2 + y} \cdot \frac{1}{h + \Delta x}$$

Simplify.

$$\Delta = \frac{\Delta y - \psi}{hx^2 + \Delta x^3 + yh + \Delta xy}$$

The definition of delta x.

$$\Delta x = x_2 - x_1$$

Apply this logic to the answer we found for delta

$$\left(\frac{\Delta y - \psi}{hx^2 + \Delta x^3 + yh + \Delta xy} \right) x = x(1 - 1)$$

Solve for x.

$$x = \frac{x(1 - 1)}{1} \div \frac{\Delta y - \psi}{hx^3 + \Delta x^3 + yh + \Delta xy}$$

Simplify.

$$x = \frac{x(1 - 1)}{1} \cdot \frac{hx^3 + \Delta x^3 + yh + \Delta xy}{\Delta y - \psi}$$

We then find the answer to be.

$$x = 0$$

Conclusion: Understanding all the aspects of the equation, makes re-substitution and simplification a lot easier. We found that although the equation seemed like a regular factoring problem, understanding the way delta works and how to isolate properly becomes a noticeable difficulty.

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1135

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Equation #8

You are given an integral and a derivative, as seen below. Add them together and solve for the products of all x values in a capital pi function.

$$\text{derivative } f(x) = \cos(x) + Ae^{ix} \left| \psi(x^2 + 4\phi) \right|^4$$

$$\int \frac{\psi\phi}{ih2mxy^2} dx + 12$$

Solve the integral.

$$\int \frac{\psi\phi}{ih2mxy^2} dx = \frac{\psi^2\phi}{4mxy^2(0 + 1i)h} + c$$

Then, find the derivative of the initial function.

$$f'(x) = \cos(x) + Ae^{ix} \left| \psi(x^2 + 4\phi) \right|^4$$

$$= iAe^{ix} + 8x\psi^{(1)}(x^2 + 4\phi)\psi^{(0)}(x^2 + 4\phi)^3 - \sin(x)$$

Create a quadratic equation with the results of both the integral, derivative, and the constant 12.

$$\left(\frac{\psi^2\phi}{4mxy^2(0 + 1i)h} \right) x^2 + \left(iAe^{ix} + 8x\psi^{(1)}(x^2 + 4\phi)\psi^{(0)}(x^2 + 4\phi)^3 - \sin(x) \right) x + 12$$

Solve for x

$$x = \frac{-\left(iAe^{ix} + 8x\psi^{(1)}(x^2 + 4\phi)\psi^{(0)}(x^2 + 4\phi)^3 - \sin(x) \right)}{2\left(\frac{\psi^2\phi}{4mxy^2(0 + 1i)h} \right)}$$

$$\pm \sqrt{\left(iAe^{ix} + 8x\psi^{(1)}(x^2 + 4\phi)\psi^{(0)}(x^2 + 4\phi)^3 - \sin(x) \right)^2}$$

$$2\left(\frac{\psi^2\phi}{4mxy^2(0 + 1i)h} \right)}$$

Add this under the square root

$$\frac{-4\left(\frac{\psi^2\phi}{4mxy^2(0 + 1i)h} \right) (12)}{2\left(\frac{\psi^2\phi}{4mxy^2(0 + 1i)h} \right)}$$

We then expand part of the function.

$$\left(iAe^{ix} + 8x\psi^{(1)}(x^2 + 4\phi)\psi^{(0)}(x^2 + 4\phi)^3 - \sin(x) \right)^2$$

Expanded Form.

$$\begin{aligned}
 & -A^2 e^{2ix} + 16iAe^{ix}\psi x^9 + 256iAe^{ix}\psi x^7\phi + 1536iAe^{ix}\psi x^5\phi^2 \\
 & + 4096iAe^{ix}\psi x^3\phi^3 + 4096iAe^{ix}\psi x\phi^4 - 2iAe^{ix}\sin(x) + 64\psi^2 x^{18} \\
 & + 2048\psi^2 x^{16}\phi + 28672\psi^2 x^{14}\phi^2 + 229376\psi^2 x^{12}\phi^3 \\
 & + 1146880\psi^2 x^{10}\phi^4 - 16\psi x^9 \sin(x) + 3670016\psi^2 x^8 \phi^5 \\
 & - 256\psi x^7 \phi \sin(x) + 7340032\psi^2 x^6 \phi^6 - 1536\psi x^5 \phi^2 \sin(x) \\
 & + 8388608\psi^2 x^4 \phi^7 - 4096\psi x^3 \phi^3 \sin(x) + 4194304\psi^2 x^2 \phi^8 \\
 & - 4096\psi x \phi^4 \sin(x) + \sin^2(x)
 \end{aligned}$$

Expand.

$$-4 \left(\frac{\psi^2 \phi}{4mxy^2(0+1i)h} \right) (12)$$

Simplify

$$\begin{aligned}
 & -4 \left(\frac{\psi^2(0-4.854i)h}{mxy^2} \right) \\
 & = \frac{\psi^2(0+19.42i)h}{mxy^2}
 \end{aligned}$$

Solve.

$$\begin{aligned}
 & 2 \left(\frac{\psi^2 \phi}{4mxy^2(0+1i)h} \right) \\
 & = \frac{\psi^2(0-0.809i)h}{mxy^2}
 \end{aligned}$$

Expansion is an expanded form of.

$$\left(iAe^{ix} + 8x\psi^{(1)}(x^2 + 4\phi) \psi^{(0)}(x^2 + 4\phi)^3 - \sin(x) \right)^2$$

Continue solving.

$$\begin{aligned}
 x &= \frac{-Ae^{ix} - 8x\psi(x^2 + 4\phi)^4 + \sin(x) \pm}{\frac{\psi^2(0-0.809i)h}{mxy^2}} \\
 &+ \sqrt{\frac{[Expansion] - \frac{\psi^2(0+19.42i)h}{mxy^2}}{\frac{\psi^2(0-0.809i)h}{mxy^2}}}
 \end{aligned}$$

This gives us.

$$\left[x = \frac{1}{\psi^2} mxy^2(0+1.236i)h \right]$$

Multiplied by.

$$\left[\frac{\psi^2(0-19.42i)h}{mxy^2} + \sqrt{\left(iAe^{ix} + 8x\psi(x^2 + 4\phi)^4 - \sin(x) \right)^2} \right]$$

$$-Ae^{ix} - 8x\psi(x^2 + 4\phi)^4 + \sin(x)$$

Simplify

$$\left[x = \frac{1}{\psi^2} mxy^2(0 + 1.236i)h \right]$$

$$\left[\frac{\psi^2(0 - 19.42i)h}{mxy^2} + \sqrt{(iAe^{ix} + 8x\psi(x^2 + 4\phi)^4 - \sin(x))^2} \right]$$

$$-Ae^{ix} - 8x\psi(x^2 + 4\phi)^4 + \sin(x)$$

Simplify.

$$x = \frac{1}{\psi^2} mxy^2(0 + 1.236i)h$$

$$\left\{ \frac{\psi^2(0 - 19.42i)h}{mxy^2} + 8x\psi(x^2 + 4\phi)^4 \exp[i\pi[T]] \right\}$$

$$T = \frac{1}{2} - \frac{\arg(8x\psi(x^2 + 4\phi)^4 + iAe^{ix} - \sin(x))}{\pi}$$

$$+ iA \exp[i\pi(T) + ix] + \sin(x) \exp[i\pi(1 + T)]$$

$$-Ae^{ix} - 8x\psi(x^2 + 4\phi)^4 + \sin(x)$$

Simplify.

$$x = \frac{1}{\psi^2} mxy^2(8.18974 \cdot 10^{-34}i)$$

Simplify.

$$\left[\frac{\psi^2(1.28677 \cdot 10^{-32}i)}{mxy^2} + \text{Expand}(\exp)(i(3.14)(U)) \right]$$

Simplify.

$$U = \frac{1}{2} - \frac{\arg(\text{Expand} + iAe^{ix} - \sin(x))}{(3.14)}$$

Simplify.

$$+ iA \exp(i(3.14)(U) + ix) + \sin(x) \exp(i(3.14)(1 + (U)))$$

$$-Ae^{ix} - (\text{Expand}) + \sin(x)$$

1139

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Equation #9

You are given the following equation, create a series representation.

