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Measuring consumer satisfaction to restaurant service: Type -2 Fuzzy Logic Approach

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Abstract: The type reducer step of “Type-2 Fuzzy Logic Systems (T2FLS)” is an additional step that distinguishes it from a conventional “Type-1 fuzzy logic system (T1FLS)”. In this paper the underlying processes of fuzzy inference systems are reviewed and some substantial terms are redefined. The “Sugeno” type “Interval Type 2 Fuzzy Logic System (IT2FLS) ” is used to measure overall customer satisfaction after entertaining foods and services from restaurant. IT2FLS is modeled using a variety of triangular and Gaussian membership functions and produces outputs in the form of three-dimensional surface views. Two distinct systems produce outputs that appear slightly different but contain the same numerical values. The contribution of the study is to redefine the substantial components of T2FLS. The endeavor will aid in the development of computer programs for T2FLS and generate interest in their use..
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Keywords: Type-2 fuzzy sets, fuzzy logic system, linguistic variables, restaurant tipping, measuring foods quality and service quality

1. Introduction

After the inception of fuzzy set theory in 1965 by its founder L. A Zadeh [1] applications approach of fuzzy set theory have been expanded. Researchers of mathematics have been studied and extended the concept of fuzzy sets in many branches of mathematics and pure mathematics. Ring, Group, Semi-group, Algebraic structure, Topology, Lattice, Metric space, Soft fuzzy sets, Complex fuzzy sets have been studied by using fuzzy set theory. Application scope of fuzzy mathematics have appeared in information science, decision analysis, medical science and engineering, economics, finance and many other disciplines. Fuzzy logic systems are using for decision making process where the inputs variables are considerably has a certain degree of uncertainty. ‘Type -1 fuzzy’ logic system is the ordinary fuzzy rule base system and ‘Type-2 fuzzy’ logic introduced by Zadeh (1975) [2] for where uncertainty includes in membership functions [3]. In the problems where levels of imprecision are founded to increase “type-2 fuzzy logic system provides a powerful paradigm for tackling the problem” [4]. John and Coupland, (2007) [4] analyzed ‘Type-2 fuzzy' logic and explored the past advances and suggestions about the applications of type-2 fuzzy sets, as well as highlighting various areas of type-2 fuzzy logic scheme (T2FLS). Mendel M. Jerry, (2013) [5] addressed various representations of ‘type-2 fuzzy sets' and explained their operations. The study proposes a simplified T2FLS and compares it to type 2 fuzzy logic systems. Mendel and John , (2002) examined “type-2 fuzzy sets” and suggested a representation for deriving union, intersections, and complement without using of the extension theory. Zhang and S. Zhang, (2012) [6] researched and presented the idea of a ‘type-2 fuzzy' soft set and applied it to decision making. Seda Türk, et al. (2014) [7] introduced ‘type-2 fuzzy sets’ in supply chain management systems. Ardakani S.S. and et al. (2014) [14] defined various aspects of service efficiency to assess air carrier operation. The study used fuzzy mathematics to rank the factors in air carrier services and discovered that “safety, timeliness, variety and types of airplane” are the most important criteria, while “tangibles” and “responsiveness” are the least important. Nagarajan and et al. (2020) [8] explored triangular ‘type-2 fuzzy soft' sets and suggested “a triangular interval type-2 fuzzy soft weighted arithmetic (TIT2FSWA)” operator with various mathematical properties. Profit analysis has been carried out using this approach. R. H Abiyev and et al. (2016) [9] developed a framework for forecasting food security risk levels using a “interval type-2 fuzzy set”. The proposed security scheme was evaluated and found to be applicable using Turkish statistical evidence. Ahmad Esmaeili and et al. (2015) [10] suggested a fuzzy framework for determining the service quality of a logistics company based on eight attributes. The system was able to recommend customers based on knowledge about the industry's service quality. The MathWorks Developer Team and the Scikit-fuzzy development team explored how to build a fuzzy expert system using the example of "the Basic tipping problems."[11, 12]. G Acmpora et al. (2016) introduced an interval type-2 fuzzy logic-based architecture for reputation management in P2P e-commerce that is effective at dealing with uncertainty, and then performed a series of experiments using eBay-like transaction datasets to show that the proposed type-2 fuzzy logic-based method outperforms other well-known and commonly used approaches [13] .

