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Mathematical modeling and analysis of teenage pregnancies in Kenya incorporating contraception and education.

O. M. Nathan [1]
[1] Department of Mathematics Masinde Muliro University of Science and Technology, Kenya
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Abstract

A mathematical model for teenage pregnancies is proposed in this study by considering information available about sexual activities and adherence to contraception measures. The population is subdivided into three (3) different compartments according to their level of information on sexual matters. The model is proved to be both epidemiologically and mathematically well posed. The existence of unique teenage pregnancy free and endemic equilibrium points are investigated. The basic reproduction number is obtained to monitor the dynamics of teenage pregnancies and ascertain its level in order to suggest effective intervention strategies to control this problem. The local as well as global asymptotic stability of these equilibrium points are studied. The analysis revealed a globally asymptotically stable teenage pregnancy free equilibrium whenever $\mathcal{R}_0 \leq 1$ and a globally asymptotically stable endemic equilibrium, if otherwise.

Keywords: Reproduction number; Teenage pregnancy; Contraceptive; Sexual reproductive education.

1 Introduction

Teenage pregnancies have become a malaise in Kenya, with some areas in western and coastal regions having 1 in every 4 girls affected as per Kenya demographics and health survey(KDHS). Teenage pregnancies pose serious health risk, psychosocial and economic dangers to the teenage girls, including thwarting their reproductive health, child birth, schooling, career growth, keeping them in vicious cycle of poverty (many come from already poor family), and overall limiting of their capabilities, opportunities and choices. In Kenya 18 percent of adolescent girls between the age of 12 and 19 years are mothers. The rate of child bearing varies across Kenya. While progress has been made

[1]Corresponding author's email:nathanoigo@yahoo.com

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to decrease the numbers in some regions, in others, it is still alarmingly high. Over 13000 girls dropping out of school every year (KDHS), there is a need to look deeply into this problem.

Adolescents fertility in low and middle income countries such as Kenya presents a severe impediment to development and can lead to school dropout, lost productivity, and the intergenerational transmission of poverty. However, there is debate about whether adolescent pregnancy is a problem in and of itself or merely symptomatic of deeper, ingrained disadvantage. The sociocultural context in which adolescents in Kenya find themselves has changed considerably within the past few generations.

In Kenya, adolescents are experiencing social turmoil resulting from conflicting values as the country becomes more urbanized and industrialised [5]. In most ethnic groups, adolescence, generally commenced with circumcision right of passage that marked transition from childhood to adulthood. Over this period, initiates were secluded from community while a selected tutor explained to them their role in the society and they were taught about sexual behaviour and pregnancy matters. Such customs conferred peer-group identity and promoted a social and personal sense of belonging [6]. Clear messages regarding sexual behaviours were conveyed to young people.

The arrival of missionaries and colonists in the 1800s, and new political, social and economic orders, most sexual socialization rituals were discarded with no alternative given. Industrialization and urbanisation continues to alter the sociocultural structure of the traditional community. Education functions, which formally rested within the family and community, are increasingly being taken over by local and national governments, churches and community groups. These institutions must unite diverse ethnic groups and develop a national message dealing with personal areas such as healthy sexual activities among the teens where the conflict often arise. Leaders remain apprehensive and uncomfortable about policies and legislation related to youths, such as those those affecting sex education, access to contraceptives, expulsion of pregnant girls from schools and enforcement of regulations of age of consent and marriage.

In kenya, only about 10 percent of young people who are sexually active reported regular use of birth control and were aware of those birth control measures [9]. These low level of contraceptives use is linked to barriers young people face when they attempt to obtain contraceptive information and usage. These barriers probably reflect the spontaneity of adolescent sexual activity and pregnancies among them.

To use a birth control in Kenya an adolescent must raise the issue with a possibly suspicious or resistant partner, obtain funds for supplies, overcome fear about rumored side effects and bargain with a health system that is not accommodating to adolescent clients [10]. On the other hand, use of natural family planning can rescue some of these issues. However, it requires meticulous charting and comprehension of the female menstrual cycle that few young people possess. It isn't surprising, then, that few adolescents use contraceptives and that many rely on their peers for information and supplies. Almost a quarter of Kenyan women give birth by the age of 18, and nearly half by the age of 20 years.