$$\eta(\tau) \zeta^{(\infty)} + \zeta(1+2)$$

First Series Representation

$$\eta(\tau) \sum_{k=0}^A \frac{((\infty - s_0)^k \zeta^{(k)}(s_0))}{k!} + \sum_{k=0}^A \left(\frac{(3 - s_0)^k \zeta^{(k)}(s_0)}{k!} \right)$$

For $s_0 \neq 1$

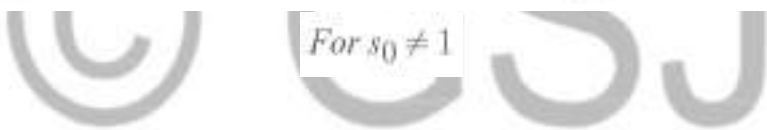
Second Series Representation

$$\eta(\tau) \zeta^{(\infty)} + \zeta(1+2) = \frac{1}{2} \left(2 \left(e^{\frac{i\pi\tau}{12} - \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \frac{e^{2ikn\pi\tau}}{k}} \right)^{\sum_{n=0}^{\infty} \frac{\sum_{k=0}^n (-1)^k (1+k)^{1-n} \binom{n}{k}}{1+n}} \right)^{(-1+\omega)} + \sum_{n=0}^{\infty} \frac{\sum_{k=0}^n \frac{(-1)^k \binom{n}{k}}{(1+k)^2}}{1+n}$$

Third Series Representation

$$\eta(\tau) \zeta^{(\infty)} + \zeta(1+2) = \left(e^{\frac{i\pi\tau}{12} - \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \frac{e^{2ikn\pi\tau}}{k}} \right)^{\sum_{k=0}^{\infty} \frac{((\infty - s_0)^k \zeta^{(k)}(s_0))}{k!}} + \sum_{k=0}^{\infty} \frac{(3 - s_0)^k \zeta^{(k)}(s_0)}{k!}$$

For $s_0 \neq 1$



1140

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Equation #10

Solve the following integral:

$$\int_0^{\frac{\pi}{2}} \cos^{-1}\left(\frac{\cos(x)}{1 + 2\cos(x)}\right) dx$$

Multiply by 2.

$$2 \cdot \int_0^{\frac{\pi}{2}} \cos^{-1}\left(\sqrt{\frac{1 + \frac{\cos(x)}{1 + 2\cos(x)}}{2}}\right) dx$$

Simplify.

$$2 \cdot \int_0^{\frac{\pi}{2}} \cos^{-1}\left(\sqrt{\frac{1 + 3\cos(x)}{2 + 4\cos(x)}}\right) dx$$

Keep in mind that.

$$\tan(z) = \sqrt{\frac{\sin^2(z)}{\cos^2(z)}}$$

We can apply this.

$$\sqrt{\frac{1 - (\cos^2(z))^2}{(\cos^2(z))^2}}$$

Expand and simplify

$$\sqrt{\frac{1 - \frac{1 + 3\cos(x)}{2 + 4\cos(x)}}{\frac{1 + 3\cos(x)}{2 + 4\cos(x)}}}$$

Use identities.

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1$$

We can then find this expression.

$$2x = \cos^{-1}(2\cos^2(x) - 1)$$

$$\text{let } t = \cos(x)$$

Apply.

$$2x = \cos^{-1}(2t^2 - 1) = 2\cos^{-1}(t)$$

$$\cos^{-1}(z) = 2\cos^{-1}\left(\sqrt{\frac{z+1}{2}}\right)$$

$$z = 2t^2 - 1$$

$$t = \pm \sqrt{\frac{z+1}{2}}$$

$$\cos(\phi) = z$$

$$\phi = \cos^{-1}(z)$$

$$\tan(\phi) = t$$

$$\phi = \tan^{-1}(t)$$

Simplify within the initial expression.

$$\sqrt{\frac{1 + \cos(x)}{1 + 3\cos(x)}}$$

$$2 \cdot \int_0^{\frac{\pi}{2}} \tan^{-1}\left(\sqrt{\frac{1 + \cos(x)}{1 + 3\cos(x)}}\right) dx$$

$$4 \int_0^{\frac{\pi}{4}} \tan^{-1}\left(\sqrt{\frac{1 + \cos(2t)}{1 + 3\cos(2t)}}\right) dt$$

$$4 \int_0^{\frac{\pi}{4}} \tan^{-1}\left(\sqrt{\frac{2\cos^2(t)}{1 + 3 - 6\sin^2(t)}}\right) dt$$

$$\begin{array}{l} \text{let } x = 2t \\ dx = 2dt \end{array}$$

$$1 + \cos(2t) = 2\cos^2(t)$$

$$\cos(2t) = 1 - 2\sin^2(t)$$

Simplify.

$$4 \int_0^{\frac{\pi}{4}} \tan^{-1}\left(\sqrt{\frac{\cos^2(t)}{2 - 3\sin^2(t)}}\right) dt$$

$$4 \int_0^{\frac{\pi}{4}} \tan^{-1}\left(\frac{\cos(t)}{\sqrt{2 - 3\sin^2(t)}}\right) dt$$

Evaluate from 1 to 0

$$\frac{1}{a} \tan^{-1}\left(\frac{1}{a}\right) = \frac{1}{a} \tan^{-1}\left(\frac{k}{a}\right)$$

$$a \int_0^1 \frac{dx}{x^2 + a^2}$$

$$4 \int_0^{\frac{\pi}{4}} \int_0^1 \frac{\sqrt{2-3\sin^2(t)}}{\cos(t)} \frac{1}{x^2 + \frac{2-3\sin^2(t)}{\cos^2(t)}} dx dt$$

Simplify.

$$4 \int_0^{\frac{\pi}{4}} \int_0^1 \frac{\sqrt{2-3\sin^2(t)} \cos(t)}{x^2 \cos^2(t) + 2 - 3\sin^2(t)} dx dt$$

$$4 \int_0^{\frac{\pi}{4}} \int_0^1 \frac{\sqrt{2-3\sin^2(t)} \cos(t)}{x^2 - x^2 \sin^2(t) + 2 - 3\sin^2(t)} dx dt$$

$$4 \int_0^{\frac{\pi}{4}} \int_0^1 \frac{\sqrt{2-3\sin^2(t)} \cos(t)}{(x^2 + 2) - \sin^2(t)(x^2 + 3)} dx dt$$

$$\text{let } \sin(t) = \sqrt{\frac{2}{3}} \sin(\phi)$$

$$\cos(t) dt = \sqrt{\frac{2}{3}} \cos(\phi) d\phi$$

Simplify.

$$I = 4 \int_0^{\frac{\pi}{3}} \int_0^1 \frac{\sqrt{2-3 \cdot \frac{2}{3} \sin^2(\phi)} \sqrt{\frac{2}{3}} \cos(\phi)}{(x^2 + 2) - \frac{2}{3} \sin^2(\phi)(x^2 + 3)} dx d\phi$$

$$4 \int_0^{\frac{\pi}{3}} \int_0^1 \sqrt{3} \frac{\sqrt{2} \cos(\phi) \sqrt{2} \cos(\phi)}{3x^2 + 6 - 2(1 - \cos^2(\phi))(x^2 + 3)} dx d\phi$$

$$I = \int_0^{\frac{\pi}{3}} \int_0^1 8\sqrt{3} \frac{\cos^2(\phi)}{x^2 + 2\cos^2(\phi)(x^2 + 3)} dx d\phi$$

$$\text{let } u = \tan(\phi)$$

$$du = \sec^2(\phi) d\phi$$

$$(1 + \tan^2(\phi)) d\phi$$

$$d\phi = \frac{du}{1 + u^2}$$

$$\begin{aligned}
 & 8\sqrt{3} \int_0^{\sqrt{3}} \int_0^1 \frac{\frac{1}{1+u^2}}{x^2 + 2 \frac{1}{1+u^2} (x^2 + 3)} \frac{1}{1+u^2} dx du \\
 & 8\sqrt{3} \int_0^{\sqrt{3}} \int_0^1 \frac{1}{(1+u^2)(u^2 x^2 + x^2 + 2x^2 + 6)} dx du \\
 & 8\sqrt{3} \int_0^1 \frac{1}{2x^2 + 6} \left[\int_0^{\sqrt{3}} \frac{du}{u^2 + 1} - \frac{x^2}{x^2} \int_0^{\sqrt{3}} \frac{du}{\frac{u^2 x^2}{x^2} + \sqrt{\frac{3x^2 + 6}{x^2}}} \right] \\
 & = \frac{x}{\sqrt{3x^2 + 6}} \tan^{-1} \left(\frac{ux}{\sqrt{3x^2 + 6}} \right) \Big|_0^{\sqrt{3}} \\
 & 8\sqrt{3} \int_0^1 \frac{\pi}{3} \frac{1}{2x^2 + 6} dx - \int_0^1 \frac{8x}{\sqrt{x^2 + 2}} \tan^{-1} \left(\frac{x}{\sqrt{x^2 + 2}} \right) dx \\
 & I = \frac{4\pi}{\sqrt{3}} \int_0^1 \frac{dx}{x^2 + \sqrt{3}^2} - 4 \int_0^1 \frac{x}{(x^2 + 3)\sqrt{x^2 + 2}} \tan^{-1} \left(\frac{x}{\sqrt{x^2 + 2}} \right) dx
 \end{aligned}$$