In this paper, we review Type-2 fuzzy logic systems and attempt to describe various FLS processes such as (fuzzifier, implication, aggregation, type-reducer and defuzzify). A type-2 fuzzy logic system is used to determine customer loyalty after dining at a restaurant.

This paper is divided into five parts. Section 1 addresses the introduction as well as prior studies. Section 2 discusses the theoretical background; Section 3 depicts the fuzzy logic system in both type 1 and type 2 configurations. In section 4, we addressed the use of a fuzzy logic system to infer customer tips after consuming the restaurant's food and service. Finally, section 5 is mainly composed of conclusions.

2. Theoretical Background

The section describes certain fuzzy sets and fuzzy logic concepts and terms that are relevant to the study. We refer readers to [1, 14] for more information on the fundamentals of fuzzy sets and systems.

2.1 Type-1 Fuzzy sets

Definition 2.1. [14] Assume that X is not empty set and I = [0, 1] is a unit interval. We use the term "fuzzy set on X" to refer to a membership function.

$$\mu(x): X \rightarrow I, I \mapsto \mu(x)$$

where $\mu(x)$ denotes membership grade of $x \in X$ under μ. .

Simply, any function $A: X \rightarrow I$ is referred to as a Fuzzy set (ordinary fuzzy set or Type -1 fuzzy set) where X denotes the referential set and I= [0, 1] denotes the valuation set.
Set of all fuzzy sets on X is denoted by I^X and $A \subset I^{X\cdot}$ Membership function of a fuzzy set is normally denoted by $\mu_A(x)$ or simply (x) .
By I^X we denote set of all fuzzy set on X and any the membership fuzzy of a fuzzy set set $A \subset I^{X\cdot}$ is normally denoted by $\mu_A(x)$ or simply $\mu(x)$.

Definition 2.2. [3] Let $A \subset I^X$ be a fuzzy set the support of A is the set of all points x in X defined by

$$\mathrm{support}(A) = \{x \in X \mid A(x) > 0\}.$$

Definition 2.3. [3] Let $A(x)$ and $B(x)$ be two membership functions of fuzzy sets $A, B \in I^X$. The union or max operation on these functions is a function $C(x)$ which is defined by

$$C(x) = \mu_{A\cup B}(x) = \max\big(A(x), B(x)\big).$$

Definition 2.4. [3] Let $A(x)$ and $B(x)$ be two membership functions of fuzzy sets $A, B \in I^X$. The intersection or min operation on these functions is a function $C(x)$ which is defined by $C(x) = \mu_{A\cap B}(x) = \min\big(A(x), B(x)\big)$.

Definition 2.5. [3] Let $A \in I^X$ and $B \in I^Y$ be two fuzzy sets. A fuzzy relation R from A to B is defined by $R: A \times B \to I$, where $I = [0,1]$ is unit interval. Set of all fuzzy relations from A to B is denoted by $I^{A\times B}$.

Definition 2.6. [3] Let $\in I^X$, $B \in I^Y$, and $C \in I^Z$ be three fuzzy sets. If R_1 and R_2 are two fuzzy relations defined from $A \times B$ and $B \times C$ respectively. The "max-min" composition of R_1 and R_2 is also a fuzzy set defined by

$$R_1 \circ R_2 = \{(x, z);\ \max(\min(R_1(x, y), R_2(y, z))) | x \in X, y \in Y, z \in Z\}.$$

2.2 Type-2 Fuzzy sets

Zadeh (1975) [15] suggested the concept of 'Type-2 Fuzzy Sets' as an expansion of 'Type-1 Fuzzy Sets,' also known as ordinary fuzzy sets. We refer readers to [3, 16, 17] for further development of the concept of 'type-2 fuzzy' sets. Operations of 'Type-2 fuzzy' sets such as union, intersection, complement and different type and figures of type-2 fuzzy set are found in [3]. Applications of 'Type-2 Fuzzy sets' and 'Fuzzy logic systems' include control systems, decision-making systems, forecasting and approximation of time series data, robot control systems, image processing, medical diagnosis, and intelligent systems, management systems among others.