To realise the SDGs in Kenya, it is important to reduce the number of teenage pregnancies in the country. If young girls grow up healthy and are able to go to school, then, they are more likely to escape the cycle of poverty and facilitate the upward social and economic mobility of their families and societies.

In 2007, the Committee on the Rights of the Child (CRC) raised an alarm over the high rates of teenage pregnancies. According to (CRC), teenage pregnancies and motherhood rates stood at 18 percent in the year 2018. About 1 in every 5 adolescent girls has either live birth, or pregnant with the first child. Over a decade now, there is no solution yet other than blame games. Therefore, there is a need to take that bold step and let our young generation have access to this vital information on sexual related activities, available pregnancy prevention methods and appropriate contraception methods.

In 2015, Kenya developed a National Adolescent Sexual Reproductive Health Policy(ASRH). The policy aims at enhancing the sexual and reproductive health of adolescents in order to realise adolescents' full potential, as well as to benefit national development agenda [11]. The National Adolescent Sexual and Reproductive Health policy (2015) directly contributes to realising Kenya's Vision 2030 agenda and its demographic dividend targets. The policy addresses young people's health and well being, helps realise gender equality, as well as reduce other forms of inequalities.

Under this policy, the government needs to empower and develop well-informed and healthy adolescents to expand their access to educational and social economic opportunities, and to ensure that they grow into resilient adults. To achieve this, barriers to access contraceptives, coupled with misleading and inaccurate or incomplete information on reproductive health rights among the school-going youth should be addressed.

The Convention on the Elimination of Discrimination against Women (CEDAW) requires state parties to eliminate discrimination against women in education and to provide women equal access to educational materials and advice on family planning methods [15]. It further protects the right to access information on family planning.

Healthy sexual and reproductive health remains a sensitive topic among Kenyan youths. A research by Aids Control Council reveals that girls aged between 15 and 24 years now account for the highest number of new infections. The group accounts for one-third of the 44,789 new HIV adult infections. In turn, this means 14,929 girls aged between 15 and 24 years were infected in 2017 alone. Nascop 2018 estimates shows that 105,230 adolescents between

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10-19 years ware living with HIV, in addition, 184,700 young adults (15-24) are also infected. Sadly, the data indicates that adolescents account for one in every 10 Aids related deaths. Therefore, adolescents and young people also need access to comprehensive information on sex, contraceptive use and HIV prevention methods.

The adolescents need to be educated about sexual activities, contraceptive use, puberty changes and HIV/AIDS preventions ways so that, they can make informed choice when faced with any difficulty. There are many risks connected with sexual activity, including healthy one (e.g feeling close to another person, enjoying physical pleasure and learning about yourself) and unhealthy one including (e.g becoming pregnant or getting someone else pregnant, catching a sexually transmitted disease such as herpes, venereal warts, or HIV among others. Therefore, sex education will not only save our young generation from early pregnancies but also from sexually related terminal diseases.

In this paper, a compartmental model to study teenage pregnancies in Kenya is built. Teenage sexual activities are assumed to a nonstandard epidemic process that rarely emerges out of nothing but is usually related to some already sexually infected environment (such as uncontrolled media contents, uninformed generation and naive society which is not willing to address the matter) which may affect susceptible individuals. In this paper, a mathematical model for the spread of teenage pregnancies in the spirit of epidemiology which describes the dynamic behavior of unhealthy sexual activities, as a disease incorporating use of contraception and awareness through sex education is formulated.

The total population of young people is divided into three compartments: susceptible class (S) consists of innocent young people who are not sexually corrupted yet and have no information on sexual activities, (I) Infected/Corrupted class consists of individuals who are sexually corrupted (practicing unhealthy sexual activities) and those who take advantage of innocence and naivety of teenage population and (T) Treatment class consists of teens who are informed about sexual activities. This population is well versed with healthy sexual reproduction information, measures to prevent pregnancies and sexual related diseases (See figure 1). Using this model, we attempt to test the effective ways of preventing and countering teenage pregnancies.

The basic reproduction number $\mathcal{R}_0$ is ued to evaluate strategies for countering teenage sexuality and pregnancies. For $\mathcal{R}_0 < 1$ the system is shown to have one locally asymptotically stable equilibrium where no Infected/Corupted individual exists and for $\mathcal{R}_0 > 1$, the system has an additional equilibrium point where Infected/Corupted population are endemic in the population and latter equilibrium is asymptotically stable for $\mathcal{R}_0 > 1$. Thus, if $\mathcal{R}_0 < 1$ the teenage pregnancies will be eradicated, that is, Infected/Corrupt population will reduce to zero with time. When $\mathcal{R}_0 > 1$ the menace becomes endemic, that is, Infected/Corrupted individuals establish themselves in the population

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with time.