Use DI method.

$$D: \tan^{-1} \left(\frac{x}{\sqrt{x^2 + 2}} \right) = \frac{1}{(x^2 + 1)\sqrt{x^2 + 2}}$$

$$I: \frac{x}{(x^2 + 3)\sqrt{x^2 + 2}} = \tan^{-1} \left(\sqrt{x^2 + 2} \right)$$

$$\frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x}{\sqrt{3}} \right) \Big|_0^1$$

$$= \frac{1}{\sqrt{3}} \frac{\pi}{6}$$

$$\frac{2\pi^2}{9} - 4 \tan^{-1} \left(\frac{x}{\sqrt{x^2 + 2}} \right) \tan^{-1} \left(\sqrt{x^2 + 2} \right) \Big|_0^1$$

Evaluate.

$$4 \int_0^1 \frac{\tan^{-1} \left(\sqrt{x^2 + 2} \right)}{(x^2 + 1)\sqrt{x^2 + 2}} dx$$

$$I = 4 \cdot \frac{5\pi^2}{96}$$

Final answer.

$$I = \frac{5\pi^2}{24}$$

Conclusion: Integration can be extremely difficult and time consuming; however, understanding fundamental theorems and concepts can help make the process a lot more digestible.

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Equation #11

Find the most precise volume for a 4th dimensional Euclidean ball with Radius:

$$\sum x\Psi^{1-i} \cos(x)$$

$$= \frac{\pi^2}{2} R^4 \approx 4.935 \cdot R^4$$

$$\frac{(2V)^{\frac{1}{4}}}{\sqrt{\pi}} \approx 0.671 \cdot V^{\frac{1}{4}}$$

$$4.935 \cdot [R]^4 = 6.90681\psi$$

$$4^{(-i+1)} \left(-0.95885n \cos\left(\frac{2n+1-\pi}{2}\right) - (A) + (B) \right)^4$$

$$A = 0.95885 \cos\left(\frac{2n+1-\pi}{2}\right)$$

$$B = 1.49675 - \cos(n+1)$$

Find a more specific result.

$$\text{Radius} = \sum x\psi^{1-i} \cos(x)$$

Using the Partial Sum Formula, we find.

$$\sum_{x=2}^n \left(\psi^{1-i} x \cos(x) \right)$$

$$-\frac{1}{4} \csc\left(\frac{1}{2}\right) \psi$$

$$1^{-i} \left(-2n \cos\left(\frac{1}{2}(2n-\pi+1)\right) - (A) - (B) + (C) + (D) \right)$$

$$A = 2 \cos\left(\frac{1}{2}(2n-\pi+1)\right)$$

$$B = \csc\left(\frac{1}{2}\right) \cos(n+1)$$

$$C = 4 \cos\left(\frac{3-\pi}{2}\right)$$

$$D = \cos(2) \csc\left(\frac{1}{2}\right)$$

Simplify.

$$R_{2k+1}(V) = (2k+1)^{\frac{1}{(2k+1)}} \left(V \cdot (2k-1)!! \sqrt{\frac{\pi}{2}} \right)^{-\frac{2}{(4k^2-1)}} R_{2k-1}(V)$$

$$f_n \doteq 2f_n - \frac{2}{n}$$

Where $f_0 \doteq 1$ and $f_1 \doteq 2$

$$V_n(R) = \pi \left[\frac{n}{2} \right] f_n R^n$$

$$S_n(R) = n\pi \left[\frac{n}{2} \right] f_n R^{n-1}$$

$$V_n \sim \frac{1}{\sqrt{n\pi}} \left(\frac{2\pi e}{n} \right)^{\frac{n}{2}} R^n$$

$$V_4(R) \sim \frac{1}{\sqrt{4\pi}} \left(\frac{2\pi e}{4} \right)^{\frac{4}{2}} R^4$$

Simplify.

$$\frac{1 \cdot \left(\frac{2\pi e}{4} \right)^{\frac{4}{2}}}{\sqrt{4\pi}} \left(-\frac{1}{4} \csc\left(\frac{1}{2}\right) \psi^{-i+1} (E - (A) + (B) + (C) - D) \right)$$

$$E = -2n \cos\left(\frac{1}{2}(2n + 1 - \pi)\right)$$

(A, B, C, and D Have already been defined)

$$\frac{e^{2\pi\frac{3}{2}}}{8} \left(-\frac{1}{4} \csc\left(\frac{1}{2}\right) \psi^{-i+1} (E - (A) + (B) + (C) - (D)) \right)^4$$

$$= \frac{e^{2\pi\frac{3}{2}}}{8} \cdot \frac{\psi^{4(-i+1)} (A^i - B^i - C^i + D^i - E^i)^4}{4^4 \sin^8\left(\frac{1}{2}\right)}$$

$$A^i = 4 \sin\left(\frac{1}{2}\right) \cos\left(\frac{-\pi + 3}{2}\right)$$

$$B^i = 2 \sin\left(\frac{1}{2}\right) n \cos\left(\frac{2n + 1 - \pi}{2}\right)$$

$$C^i = 2 \sin\left(\frac{1}{2}\right) \cos\left(\frac{2n + 1 - \pi}{2}\right)$$

$$D^i = \cos(2)$$

$$E^i = \cos(n + 1)$$

(A, B, C, D, and E Have already been defined)

$$\frac{e^{2\pi\frac{3}{2}} \psi^{4(-i+1)} (A^i - B^i - C^i + D^i - E^i)^4}{2048 \sin^8\left(\frac{1}{2}\right)}$$

1148

Jake Brockbank

Equation #12

You are given three integrals, listed below: 1, 2, 3. You must simply simplify all three then multiply the products yielded in the form abc. (Each one will be labelled a, b, and c).

$$\textcircled{1} \iiint \tanh^2(x) dx$$

$$\textcircled{2} \int \frac{dx}{2x\sqrt{1-x}\sqrt{2-x+\sqrt{1-x}}}$$

$$\textcircled{3} \int_{4\pi r^2}^{\psi(-ih)} -\ln(2x+\psi)\Omega^{i\sqrt{3}} dx$$

First Integral.

$$\int \tanh^2(x) dx = x - \tanh(x) + C$$

$$\int \int \tanh^2(x) dx = \frac{1}{2}x(2C+x) - \log(\cosh(x)) + C$$

$$\int \int \int \tanh^2(x) dx = \frac{1}{6} \left(x(3Cx + 6C + x^2 + 3x + 6\log(e^{-2x} + 1) - 6\log(\cosh x)) \right)$$

$$-3Li_2(-e^{-2x}) + C$$

Second Integral.

$$\frac{1}{2} \left[\tanh^{-1} \left(\frac{1 - \sqrt{1-x}}{2\sqrt{-x + \sqrt{1-x} + 2}} \right) - A \right] + C$$

$$A = \frac{\tanh^{-1} \left(\frac{\sqrt{3}(\sqrt{1-x} + 1)}{2\sqrt{-x + \sqrt{1-x} + 2}} \right)}{\sqrt{3}}$$

Third Integral.