Definition 2.7. [18] The membership function $\widetilde{A} = \{((x, u), \mu_{\widetilde{A}}(x, u)) | \forall x \in X, \forall u \in J_x \subseteq [0,1]\}$ is used to define a type-2 fuzzy set, where $0 \le \mu_{\widetilde{A}}(x, u) \le 1$, $\forall u \in J_x \subseteq [0,1]$. Indeed, $J_x \subseteq [0,1]$ is a type-1 fuzzy set that represents primary membership of x and referred to as the secondary set. The ambiguity associated with membership is referred to as the footprint of uncertainty (FOW), which is depicted in Figs. 1a and 1b as shaded reasons. 'Interval type-2 fuzzy set' is bounded by an upper and a lower membership function.

$$\widetilde{A} = \int_{x\in X} \int_{u\in J_x} \frac{\mu_{\widetilde{A}}(x, u)}{(x, u)} \ ; \quad \forall x \in X, \forall u \in J_x \subseteq [0,1]$$

Where $\int \int$ denotes union of all admissible input variables.

Definition 2.8. 'Type-2 Fuzzy sets': Assume $X \subset R$ a nonempty set. A function $\widetilde{A}: X \to J_x \subseteq [0,1]$, is called Interval 'Type-2 fuzzy' set, if $[a, b] \subseteq J_x$ where $a \le b$ and if $a = b$ then $\widetilde{A}$ will reduce to a 'type-1 fuzzy set'.

Definition 2.9. Type-2 Fuzzy sets membership functions: Let $x \in X \subset R$. The membership function of a type-2 fuzzy set $\widetilde{A}$ is defined by

$$\mu_A: X \to [\mu_l(x), \mu_u(x)] \subseteq [0,1]$$

where μ_l and μ_u are respectively called lower and upper member ship functions of a type-2 fuzzy sets.

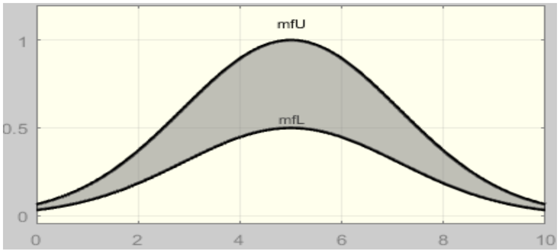


Fig 1a. Footprint of Uncertainty Type-2 Fuzzy sets Source [3]

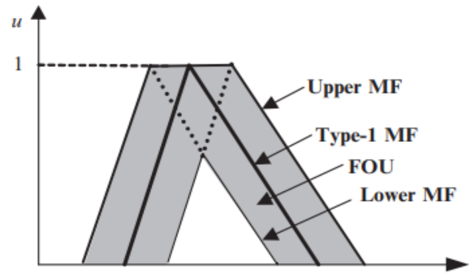


Fig 1b. Type-2 membership function. Source [19]

Gaussian type membership functions: Let A be a fuzzy set constructed by its underlying membership function defined by $\mu(x) = \exp\left\{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2\right\}$ then A is called a Gaussian fuzzy number.

3. Fuzzy Logic Systems

A type-2 fuzzy set is one that retains an ambiguous membership feature. When simulating real-world problems, a higher degree of approximation can be used to mitigate the uncertainty associated with traditional type-1 fuzzy systems. Type-2 fuzzy structures are constructed using fuzzy if-then rules that use type-2 fuzzy sets. Type-2 fuzzy logic is a generalization of classical fuzzy type-1 logic in that it includes ambiguity in the specification of the membership function as well as the linguistic variables. The fuzzy logic system (FLS), also known as fuzzy rule-based systems (FRBS), fuzzy inference system (FIS), and fuzzy expert system (FES), is based on the fuzzy concept proposed by Zadeh in 1965 [20], and it represents the reasoning of human experts in the form of construction rules (set of IF-THEN-statements) to solve real-world problems such as control, prediction, and inference, data mining, and bioinformatics data processing. Additionally, FRBS are referred to as fuzzy inference systems, fuzzy logic-based systems, fuzzy compositional rule-based systems, fuzzy controllers, and fuzzy models. One critical application of FRBS is to work with imprecise data. A fuzzy rule-based method (FRBS) is a technique for deriving a conclusion from a given set of inputs. The compositional rule of inference is the fundamental rule (law) for FRBS.