2 Model Formulation

The spread of teenage pregnancy is modeled as a contact process in this work. The transfer diagram is similar, but different to the one proposed by [15],[16] and [17] to model spread, prevention and treatment of radical ideology. The susceptibles are increased by the recruitment of young individuals into the compartment S at a rate Λ and decreased when sexually informed individuals move to informed class T at a rate γ. It is assumed that susceptible can be influenced via effective contact with infected/corrupted who are sexually corrupted in compartment I at a rate β. Unhealthy sexual influence transmission probability per contact is represented by ρ.

The parameter τ is assumed to be $0 \leq \tau < 1$, this estimates how well an individual is informed about sexual activities, $\tau = 0$ if an individual understands nothing and highly likely to be influenced into unhealthy sexual practices and vice versa. Thus $\beta = \rho(1-\tau)$ is the effective contact rate. The parameter π accounts for sexually corrupt individuals who opt to quit from the behaviour voluntarily(this group accounts for those who were sexually abused in one way or the other). The infected compartment I is increased when the susceptibles interact with sexually corrupted individuals at a rate β.

The parameter σ measures the rate at which sexually corrupted individuals may be guided positively by qualified personnel, parents or even teachers and join informed class T. The informed class T are individuals who are informed and are aware of healthy sexual activities. This compartment is increased at a rate γ and σ when susceptibles and sexually corrupted individuals join the informed compartment T respectively. All three compartments are decreased by natural death rate μ. S(t), $I(t)$, and $T(t)$ are variables that represent numbers of the individuals in the three compartments at time t in years.

This study assumes the following: homogeneous mixing of the teens and the parameter γ accounts for young individual who are brought up in an environment where healthy sexual activities information is readily available. The total population size at time t is denoted by N(t) with $N(t) = S(t) + It) + T(t)$. The transfer diagram for the system is given in figure 1.

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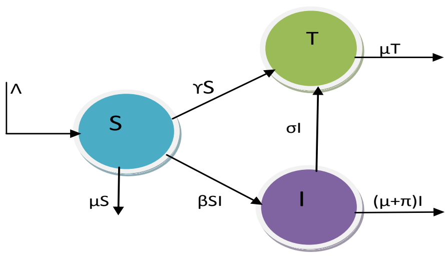

Figure 1: *Flow chart diagram of the proposed model.*

Thus, the teenage pregnancy model consist of the following differential equations, with non negative initial conditions.

$$\begin{aligned}\frac{dS}{dt} &= \Lambda - \beta S(t)I(t)) - (\gamma + \mu)S(t) \\ \frac{dI}{dt} &= \beta S(t)I(t) - (\pi + \sigma + \mu)I(t) \\ \frac{dT}{dt} &= \gamma S(t) + \sigma I(t) - \mu T(t). \end{aligned} \tag{1}$$

Where $S(0) > 0$, $I(0) \geq 0$, and $T(0) > 0$, $\beta = \rho(1-\tau)$ is the effective contact of infection and $\lambda = \beta S(t)I(t)$ is the force of recruitment.

3 Analysis of the Model

Since the study involves a human population, then all population compartments are expected to be non negative $\forall t > 0$ in the feasible region Γ where $S(t), I(t), T(t) \in \Gamma \subset \mathcal{R}^3_+$. It can be shown that all solutions are bounded in Γ, $\forall t > 0$ such that $0 < N \leq \frac{\Lambda}{\mu}$. Thus the model is epidemiologically well posed in the region Γ and can be analysed.