$$\left[\frac{1}{2} \Omega^{i\sqrt{3}} (2x - (\psi + 2x)\ln(\psi + 2x)) \right]_{4\pi r^2}^{\psi(-ih)}$$

$$\frac{1}{2} \Omega^{i\sqrt{3}} \left(2(\psi(-ih)) - (\psi + 2(\psi(-ih))) \right) \ln(\psi + 2(\psi(-ih)))$$

$$- \left[\frac{1}{2} \Omega^{i\sqrt{3}} \left(2(4\pi r^2) - (\psi + 2(4\pi r^2)) \right) \ln(\psi + 2(4\pi r^2)) \right]$$

$$\frac{1}{2}\Omega^{i\sqrt{3}} \left(2\psi^0((0-1i)h) - (\psi + 2\psi^0((0-1i)h)) \right) \ln \left(\psi + 2\psi^0((0-1i)h) \right) - \left[-4\pi r^2 \Omega^{i\sqrt{3}} \ln(\psi + 8\pi r^2) - \frac{1}{2}\psi \Omega^{i\sqrt{3}} + 4\pi r^2 \Omega^{i\sqrt{3}} \right]$$

Combine (Integral 1 is a, integral 2 is b, and integral 3 is c)
 ab equals

$$\left[\frac{1}{6} \left(x(3Cx + 6C + x^2 + 3x + 6\log(e^{-2x} + 1) - A - B) \right) \right]$$

$$A = 6\log(\cosh(x))$$

$$B = 3Li_2(-e^{-2x})$$

(Plus, constant C)

$$\cdot \left\{ \frac{1}{2} \left(\tanh^{-1} \left(\frac{1 - \sqrt{1-x}}{2\sqrt{-x + \sqrt{1-x} + 2}} \right) - A \right) \right\}$$

$$A = \frac{\tanh^{-1} \left(\frac{\sqrt{3}(\sqrt{1-x} + 1)}{2\sqrt{-x + \sqrt{1-x} + 2}} \right)}{\sqrt{3}}$$

Expanding we get.

$$\frac{Cx^2}{2} + Cx + C - \frac{1}{2}Li_2(-e^{-2x}) + \frac{x^3}{6} + \frac{x^2}{2} + x\log(e^{-2x} + 1) - x\log(\cosh(x))$$

Multiplied by

$$C + \frac{1}{2}\tanh^{-1} \left(\frac{1}{2\sqrt{-x + \sqrt{1-x} + 2}} - \frac{\sqrt{1-x}}{2\sqrt{-x + \sqrt{1-x} + 2}} \right) - \frac{\sqrt{1-x} \tanh^{-1} x}{4\sqrt{-x + \sqrt{1-x} + 2}} - \frac{\tanh^{-1} x}{4\sqrt{-x + \sqrt{1-x} + 2}}$$

We then get.

$$Cx^2 + x^2 - x \tanh^{-1} \left(\frac{(-\sqrt{-x+1} + 1)\sqrt{-x + \sqrt{-x+1} + 2}}{-2x + 2\sqrt{-x+1} + 4} \right) \ln(\cosh(x)) + Li_2 e^{-2x}$$

$$+Cx + C + \frac{x^3}{6} + x \ln(e^{-2x} + 1) - Cx \ln(\cosh(x)) + \frac{x \tanh^{-1}(x) \ln(\cosh(x)) (\sqrt{-x+1} + 1) \sqrt{-x + \sqrt{-x+1} + 2}}{-4x + 4\sqrt{-x+1} + 8}$$

Multiply by third integral result (abc)

$$\text{let } \psi = a$$

$$\text{let } h = b$$

$$\text{let } \Omega = c$$

Apply.

$$\begin{aligned} & -(\psi + (0 - 2i)h) \ln(\psi + (0 - 2i)h) + 4\pi r^2 \Omega^{i\sqrt{3}} \ln(\psi + 8\pi r^2) \\ & + \frac{1}{2} \psi \Omega^{i\sqrt{3}} + \Omega^{i\sqrt{3}} (0 - 1i)h - 4\pi r^2 \Omega^{i\sqrt{3}} \end{aligned}$$

Simplify.

$$\begin{aligned} & -\ln(a - 2bi)(a - 2bi) + 4\pi c^{i\sqrt{3}} r^2 \ln(8\pi r^2 + a) + \\ & \frac{ac^{i\sqrt{3}}}{2} - bic^{i\sqrt{3}} - 4\pi c^{i\sqrt{3}} r^2 \end{aligned}$$

$$\text{let } A_2 = Li_2$$

$$\begin{aligned} & Cx^2 + x^2 - Li_2 e^{-2x} x \tanh^{-1} \left(\frac{-\sqrt{-x+1} \sqrt{-x + \sqrt{-x+1} + 2}}{-2x + 2\sqrt{-x+1} + 4} \right) \ln \\ & (\cosh(x)) (-x + \sqrt{-x+1} + 2) + x \tanh^{-1}(x) \ln(\cosh(x)) \\ & \left(\frac{\sqrt{-x+1} \sqrt{-x + \sqrt{-x+1} + 2}}{-a \ln(a - 2bi) + 2bi \ln(a - 2bi)} \right) \\ & \frac{2(-x + \sqrt{-x+1} + 2)}{2(-x + \sqrt{-x+1} + 2)} \end{aligned}$$

Simplify.

$$\begin{aligned} & 2\pi c^{i\sqrt{3}} r^2 \ln(8\pi r^2 + a) \left(Cx^2 + x^2 + x \operatorname{arctanh}(x) \ln(\cosh(x)) (\sqrt{-x+1} + 1) \right. \\ & \left. \sqrt{-x + \sqrt{-x+1} + 2} - a_2 e^{-2x} x \operatorname{arctanh} \left(\frac{-\sqrt{-x+1} \sqrt{-x + \sqrt{-x+1} + 2}}{-2x + 2\sqrt{-x+1} + 4} \right) \ln(\cosh(x)) \right. \\ & \left. (-x + \sqrt{-x+1} + 2) \right) \div (-x + \sqrt{-x+1} + 2) \end{aligned}$$

$$\text{let } Li_2 = D_2$$

Simplify.

$$\begin{aligned}
 & cx^2 + x^2 - d_2 e^{-2x} x \operatorname{arctanh} \left(-\frac{\sqrt{-x+1} \sqrt{-x+\sqrt{-x+1}+2}}{-2x+2\sqrt{-x+1}+4} \right) \ln(\cosh(x)) \\
 & (-x + \sqrt{-x+1} + 2) + x \operatorname{arctanh}(x) \ln(\cosh(x)) (\sqrt{-x+1} + 1) \sqrt{-x+\sqrt{-x+1}+2} \\
 & \quad \left(-a \ln(a - 2bi) + 2bi \ln(a - 2bi) \right) \\
 & 2 \left(2\pi c^{i\sqrt{3}} r^2 \ln(8\pi r^2 + a) (cx^2 + x^2 + x \operatorname{arctanh}(x) \ln(\cosh(x))) (\sqrt{-x+1} + 1) \right. \\
 & \left. \sqrt{-x+\sqrt{-x+1}+2} \right) - d_2 e^{-2x} x \operatorname{arctanh} \left(-\frac{\sqrt{-x+1} \sqrt{-x+\sqrt{-x+1}+2}}{-2x+2\sqrt{-x+1}+4} \right) \ln(\cosh(x)) \\
 & \quad (-x + \sqrt{-x+1} + 2)
 \end{aligned}$$

Everything divided by

$$\div (2(-x + \sqrt{-x+1} + 2))$$

Simplify.

$$4\pi c^{i\sqrt{3}} r^2 \ln(8\pi r^2 + a) (cx^2 + x^2 + x \operatorname{arctanh}(x) \ln(\cosh(x))) (\sqrt{-x+1} + 1)$$

$$\begin{aligned}
 & \sqrt{-x+\sqrt{-x+1}+2} \\
 & \text{Everything divided by} \\
 & \div (2(-x + \sqrt{-x+1} + 2))
 \end{aligned}$$

$$\frac{2d_2 x \operatorname{arctanh} \left(-\frac{\sqrt{-x+1}}{2\sqrt{-x+\sqrt{-x+1}+2}} \right) \ln(\cosh(x)) (-x + \sqrt{-x+1} + 2)}{e^{2x}}$$

Simplify.