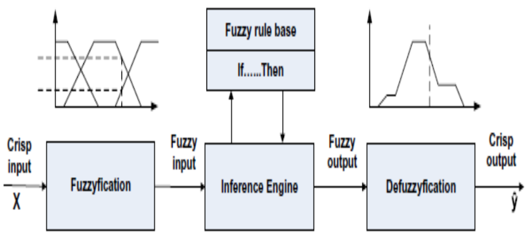


Fig 2a. ‘Type 1 Fuzzy Logic’ system. Source [21]

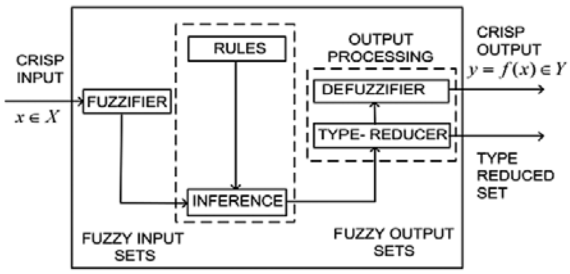


Fig 2b. ‘Type-2 Fuzzy Logic’ system. Source [3]

The underling steps in Type-1 and ‘Type-2 Fuzzy logic’ systems are seen in figure 2(a) and 2(b) and the steps are accordingly Setp 1: Crisp Input, Step 2: Fuzzifier Step 3: Fuzzy Input Sets, Step 4: Rules Block to Inference, Step 5: Fuzzy Output Step 6: Type Reducer and Finally Step 7: Defuzzifier for crisp output

3.1 Type -1 Fuzzy Inference Systems

Fuzzy inference system is the process of obtaining a decision from given set of ambiguous inputs. The basic rule (law) of fuzzy inference system is the compositional rule of inference [22, 23] . A single fuzzy relation is not sufficient to make a decision. Real life applications require a finite collection of statements that can express in terms of fuzzy logics or rules. The collection of rules is called a rule block of the fuzzy inference system.

Definition 3.1. [23] A fuzzy rule is defined as triplet (A; B; R) defined by $R: A \times B \rightarrow I$, where $A \in I^X$; $B \in I^Y$, are connected by the fuzzy relation $R \subset I^{X\times Y}$.

Definition 3.2. [23] Fuzzy rule of "If x is A then y is B" is be defined as a fuzzy relation as follows:

$$\mu_R(x,y) = A(x) \wedge B(y);$$

Where the inference is obtained by

$$\mu_B(y) = \max(\min(\mu_A(x), \mu_R(x,y)))$$

Definition 3.3.[23] Fuzzy logic system for a single input is "If x is A_i then y is B_i; i = 1,…,n", is defined by the fuzzy relation by

$$R(x,y) = \vee_{i=1}^{n} \left(A_i(x) \wedge B_i(y)\right)$$

Suppose $A_i \in I^X$; $B_i \in I^Y$ and $C_i \in I^Z$; i = 1, … , n; are fuzzy sets. A fuzzy inference system constructed two inputs are "if (x is A_i) and (y is B_i) then z is C_i" shown in tabular form in Table 1.

Antecedent	:	$x \in A_i$ and $x \in B_i$		
Rule 1	:	If $x \in A_1$	and $y \in B_1$	then $z \in C_1$
Rule 2	:	If $x \in A_2$	and $y \in B_2$	then $z \in C_2$
.	⋮	⋮	⋮.	⋮
Rule n		If $x \in A_n$	and $y \in B_n$	then $z \in C_n$
Conclusion	:			$z \in C_i$

Table 1: Fuzzy Inference system

The process of conflict resolution [24] is applied to decide which consequence has taken place.

3.2 Operational overview of fuzzy inference:

If A and B are two inputs and C is the output in a fuzzy inference system, then rule wise operational overview of a fuzzy inference system is shown in Table 2.

Antecedent (Facts)		$\min(x \in A, y \in B) \rightarrow z \in C$		**Inference/ Output** $z \in C_i$
Rule 1	**:**	$x_1 \in A_1$ and $y \in B_1$	then $z_1 \in C_1$	$z_1=\min(\min(A_1,B_1),C_1)$
Rule 2	**:**	$x_2 \in A_2$ and $y_2 \in B_2$	then $z_2 \in C_2$	$z_2= \min(\min(A_2,B_2),C_2)$
⋮		⋮	⋮	⋮
Rule i	**:**	$x_i \in A_i$ and $y_i \in B_i$	then $z_i \in C_i$	$z_i= \min(\min(A_i,B_i),C_i)$
Rule n	**:**	$x_i \in A_n$ and $y \in B_n$	then $z_i \in C_n$	$z_n= \min(\min(A_n,B_n),C_n)$
Conclusion		Aggregation of all outputs z_i		$z=\max(z_1, z_2, ..., z_n)$

Table 2: Generalized Fuzzy Inference system operational overview

3.3 Review and define different component of fuzzy inference system

This section contributes some definitions (3.4-10) of the substantial terms and components which are used in fuzzy inference system.