3.1 Corrupt Free Equilibrium (CFE) point

The Corrupt Free Equilibrium point $E^0(S_0, 0, T_0)$ of system of equations (1) is obtained by setting the Infected class to zero. This gives;

$$\begin{aligned} 0 &= \Lambda - \beta S(t)I(t) - (\gamma + \mu)S(t) \\ 0 &= \beta S(t)I(t) - (\pi + \sigma + \mu)T(t) \\ 0 &= \gamma S(t) + \sigma I(t) - \mu T(t). \end{aligned} \tag{2}$$

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At the $E^0(S_0, 0, T_0)$, the system of equations (1) reduces to;

$$\begin{aligned} \Lambda - \beta S(t)I(t) - (\gamma+\mu)S(t) &= 0 \\ \gamma S(t) + \sigma I(t) - \mu T(t) &= 0 \end{aligned} \tag{3}$$

The $E^0(S_0, 0, T_0)$ point for system of equations (1) is thus given by;

$$E^0\{S_0, 0, T_0\} = \{\frac{\Lambda}{(\gamma+\mu)}, 0, \frac{\gamma\Lambda}{\mu(\gamma+\mu)}\} \tag{4}$$

3.2 The Basic Reproduction Number $\mathcal{R}_0$

The basic reproduction number $\mathcal{R}_0$ is the spectral radius of the next generation matrix calculated at $E^0(S_0, 0, T_0)$. The basic reproduction number is important since it is directly related to the effort required to eliminate this problem. $\mathcal{R}_0$ can be calculated as follows (see Watmough *et al.*[13] for more details).

Definition 3.2.1. The basic reproduction number $\mathcal{R}_0$ is the average number of population at puberty, one sexually corrupt person can recruit in a purely susceptible population throughout the contact period. The basic reproduction number of the system is determined using next generation matrix approach, Watmough *et al.*[13].

Lemma 3.2.1. The basic reproduction number of the System of equations (1) is given by;

$$\mathcal{R}_0 = \frac{\beta S_0}{\pi + \sigma + \mu} \tag{5}$$

Proof. Consider that $\mathcal{F}_i$ is the rate of appearance of new Infected individuals in compartment associated with index i and $\mathcal{V}^-{}_i$ is the rate of transfer of individuals out of of compartment associated with index i. In this way, the matrices $\mathcal{F}_i$ and $\mathcal{V}^-{}_i$ associated with model (1) are given by;

$$\mathcal{F}_i = \begin{bmatrix} \beta S(t)I(t) \\ 0 \end{bmatrix} and \mathcal{V}^-{}_i = \begin{bmatrix} (\pi+\sigma+\mu)I(t) \\ -\gamma S(t) - \sigma I(t) + \mu T(t) \end{bmatrix} \tag{6}$$

The Jacobian matrix of $\mathcal{F}_i$ and $\mathcal{V}_i$ evaluated at Corrupt free equilibrium are respectively given by;

$$F = \begin{bmatrix} \beta S_0 & 0 \\ 0 & 0 \end{bmatrix} And, V = \begin{bmatrix} \pi+\sigma+\mu & 0 \\ -\sigma & \mu \end{bmatrix} \tag{7}$$

Determining the inverse of matrix V yields;

$$V^{-1} = \frac{1}{\mu(\pi+\sigma+\mu)} \begin{bmatrix} \mu & 0 \\ \sigma & \pi+\sigma+\mu \end{bmatrix} \tag{8}$$

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The spectral radius of system of equations (1) given by $\mathcal{R}_0 = \rho\mathcal{F}\mathcal{V}^{-1}$ is;

$$\mathcal{R}_0 = \frac{\beta S_0}{\pi + \sigma + \mu} \tag{9}$$

□

The parameters are as described in section 2. $\mathcal{R}_0$ is a useful parameter that quantifies the transmission of this unhealthy sexual influence in our model. It is clear that $\mathcal{R}_0$ is a decreasing function of σ, increasing this parameter decreases $\mathcal{R}_0$. This, according to our model is a successful strategy to counter teenage pregnancies. This corresponds to increasing the awareness of different methods of contraception and information about teenage reproductive health and be accorded with necessary help and information concerning healthy sexual activities.

Another different successful strategy will be achieved by increase parameter τ by encouraging teenagers to shun a way from unhealthy sexual activities or engage in safe and protected sexual activities. Similarly, effective control of harmful media content and other harmful content they access through social media is certainly a positive step to reduction of parameter ρ. To increase σ, healthy sexual information is needed to be provided these young people so that they make informed decisions or incase assaulted they have an idea of what to do [15]. Availability of healthy sexual discussion from parents at home is a milestone in overcoming unhealthy sexual behaviours among the teenagers.

3.3 Local stability of Corrupt Free Equilibrium (CFE) point.

The stability of an equilibrium point determines whether or not solutions near the equilibrium point remains nearby, get closer or get further a way.