$$\begin{aligned}
 & cx^2 + x^2 - d_2 e^{-2x} x \operatorname{arctanh} \left(-\frac{\sqrt{-x+1} \sqrt{-x+\sqrt{-x+1}+2}}{-2x+2\sqrt{-x+1}+4} \right) \ln(\cosh(x)) (-x + \sqrt{-x+1} + 2) \\
 & + x \operatorname{arctanh}(x) \ln(\cosh(x)) (\sqrt{-x+1} + 1) (\sqrt{-x+\sqrt{-x+1}+2}) \\
 & \quad (-a \ln(a - 2bi) + 2bi \ln(a - 2bi))
 \end{aligned}$$

Everything divided by

$$\div (2(-x + \sqrt{-x+1} + 2))$$

$$2 \left(2\pi c^{i\sqrt{3}} r^2 \ln(8\pi r^2 + a) (cx^2 + x^2 + x \operatorname{arctanh}(x) \ln(\cosh(x))) (\sqrt{-x+1} + 1) \right.$$

$$\left. \sqrt{-x+\sqrt{-x+1}+2} \right)$$

Everything divided by

$$\div (-x + \sqrt{-x+1} + 2)$$

$$+ \frac{d_2 e^{-2x} x \tanh^{-1} \left(\frac{-\sqrt{-x+1} \sqrt{-x + \sqrt{-x+1} + 2}}{-2x + 2\sqrt{-x+1} + 4} \right) \ln(\cosh(x))}{2}$$

Simplify.

$$cx^2 + x^2 - d_2 e^{-2x} x \operatorname{arctanh} \left(\frac{-\sqrt{-x+1} \sqrt{-x + \sqrt{-x+1} + 2}}{-2x + 2\sqrt{-x+1} + 4} \right) \ln(\cosh(x)) (-x + \sqrt{-x+1} + 2)$$

$$+ x \operatorname{arctanh}(x) \ln(\cosh(x)) (\sqrt{-x+1}) (-x + \sqrt{-x+1} + 2) (-a \ln(a - 2bi) + 2bi \ln(a - 2bi))$$

$$4\pi\omega^{j\sqrt{3}} r^2 \ln(8\pi r^2 + \psi) (cx^2 + x^2 + x \operatorname{arctanh}(x) \ln(\cosh(x))) (\sqrt{-x+1} + 1)$$

$$1 + d_2 e^{-2x} x \operatorname{arctanh} \left(\frac{\sqrt{-x + \sqrt{-x+1} + 2}}{-2x + 2\sqrt{-x+1} + 4} \right) \ln(\cosh(x)) (-x + \sqrt{-x+1} + 2)$$

Everything divided by

$$\div (2(-x + \sqrt{-x+1} + 2))$$



1154

Jake Brockbank

Equation #13

$$\int \aleph \Xi \Phi^{\psi^\phi} d\psi$$

$$= \frac{\Xi \psi \aleph \left(-\psi^\phi \log(\Phi) \right)^{-\frac{1}{\phi}} \Gamma \left(\frac{2}{1+\sqrt{5}}, -\psi^\phi \log(\Phi) \right)}{\phi} + C$$

$$f(x)^{-1} = -\frac{\phi \aleph^\phi \sqrt{-\psi^\phi \log(\Phi)}}{\Xi \psi \Gamma \left(\frac{2}{1+\sqrt{5}}, -\psi^\phi \log(\Phi) \right)} = R$$

let $V_n = \text{volume}$

$R = \text{radius}$

$$V_n = \int_0^R S_n r^{n-1} dr = \frac{S_n R^n}{n}$$

$$S_n \int_0^\infty e^{-r^2} r^{n-1} dr$$

$$= \int_{-\infty}^\infty \dots \int_{-\infty}^\infty e^{-(x_1^2 + \dots + x_n^2)} dx_1 \dots dx_n$$

$$= \left(\int_{-\infty}^\infty e^{-x^2} dx \right)^n$$

Hyper Sphere.

$$x_1^2 + x_2^2 + \dots + x_n^2 = R^2$$

Set of all points.

$$x = (x_1, x_2 \dots x_{n+1}) \text{ in } E^{n+1}$$

$$x_1^2 + \dots + x_{n+1}^2 = 1$$

$$\{x \in R^n | d(x,0) = 1\}$$

$$x_1^2 + x_2^2 = R^2$$

$$x_1^2 + x_2^2 + x_3^2 = R^2$$

$$r = R \sin \psi$$

$$d_s^3 = \frac{dr^2}{\left(1 - \frac{r^2}{R^2}\right)} + r^2 (d\phi^2 + \sin^2 \phi d\theta^2)$$

$$\frac{dS_n}{dn} = \frac{\pi^{\frac{n}{2}} \left[\ln \pi - \psi_0 \left(\frac{1}{2}n \right) \right]}{\Gamma \left(\frac{1}{2}n \right)} = 0$$

$$\psi_0(x) = \Psi(x)$$

Spherical coordinate.

$$x_1 = R \sin(\psi) \sin(\phi) \cos(\theta)$$

$$x_2 = R \sin(\psi) \sin(\phi) \sin(\theta)$$

$$x_3 = R \sin(\psi) \cos(\phi)$$

$$x_4 = R \cos(\psi)$$

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = R^2$$

$$ds^2 = R^2 \left[d\psi^2 + \sin^2 \psi (d\phi^2 + \sin^2 \phi d\theta^2) \right]$$

SA reaches maximum, then decreases towards zero as n increases

Apply Gamma.

$$\Gamma(m) = 2 \int_0^\infty e^{-r^2} r^{2m-1} dr$$

Therefore,

$$\begin{aligned} \frac{1}{2} S_n \Gamma \left(\frac{1}{2}n \right) &= \left[\Gamma \left(\frac{1}{2} \right) \right]^n \\ &= \left(\pi^{\frac{1}{2}} \right)^n \end{aligned}$$

$$S_n = \frac{2 \pi^{\frac{n}{2}}}{\Gamma \left(\frac{1}{2}n \right)}$$

$$S_n = \left\{ \begin{array}{l} \frac{2^{\frac{(n+1)}{2}} \pi^{\frac{(n-1)}{2}}}{(n-2)!!} \text{ for } n \text{ odd} \\ \frac{2n^{\frac{1}{2}}}{|\frac{1}{2}n-1|!} \text{ for } n \text{ even} \end{array} \right\}$$

Hyper Surface Area

$$\begin{aligned} S_3 &= \int_0^\pi R d\psi \int_0^\pi R \sin \psi d\phi \int_0^{2\pi} R \sin \psi \sin \phi d\theta \\ &= 2 \pi^2 R^3 \end{aligned}$$

$$2\pi^2 \left[\frac{\phi^2 \sqrt{-\psi^{\phi} \log(\Phi)}}{\Xi \psi \Gamma\left(\frac{2}{1+\sqrt{5}}, -\psi^{\phi} \log(\Phi)\right)} \right]$$

$$2\pi^2 \left[\frac{\phi^3 \aleph^3 (-\psi^{\phi} \log(\Phi))^{\frac{3}{2}}}{\Xi^3 \psi^3 \Gamma\left(\frac{1}{2}(\sqrt{5}-1), -\psi^{\frac{1}{2}(1+\sqrt{5})} \log(\Phi)\right)^3} \right]$$

3 - sphere to dimensions $n \geq 4$

Final answer.

$$SA = \frac{2\pi^2 \phi^2 \aleph^3 \psi^{3\phi-3} \log^3(\Phi)}{\Xi^3 \Gamma\left(\frac{1}{2}(-1+\sqrt{5}), -\frac{1}{2}\psi^{1+\sqrt{3}} \log(\Phi)\right)^3}$$

Conclusion: The concept of higher dimensional objects has been something pondered by physicists and mathematicians for centuries. Conceptually the idea may seem confusing; however, once you begin to calculate, you begin to understand and notice slight patterns.



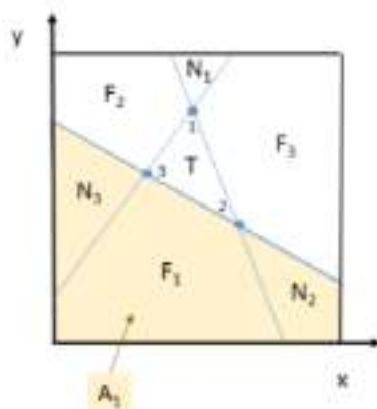
1157

Jake Brockbank

Equation #14

Consider a random triangle illustrated below. Regions are labelled near (N_i) and far (F_i) from point i . T is the triangle. If the 4th point falls in regions F_1 , F_2 , or F_3 a convex quadrilateral will result. The probability that 4 points produce a convex shape is equivalent to the probability that the 4th point will be in one of those regions. That is, $p = E[F_1 + F_2 + F_3]$ where F_i is the random variable expressing the area of that region.

Also keep in mind: that $35/48\pi^2 \leq \kappa \leq 1/12$, where the minimum is attained only when K is an ellipse and the maximum only when K is a triangle. The upper and lower bounds of κ only differ by about 13%. It has been shown, [2] that $\kappa = 11/144$ for K a square.



Consider shaded region $[A_1]$ and $[A_2][A_3]$

$$F_1 + N_2 + N_3 = A_1$$

$$N_1 + F_2 + F_3 + T = 1 - A_1$$

Combine both.

$$F_1 + F_2 + F_3 = 2 - 2T - A_1 - A_2 - A_3$$

Therefore,

$$P = E[F_1 + F_2 + F_3] = 2 - 2E[T] - 3E[A]$$

$$E[T] = \frac{11}{144}$$

$$f(a_{min}) = 4a_{min}$$

$$\text{For } 0 < a_{min} < \frac{1}{2}$$

$$E[A_{app}] = 16 \int_0^{\frac{1}{2}} a_{min}^2 (1 - a_{min}) da_{min} = \frac{5}{12}$$

Given points (2,3)

$$A_1 = 1 - a_{min}$$

Distribution for a_{min} is given by $f(x) = 2x$

$$0 < x < 1$$

$$f(a_{min}) = -16a_{min} \log(2a_{min})$$

$$\text{For } 0 < a_{min} < \frac{1}{2}$$

$$\begin{aligned} E[A_{corner}] &= -32 \int_0^{\frac{1}{2}} a_{min}^2 (1 - a_{min}) \log(2a_{min}) da_{min} \\ &= \frac{23}{72} \end{aligned}$$

$$\begin{aligned} E[A] &= P(A_{opp})E[A_{opp}] + P(A_{corner})E(A_{corner}) \\ &= \frac{2}{3} \frac{5}{12} + \frac{1}{3} \frac{23}{72} = \frac{83}{216} \end{aligned}$$

$$P = 2 - 2 \frac{11}{144} - 3 \frac{83}{216} = \frac{25}{36} = \left(\frac{5}{6}\right)^2 = 0.694$$

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Equation #15

Integrate. Brute force calculates (No formulas) (Using Taylor series) (Then puiseux) sum from psi equals zero:

$$\int \psi^\psi d\psi$$

Series expansion of the integral at $\psi = 0$

Some information prerequisites to know

Taylor series E.g.

$$\sin(x) = \sum_{n=0}^{\infty} \left(\frac{(-1)^n}{(2n+1)!} x^{2n+1} \right) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$$

$$f(x) = e^x = \sum_{n=0}^{\infty} \left(\frac{x^n}{n!} \right) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots$$

$$x^x \rightarrow e^u = 1 + u + \frac{u^2}{2} + \dots$$

$$x^x \sim \sum_{k=0}^{\infty} \left(\frac{1}{k!} (x \log(x))^k \right) = 1 + x \log x + \frac{1}{2} x^2 \log^2 x + 0 (x^3 \log^3 x)$$

$$\left(\left(\text{Digamma Function } \psi(x) = \ln x - \frac{1}{2x} \right) \right)$$

$$\psi(z) =$$

$$\left[-\gamma - \sum_{k=0}^{\infty} \zeta(k) (-z)^k \right] \left[\psi(z) = \left[-\gamma - \sum_{k=0}^{\infty} \zeta(k) (-z)^k \right] \right]$$

$$\psi(z)^{\psi(z)} =$$

$$\left[-\gamma - \sum_{k=0}^{\infty} \zeta(k) (-z)^k \right] \left[-\gamma - \sum_{k=0}^{\infty} \zeta(k) (-z)^k \right]$$

$$\sum_{k=0}^{\infty} (\zeta(k) (-z)^k)$$

$$(-z)^k \zeta(k) = \frac{(-z)^k \sum_{n=0}^{\infty} \frac{\sum_{j=0}^n (-1)^j (1+j)^{1-k} \binom{n}{j}}{1+n}}{-1+k}$$

Taylor series of $\psi(x)$ at $z = 1$

$$\psi(z+1) = -\gamma - \sum_{k=1}^{\infty} \zeta(k+1) (-z)^k$$

at $z = 0$

$$\psi(z) = -\gamma - \sum_{k=0}^{\infty} \zeta(k) (-z)^k$$

I simplified the information into the following equation of psi z to the psi z at psi equals zero.

$$\psi(z)\psi(z) = \left[-Y - \sum_{k=0}^{\infty} \left[\frac{(-z)^k \sum_{n=0}^{\infty} \frac{\sum_{j=0}^n (-1)^j (1+j)^{1-k} \binom{n}{j}}{1+n}}{-1+k} \right] \right]$$

To the power of

$$\left[-Y - \sum_{k=0}^{\infty} \left[\frac{(-z)^k \sum_{n=0}^{\infty} \frac{\sum_{j=0}^n (-1)^j (1+j)^{1-k} \binom{n}{j}}{1+n}}{-1+k} \right] \right]^{2^{-s}}$$

$$\sum_{j=0}^n (-1)^j (1+j)^{1-s}$$

$$2^{-s} \left[2(-1)^n \left(\zeta\left(s-1, \frac{n+2}{2}\right) - \zeta\left(s-1, \frac{n+3}{2}\right) \right) + A \right]$$

$$A = (2^s - 4)\zeta(s-1)$$

$$\psi(z)\psi(z) = \left[-Y - \sum_{k=0}^{\infty} \zeta(k) (-z)^k \right] \left[-Y - \sum_{k=0}^{\infty} \zeta(k) (-z)^k \right]$$

$$(-z)^k \zeta(s) = \frac{(-z)^k}{-1+s} \left[\frac{(n+1)2^{-s} \left[((-1)^k + 1)\zeta\left(-1, \frac{n+2}{2}\right) + s\zeta^{(1,0)}\left(-1, \frac{n+2}{2}\right) + A \right]}{\frac{1}{2}(k+1)(k+2)} \right]$$

$$A = \frac{1}{2}s^2 \zeta^{(2,0)}\left(-1, \frac{n+2}{2}\right) + \frac{1}{6}s^3 \zeta^{(3,0)}\left(-1, \frac{n+2}{2}\right) + \frac{1}{24}s^4 \zeta^{(4,0)}\left(-1, \frac{n+2}{2}\right) + O(s^5) - (Big) +$$

$$(n+1)(2^s - 4)((n+1)\zeta(s-1))$$

$$Big = \zeta\left(-1, \frac{n+3}{2}\right) + s\zeta^{(1,0)}\left(-1, \frac{n+2}{2}\right) + \frac{1}{2}s^2 \zeta^{(2,0)}\left(-1, \frac{n+3}{2}\right) + \frac{1}{6}s^3 \zeta^{(3,0)}\left(-1, \frac{n+3}{2}\right) + \frac{1}{24}s^4 \zeta^{(4,0)}\left(-1, \frac{n+3}{2}\right) + O(s^5)$$

$$\sum_{n=0}^{\infty} (2^s - 4)$$

$$(2^s - 4),$$

$$-\frac{1}{12} + s\left(\frac{1}{12} - \log(A)\right) + \frac{1}{2}s^2\zeta''(-1) + \frac{1}{6}\zeta'''(-1)s^3 + \frac{1}{24}\zeta^{(4)}(-1)s^4 + \frac{1}{120}\zeta^{(5)}(-1)s^5 + O(s^6)$$

$$\sum_{n=0}^{\infty} \frac{2^{-s}(A)}{1+n}$$

$$A = 2(-1)^n \left(\zeta\left(s-1, \frac{n+2}{2}\right) - \zeta\left(s-1, \frac{n+3}{2}\right) \right) + (2^s - 4)\zeta(s-1)$$

$$(-z)^k \zeta(s) = \frac{(-z)^k \left[\frac{(n+1)2^{-s} \left[((-1)^k + 1) \zeta\left(-1, \frac{n+2}{2}\right) + A \right]}{\frac{1}{2}(k+1)(k+2)} \right]}{-1+s}$$

$$A = s\zeta^{(1,0)}\left(-1, \frac{n+2}{2}\right) + \frac{1}{2}s^2\zeta^{(2,0)}\left(-1, \frac{n+2}{2}\right) + \frac{1}{6}s^3\zeta^{(3,0)}\left(-1, \frac{n+2}{2}\right) + \frac{1}{24}s^4\zeta^{(4,0)}\left(-1, \frac{n+2}{2}\right) + O(s^5) - (Big) + (n+1)(2^s - 4)((n+1)\zeta(s-1))$$