Definition 3.3.1. For local stability, perturbing the CFE, the system stays in the neighborhood of equilibrium point or, for asymptotic stability, it returns to equilibrium point.

Theorem 3.3.1 .The CFE of the system of equations (1) is locally asymptotically stable whenever $R_0 < 1$ and unstable otherwise.

Proof. From the system of equations (1), the characteristic polynomial evaluated at $E^0 = (S^0, I^0, T^0)$ is given by;

$$J_{E^0} = \begin{vmatrix} -(\gamma+\mu)-\lambda_1 & \beta S_0 & 0 \\ 0 & (\pi+\sigma+\mu)(\mathcal{R}_0-1)-\lambda_2 & 0 \\ \gamma & \sigma & -\mu-\lambda_3 \end{vmatrix} = 0 \tag{10}$$

Evaluating the roots of characteristic polynomial (10), we have;

$$\begin{cases} \lambda_1 = -\{\gamma + \mu\} < 0 \\ \lambda_2 = (\pi + \sigma + \mu)(\mathcal{R}_0 - 1) \\ \lambda_3 = -\mu < 0. \end{cases} \tag{11}$$

Thus, $\lambda_2 < 0$ if and only if $\mathcal{R}_0 < 1$. Using the Routh-Hurwitz criterion [32], it can be seen that all the eigenvalues of the characteristic polynomial (10) have negative real part if and only if $\mathcal{R}_0 < 1$. Hence E^0 is LAS on Γ provided the inequality $\mathcal{R}_0 < 1$ is satisfied. This completes the proof. □

3.4 Global stability of the Corrupt Free Equilibrium (CFE) Point.

Definition 3.4.1. An equilibrium point is global (asymptotically) stable if it is unique equilibrium of the dynamical system and the property hold globally (its domain of attraction is entire state space).

Theorem 3.4.1. *$\mathcal{R}_0 < 1$, then the corrupt free equilibrium point $E^0 = (S^0, I^0, T^0)$ of system of equations (1) is globally asymptotically stable in Γ and unstable if $\mathcal{R}_0 > 1$*

Proof. To prove this the following Lyapunov function is used;

$$K(S, I, T) = \{S(t) - S_0(t) - S_0(t)ln\frac{S(t)}{S_0(t)}\} + I(t) \tag{12}$$

Determining the time derivative of the lyapunov function (12) along the trajectories of system of equations (1) gives;

$$\frac{dK(S, I, T)}{dt} = \{1 - \frac{S_0}{S}\}\frac{dS}{dt} + \frac{dI}{dt} \tag{13}$$

Substituting in for $\frac{dS}{dt}$ and $\frac{dI}{dt}$ yields;

$$\begin{cases} \frac{dK(S,I,T)}{dt} = \{1 - \frac{S_0}{S}\}\{(\gamma + \mu)S_0 - \beta SI - (\gamma + \mu)S\} \\ +\{\beta SI - (\pi + \sigma + \mu)I\} \end{cases} \tag{14}$$

Expanding and simplifying equation (14) results to;

$$\frac{dK(S, I, T)}{dt} = -(\gamma + \mu)\frac{(S - S_0)^2}{S} + (\pi + \sigma + \mu)\{\mathcal{R}_0 - 1\}I \tag{15}$$

Which is negative definite for $\frac{dK}{dt} |_3 \leq 0$ for $\mathcal{R}_0 \leq 1$. When $\mathcal{R}_0 = 1$, $\frac{dK}{dt} |_3 = 0$ if and only if $I(t) = 0$. When $\mathcal{R}_0 \leq 1$, it is easy to verify that the largest invariant set of system (1) on the set $\{(S, I, T) \in \Gamma : (\frac{dK}{dt} |_3 = 0\}$ is a singleton (E^0).Therefore, by the LaSalles Invariance Principle [3], the Corrupt free equilibrium (E_0) is globally stable on the set Γ whenever $\mathcal{R}_0 \leq 1$. □

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3.5 Existence of the Corrupt Persistence Equilibrium (CPE) point.

The endemic equilibrium state is the state where the teenage Pregnancy issue cannot be totally eradicated but remains in the population. For Teenage pregnancies issue to persist in the population, the Treated class, the Susceptible class and Infected class must not be zero at equilibrium point. In other words, if $E^*(S^*, I^*, T^*)$ is endemic equilibrium state, then $E^*(S^*, I^*, T^*) \neq (0, 0, 0)$
Lemma 3.5.1 .The system of equations (1) has a unique endemic (positive) equilibrium whenever $\mathcal{R}_0 > 1$, and no positive equilibrium otherwise.