We can represent that entire equation as Ans. 1

$$\psi(z)\psi(z) = \left[-Y - \sum_{k=0}^{\infty} [Ans. 1]\right] \left[-Y - \sum_{k=0}^{\infty} [Ans. 1]\right]$$

Only partial sum

At $\psi(z) = 0$ function diverges

$$\left\{ \zeta(-s), \frac{1}{2}(n+2)s \right\}$$

Summation of both gives us.

$$\zeta(-s) = (n+1)\zeta(-s)$$

$$\frac{1}{2}(n+2)s = \frac{1}{2}(k+1)(n+2)s$$

$$\psi(z)\psi(z) = \frac{\left[-Y - \left[\frac{z(-z)^n + 1}{z+1}\right] \left[(k+1)(n+1)2^{-s} \left[\frac{A}{\frac{1}{2}(k+1)(k+2)} \right] \right] \right]}{-1+s}$$

$$\frac{1}{2}s^2\zeta^{(2,0)}\left(-1, \frac{n+3}{2}\right)$$

$$= \left\{ -\frac{1}{2}s^2\zeta(u; g_2, g_3)^2, \frac{1}{4}(n+3)s^2 \right\}$$

First part can be broken down into the following.

$$\left\{ -\frac{1}{2}s^2\zeta(u)^2, -\frac{1}{2}s^2\zeta(g_2)^2, -\frac{1}{2}s^2\zeta(g_3)^2 \right\}$$

$$First = N_1 \quad Second = N_2 \quad Third = N_3$$

Second part of the original function with $\frac{1}{4}(n+3)s^2$

$$\begin{aligned} & \frac{1}{4}(k+1)(n+3)s^2 \\ & s\zeta^{(1,0)}\left(-1, \frac{n+3}{2}\right) \\ & = \left\{ \zeta(-s), \frac{1}{2}(n+3)s \right\} \\ & \{A\} \end{aligned}$$

$$A = \frac{1}{2}\ln(2\pi) + \frac{36 + (3 - 6\ln(2\pi))}{48} - 56 - 12\ln(2\pi) + \frac{5}{144} + O(s^7), \frac{1}{2}(k+1)(n+3)s$$

Anything after big O notation is in brackets. So, when subtracting, you must subtract the whole function.

$$\begin{aligned} & (n+1)2^{-s}[A] \\ A = & \left((-1)^k + 1 \right) \zeta\left(-1, \frac{n+2}{2}\right) + s\zeta^{(1,0)}\left(-1, \frac{n+2}{2}\right) + \frac{1}{2}s \\ & 2\zeta^{(2,0)}\left(-1, \frac{n+2}{2}\right) + \frac{1}{6}s^3\zeta^{(3,0)}\left(-1, \frac{n+2}{2}\right) + \frac{1}{24}s^4\zeta^{(4,0)}\left(-1, \frac{n+2}{2}\right) \\ & + O(s^5) - \zeta\left(-1, \frac{n+3}{2}\right) + s\zeta^{(1,0)}\left(-1, \frac{n+3}{2}\right) \\ & + \frac{1}{2}s^2\zeta^{(2,0)}\left(-1, \frac{n+3}{2}\right) + \frac{1}{6}s^3\zeta^{(3,0)}\left(-1, \frac{n+3}{2}\right) \\ & + \frac{1}{24}s^4\zeta^{(4,0)}\left(-1, \frac{n+3}{2}\right) + O(s^5) + \\ & (n+1)(2^s - 4)((n+1)\zeta(s-1)) \\ & \left[\frac{1}{6}s^3\zeta^{(3,0)}\left(-1, \frac{n+3}{2}\right) \right] \\ & \left\{ -\frac{1}{6}s^3\zeta(u; g_2, g_3)^3, \frac{1}{12}(n+3)s^3 \right\} \end{aligned}$$

Evaluating the summation of the first part gives us.

$$\left\{ \frac{s^3\zeta(u)^3}{6}, \frac{1}{6}s^3\zeta(g_2)^3, \frac{1}{6}s^3\zeta(g_3)^3 \right\}$$

Evaluating the summation of the second part gives us.

$$\frac{1}{12}(k+1)(n+3)s^3$$

Continuing the expansion of the first part yields.

$$-\frac{1}{48} - \frac{\ln(2\pi)}{16} + \frac{2(36)}{384} + \frac{2(3-18\ln(2\pi))}{384} + 56 + 36\ln(2\pi) - \frac{(12-12\log_2(\pi)-24\ln(2)\ln(\pi))}{9216} - 12 - (-12\ln(2\pi)) + 24\ln(2\pi) + 46 - 2304\ln(2\pi) + 144 + 384\ln(2\pi) + O(u^5)$$

Going back to the first function.

$$\left\{ -\frac{1}{2}s^2\zeta(u)^2, -\frac{1}{2}s^2\zeta(g_2)^2, -\frac{1}{2}s^2\zeta(g_3)^2 \right\}$$

$$First = N_1 \quad Second = N_2 \quad Third = N_3$$

Now evaluating, we find.

$$N_1: -\frac{1}{8} - \frac{\ln(2\pi)}{4} + \frac{(n+1)s^2u^2(36)}{96} + \frac{(n+1)s^2u^3(3-12\ln(2\pi))}{48} + (12(n+1)y+12) + 4 + 12\ln(2\pi) + (n+1)O(u^4)$$

$$N_2: -\frac{1}{8} - \frac{\ln(2\pi)}{4} + \frac{(n+1)s^2g_2^2(36)}{96} + \frac{(n+1)s^2g_2^3(3-12\ln(2\pi))}{48} + (12(n+1)y+12) + 4 + 12\ln(2\pi) + (n+1)O(g_2^4)$$

$$N_3: -\frac{1}{8} - \frac{\ln(2\pi)}{4} + \frac{(n+1)s^2g_3^2(36)}{96} + \frac{(n+1)s^2g_3^3(3-12\ln(2\pi))}{48} + (12(n+1)y+12) + 4 + 12\ln(2\pi) + (n+1)O(g_3^4)$$

If something diverges it becomes obsolete.

Series expansion at $u = 0$.

$$\sum_{k=0}^{\infty} [A]$$

$$A = -\frac{s^2}{8} - \frac{u(s^2 \log(2\pi))}{4} + \frac{s^2u^2(B)}{96} + \frac{s^2u^3(C)}{48} + O(u^4)$$

$$B = 24y_1 + 12y^2 - \pi^2 - 24\log^2(2) - 24\log^2(\pi) - 2\log(16777216)\log(\pi)$$

$$C = \left(-2(-3y_2 - 12y_1 \log(2\pi)) + 2\zeta(3) + D \right)$$

$$D = \log^3(2\pi) + 3\log^2(2)\log(2\pi) + 3\log^2(\pi)\log(2\pi) + 6\log(2)\log(\pi)\log(2\pi)$$

$$\psi(z)\psi'(z) = \frac{\left[-\gamma - \left[\frac{z(-z)^n + 1}{z+1} \right] \left[\frac{A}{\frac{1}{2}(k+1)(k+2)} \right] \right]}{-1+s}$$

$$A = (k+1)(n+1)2^{-s}[B]$$

$$\begin{aligned} B = & \frac{1}{2} \left(2m + (-1)^m + 3 \right) \zeta \left(-1, \frac{n+2}{2} \right) + \\ & \left\{ (n+1)\zeta(-s), \frac{1}{2}(k+1)(n+2)s \right\} - \frac{1}{8} - \frac{\ln(2\pi)}{4} + \\ & \frac{(n+1)s^2 u^2(36)}{96} + \frac{(n+1)s^2 u^3(3-12\ln(2\pi))}{48} + \\ & (12(n+1)y+12) + 4 + 12\ln(2\pi) + (n+1)O(u^4), -\frac{1}{8} - \\ & \frac{\ln(2\pi)}{4} + \frac{(n+1)s^2 g_2^2(36)}{96} + \frac{(n+1)s^2 g_2^3(3-12\ln(2\pi))}{48} + \\ & (12(n+1)y+12) + 4 + 12\ln(2\pi) + (n+1)O(g_2^4), - \\ & \frac{1}{8} - \frac{\ln(2\pi)}{4} + \frac{(n+1)s^2 g_3^2(36)}{96} + \\ & \frac{(n+1)s^2 g_3^3(3-12\ln(2\pi))}{48} + (12(n+1)y+12) + 4 + 12\ln \\ & (2\pi) + (n+1)O(g_3^4), \frac{1}{4}(k+1)(n+2)s^2 \end{aligned}$$

Adding the following sum.

$$\left\{ -\frac{1}{6}s^3 \zeta(u; g_2, g_3)^3, \frac{1}{12}(n+3)s^3 \right\}$$

We get the following.

$$\frac{1}{4}(k+1)(n+2)s^2 + [A]$$

$$\begin{aligned}
 A = & \frac{1}{48} + \left(\frac{1}{16} + \frac{\ln(2\pi)}{16} \right) + \frac{2(-36)}{384} - \\
 & \frac{(3 - 18\ln(2\pi))}{384} + 56 + 36\ln(2\pi) + \\
 & \frac{2}{9216} \left(12 + (6\log_2(\pi)) \right) - 4 - \\
 & (-12\ln(2\pi) + 24\ln(2\pi)) - 98 + O(u^5), \frac{1}{48} + \\
 & \left(\frac{1}{16} + \frac{\ln(2\pi)}{16} \right) + \frac{2(-36)}{384} - \frac{(3 - 18\ln(2\pi))}{384} + 56 + 36\ln \\
 & (2\pi) + \frac{2}{9216} \left(12 + (6\log_2(\pi)) \right) - 4 - \\
 & (-12\ln(2\pi) + 24\ln(2\pi)) - 98 + O(g_2^5), \frac{1}{48} + \\
 & \left(\frac{1}{16} + \frac{\ln(2\pi)}{16} \right) + \frac{2(-36)}{384} - \frac{(3 - 18\ln(2\pi))}{384} + 56 + 36\ln \\
 & (2\pi) + \frac{2}{9216} \left(12 + (6\log_2(\pi)) \right) - 4 - \\
 & (-12\ln(2\pi) + 24\ln(2\pi)) - 98 + O(g_3^5), \\
 & \frac{1}{12} (k+1)(n+2)s^3
 \end{aligned}$$

$$\left\{ \frac{1}{24} s^4 \zeta^{(4,0)} \left(-1, \frac{n+2}{2} \right), -\frac{1}{24} s^4 \zeta(u; g_2, g_3)^4, \frac{1}{48} (n+2)s^4 \right\}$$

Summation of the second part gives us.

$$\frac{1}{48} (k+1)(n+2)s^4$$

Summation of the first part gives us.

$$\left\{ -\frac{1}{24} s^4 \zeta(u)^4, -\frac{1}{24} s^4 \zeta(g_2)^4, -\frac{1}{24} s^4 \zeta(g_3)^4 \right\}$$

First part = N_1 Second part = N_2 Third part = N_3

$$\begin{aligned}
 N_1: & -\frac{1}{384} - \frac{\ln(2\pi)}{96} + \frac{2(6)}{384} + \\
 & \frac{2(15 + 24\ln(2\pi) - 4 + 12\ln(2\pi))}{576} - \frac{18}{36864} - 72\log(2)\log \\
 & (\pi) - 36\log(2)\log(\pi) + 18\ln(\pi) + 36\log^2(\pi) + 48 + 48\pi \\
 & ^2 - 2304\ln(2) + 2304\ln(\pi) + 144 - 384\ln(2\pi) + \\
 & \frac{2((12 - 24\ln(2\pi)) - 56 - 12\ln(2\pi))}{276480} - 480 \\
 & (-2(-3 - 12\ln(2\pi))) + 28 + 12\ln(2\pi) - \\
 & (-12 - 24\ln(2\pi)) - 56 - 12\ln(2\pi) + O(u^6)
 \end{aligned}$$

Adding this to the final equation, we get.

$$\begin{aligned}
 A = & \frac{1}{48} + \left(\frac{1}{16} + \frac{\ln(2\pi)}{16} \right) + \frac{2(-36)}{384} - \\
 & \frac{(3 - 18\ln(2\pi))}{384} + 56 + 36\ln(2\pi) + \\
 & \frac{2}{9216} \left(12 + (6\log_2(\pi)) \right) - 4 - \\
 & (-12\ln(2\pi) + 24\ln(2\pi)) - 98 + O(u^5), \frac{1}{48} + \\
 & \left(\frac{1}{16} + \frac{\ln(2\pi)}{16} \right) + \frac{2(-36)}{384} - \frac{(3 - 18\ln(2\pi))}{384} + 56 + 36\ln \\
 & (2\pi) + \frac{2}{9216} \left(12 + (6\log_2(\pi)) \right) - 4 - \\
 & (-12\ln(2\pi) + 24\ln(2\pi)) - 98 + O(g_2^5), \frac{1}{48} + \\
 & \left(\frac{1}{16} + \frac{\ln(2\pi)}{16} \right) + \frac{2(-36)}{384} - \frac{(3 - 18\ln(2\pi))}{384} + 56 + 36\ln \\
 & (2\pi) + \frac{2}{9216} \left(12 + (6\log_2(\pi)) \right) - 4 - \\
 & (-12\ln(2\pi) + 24\ln(2\pi)) - 98 + O(g_3^5), \\
 & \frac{1}{12} (k+1)(n+2)s^3 - \frac{1}{384} - \frac{\ln(2\pi)}{96} + \frac{2(6)}{384} + \\
 & \frac{2(15 + 24\ln(2\pi) - 4 + 12\ln(2\pi))}{576} - \frac{18}{36864} - 72\log(2)\log \\
 & (\pi) - 36\log(2)\log(\pi) + 18\ln(\pi) + 36\log \\
 & ^2(\pi) + 48 + 48\pi^2 - 2304\ln(2) + 2304\ln(\pi) + 144 - 384\ln \\
 & (2\pi) + \frac{2((12 - 24\ln(2\pi)) - 56 - 12\ln(2\pi))}{276480} - 480 \\
 & (-2(-3 - 12\ln(2\pi))) + 28 + 12\ln(2\pi) - \\
 & (-12 - 24\ln(2\pi)) - 56 - 12\ln(2\pi) + O(u^6), \\
 & \frac{1}{48} (k+1)(n+2)s^4
 \end{aligned}$$

Combining everything together, we get the following fully simplified expansion. When there is multiplication it means multiplying the whole function.

$$\begin{aligned}
 & \psi + \frac{1}{4}\psi^2(2\log(\psi) - 1) + \frac{1}{54}\psi \\
 & 3\left(9\log^2(\psi) - 6\log(\psi) + 2\right) + \frac{1}{768}\psi \\
 & 4\left(32\log^3(\psi) - 24\log^2(\psi) + 12\log(\psi) - 3\right) + \\
 & \frac{\psi^5(625\log^4(\psi) - 500\log^3(\psi) + 300\log^2(\psi) - 120\log(\psi) + 24)}{75000} + \\
 & \frac{\psi^6 \cdot 324\log^5(\psi) - 270\log^4(\psi) + 180\log^3(\psi) - 90\log^2(\psi)}{+ 30\log(\psi) - 5} + \\
 & \frac{1}{233280} \\
 & \frac{1}{592950960}\psi^7 \cdot 117649\log^6(\psi) - 100842\log^5(\psi) + 72030\log \\
 & 4(\psi) - 41160\log^3(\psi) + 17640\log^2(\psi) - 5040\log \\
 & (\psi) + 720 + O(\psi^8) + C
 \end{aligned}$$

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