Proof. In order to obtain the corrupt persistent equilibrium state, we solve equations $(16, 17, 18)$ simultaneously.

$$\Lambda - \beta SI - (\gamma + \mu)S = 0 \tag{16}$$

$$\beta SI - (\pi + \sigma + \mu)I = 0 \tag{17}$$

$$\gamma S + \sigma I - \mu T = 0. \tag{18}$$

From equations 16, 17 and 18 we have;

$$S^* = \frac{\Lambda}{(\gamma + \mu)\mathcal{R}_0}$$
$$I^* = \frac{\gamma + \mu}{\beta}(\mathcal{R}_0 - 1)$$
$$T^* = \frac{\gamma\Lambda}{\mu(\gamma + \mu)\mathcal{R}_0} + \frac{\sigma(\gamma + \mu)}{\mu\beta}(R_0 - 1) \tag{19}$$

The equations in the system (1) are solved in terms of the associated force of infection at steady-state, given by equation (19) to give;

$$\lambda = \Lambda(1 - \frac{1}{\mathcal{R}_0}) \tag{20}$$

Noting that $\mathcal{R}_0 < 1$ implies that the force of recruitment at steady state λ is negative (which is biologically meaningless). Therefore, the model has no positive equilibria in this case. □

3.6 Stability of Corrupt Persistence Equilibrium (CPE) point

Theorem 3.6.1 *Teenage pregnancy endemic equilibrium is locally asymptotically stable whenever* $\mathcal{R}_0 > 1$.

Proof. We set the left hand side of system (1) equal to zero and solve it to obtain the endemic equilibrium E^* given in equation (19). From the effective recruitment rate $\beta S(t)I(t)$ at endemic equilibrium we have;

$$\beta SI = (\pi + \gamma + \mu)I^* \tag{21}$$

This is equivalent to;

$$\beta SI = \frac{(\pi + \gamma + \mu)(\gamma + \mu)}{\beta}\{\frac{\beta\Lambda}{(\gamma + \mu)(\pi + \sigma + \mu)} - 1\} \tag{22}$$

By Lemma 3.2.1 $\mathcal{R}_0 = \frac{\beta\Lambda}{(\gamma+\mu)(\pi+\sigma+\mu)}$ and note that all parameters are positive. Therefore, if $\mathcal{R}_0 > 1$, then $\beta S(t)I(t) > 0$ (i.e positive). With this we can conclude the existence of teenage pregnancies endemic equilibrium point. To prove theorem 3.6.1 we compute the Jacobian matrix of the system (1) at endemic equilibrium E^* and then evaluate the eigenvalues as given in Theorem 3.3.1. Finally, we employ Routh-Hurwitzs criterion [32] to show the endemic equilibrium E^* of the system (1) is locally asymptotically stable for $\mathcal{R}_0 > 1$. With this we conclude the proof of the theorem 3.6.1. □

3.7 Sensitivity Analysis of Model the Parameters.

Sensitivity analysis help us to know the parameters that have a high impact on the reproduction number $\mathcal{R}_0$ so that efforts to curb the problem is directed to the parameters with high impact on the reproduction number $\mathcal{R}_0$. A sensitivity analysis of the model (1) is carried out in the sense of [33] and [34].

Definition 3.7.1. The normalized forward sensitivity index of a variable k that depends differentiably on a parameter l is defined as;

$$\bigsqcup_{l}^{k} = \frac{\partial k}{\partial l} \cdot \frac{l}{k} \tag{23}$$

Therefore, sensitivity indices of the basic reproduction number $\mathcal{R}_0$ with respect to the model parameters are computed as follows;

$$\begin{cases} \bigsqcup_{\beta}^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \beta} \cdot \frac{\beta}{\mathcal{R}_0} = 1 > 0 \\ \bigsqcup_{\pi}^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \pi} \cdot \frac{\pi}{\mathcal{R}_0} = \frac{-\pi}{\pi+\sigma+\mu} < 0 \\ \bigsqcup_{\sigma}^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \sigma} \cdot \frac{\sigma}{\mathcal{R}_0} = \frac{-\sigma}{\pi+\sigma+\mu} < 0 \\ \bigsqcup_{\gamma}^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \gamma} \cdot \frac{\gamma}{\mathcal{R}_0} = \frac{-\gamma}{\gamma+\mu} < 0 \\ \bigsqcup_{\mu}^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \mu} \cdot \frac{\mu}{\mathcal{R}_0} = \frac{-\mu(\gamma+\pi+\sigma+2\mu)}{(\gamma+\mu)(\pi+\sigma+\mu)} < 0 \\ \bigsqcup_{\tau}^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \tau} \cdot \frac{\tau}{\mathcal{R}_0} = \frac{-\tau}{1-\tau} < 0 \end{cases} \tag{24}$$

The positive sign of sensitivity index (S.I) of the basic reproduction number $\mathcal{R}_0$ to the model parameters shows that an increase (or decrease) in the value

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of each of the parameters in this case will lead to an increase (or decrease) in the basic reproduction number of the infection.

Similarly, the negative sign of S.I of the basic reproduction number to the model parameters implies that an increase (or decrease) in the value of each of the parameters in this case leads to a corresponding decrease (or increase) in the basic reproduction number of the infection. Thus, with sensitivity analysis, one can get insight into the appropriate intervention strategies to prevent and control the spread of teenage pregnancies described by model (1).

3.8 Discussion

The basic reproduction number is expressed in terms of the model parameters. Therefore, prediction and prevention strategies can easily be made. For instance, the level of awareness is too low in Kenya, despite momentum at the national level, girls and boys still have limited access to high-quality and comprehensive puberty, contraception information, HIV/AIDs awareness and sexual education, resulting to increased teenage pregnancy, high HIV/AIDs infection, sexual transmitted infections and high school dropout. Educationists and parents may opt to skip puberty education because communities perpetuate taboos and misconceptions about puberty and sexual education.

Healthy sexual education, awareness programs and provision of much needed support are the most crucial approaches for addressing poor puberty sexuality among young women and girls. From carried out researches, one reason given for the historical decline (1991-2005) in teen pregnancies and births is that sexually active female teenagers had significantly increased their use of contraceptives, particularly condoms. Abstinence campaigns, aimed at younger teens, were also seen as having a positive effect on pregnancy prevention. Moreover, casual sex, which may increase the risk of sexually transmitted diseases (STDs) and may prove to be fatal given the presence of HIV/AIDS, is viewed in an increasingly negative light by many teenagers. The findings are consistent with available literature. Numerical analysis to ascertain the theoretical analysis and optimal control strategies are left for future work.

3.9 Conclusion

A comprehensive approach to sex education provides today's youths with the information and decision-making skills needed to make realistic, practical decisions about whether or not to engage in sexual activities. Such an approach allows young people to make informed decisions regarding abstinence, setting relationship limits, overcoming peer pressure, contraceptives use and the prevention of sexually transmitted diseases. It is a total shame to the current society where children feel safe being at school other than home.

Owning up to this challenge and providing these youths with vital information they need will save them from early marriages, abortions, birth related deaths, fistula complications, lost opportunities and sexually transmitted diseases. Moreover, a sexually informed young generation is a plus to the government of the day in two ways. First, excelling in their academics will reduce over dependence on already struggling society and rather become agents of development. Secondly, knowledge of sexual activities among young individuals will probably reduce the high level of HIV/AIDs infections among youths thereby reducing the health burden on the government expenditures.

Given that 47.4 percent (2011 data)[1] of high school students have experienced sexual intercourse, advocacy of abstinence-only messages provide no protection against the risks of pregnancy and diseases for these young people.

3.10 Recommendation

Some of the red flags that indicates that a teen is involved in dangerous sexual risk taking include participating in unprotected intercourse or having sexual relationship in which one does not trust the partner, or feel that they are victimized or abused, or feel that that they are abusing or victimizing someone else. In such situations these teens need help. Therefore, information/knowledge is immune as far as teenage sexual activities is concerned. The information on sexual matters, sexual rights, protection against HIV/AIDs and contraceptive measures should be readily available to this group of youths. Many teens are physically ready for sexual activities before they are emotionally ready. It is therefore, essential to learn and plan for sexual activity. The decision to make this information available to these young individuals to avert the current teens parenthood situation squarely lies on the Government, parents and the concerned parties.

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