Definition 3.4. Let $x \in X \subset R$ be any real number and A be a linguistic variable consists with n membership functions $\mu_1(x_1), \mu_2(x_2), \cdots, \mu_n(x_n)$ under $X \subset R$, where each membership function represents some linguistic terms for instance of very small, small, normal, … etc.

Definition 3.5. Fuzzification means a process of finding the value of membership functions from a given input of discussed real number. Mathematically, fuzzification of a linguistic variable A is expressed by a function $f_A: x_i \in X \to]0,1]$

Definition 3.6. Let A be a linguistic variable with n terms. The fuzzification of any real input $x \in X \subset R$ is defined explicitly by

$$f_A: x_i \in X \to \min([I, \mu_i], [I, \mu_{i+1}], \cdots, [I, \mu_n]) \to I \ , \ I =]0,1] \text{ and } i = 1,2 \cdots n$$

Note: Construction of linguistic variable, membership functions intersects to the graduation of linguistic terms. So, any input may returns more values of $\alpha_i \in]0,1]$, minimum of all α_i values will be taken.

Definition 3.7. Let $x_i \in X_i \subset R_+$ be nonzero real numbers and A_i be linguistic variables, $i = 1,2 \cdots n$. For any $i = 1$ (say), the $x_1 \circ A_1 \to \alpha_1 \in]0,1]$.

Definition 3.8. Let $\alpha^j \in]0,1]$ and $y^j \subset I^Y$, $Y \subset R_+$ be the membership functions of linguistic variables B. An inference obtained from a set of fuzzy rules (a set of reasons) can defined by a function :

$$f_B: \alpha^j \circ y^j \to Y^j \ .$$

Definition 3.9. Let Y^j, $j = 1,2, \cdots, n$ are inferences obtained from n fuzzy rules. The aggregated decision of all inferences Y^j is called aggregation which is defined by the function

$$f_Y: \max(Y^j) \to Y$$

Definition 3.10. Defuzzification is a process of finding real value from membership function. Let Y be a membership function obtained after aggregating. Mathematically, the defuzzification of Y is expressed by $dFuz: Y \to x \in X \subset R$. The process is defined by the method centre of area given by

$$dFuz_{COA} = \frac{\int_y \mu_A(y) y dy}{\int_y \mu_A(y) dy}$$

Definiton 3.11 [25] Let A_i, B_i are fuzzy sets. A fuzzy inference system is called **Mamdani** type if the Rules of the system takes the following form:

$$R_i: \text{ If } (x_1 is\ A_{1i}) \ \&\ (x_2 is\ A_{2i}) \& \cdots \&\ (x_n is\ A_{ni})\ then\ y\ is\ B_i$$

In Mamdani type model can take both numeric and fuzzy inputs and generate fuzzy outputs that leads to a crisp values.

Definition 3.12 [25] Let A_i, B_i are fuzzy sets. A fuzzy inference system is called **Takagi-Sugeno-Kang type or simply Sugeno** type if the Rules of the system takes the following form:

$$R_i: \text{ If } (x_1 is\ A_{1i}) \ \&\ (x_2 is\ A_{2i}) \& \cdots \&\ (x_n is\ A_{ni})\ then\ y = f(x_1, x_2, \cdots x_n)$$

In sugeno type model the function $f(\cdots)$ takes only numerical values.

3.3.1. Steps in Fuzzy Logic System

Fuzzification of inputs: Let $x_i \in X_i \subset R_+$ be nonzero real numbers and A_i be linguistic variables. Again let y^j be the membership function of B^j the linguistic variable of decision. A fuzzy rule base system with n rules can express in the following expression:

$$x_i \circ A_i^j \to y^j \ ; i, j = 1,2, \cdots n$$

The first term is defined by $x_i \circ A_i^j = \min(\min(x_i, A_i^j)) = \alpha^j \in]0,1]$ is the fuzzification steps

Implication operator: If $\alpha^j \in]0, 1]$ and y is a membership function the implication operator on y give output of another new membership function Y, given by

$$\alpha^j \circ y^j = \min(\alpha^j, y^j) = Y^j \text{ , } j = 1,2,\cdots n$$

Aggregation: Let Y^j are membership functions obtained from implication operation. The aggregation operation provides a single membership function which is obtained the following expression

$$\max(Y^j) = f_Y \text{ , } j = 1,2,\cdots n$$

Defuzzification: Obtain real number form aggregated functions. $\text{dFuz}: f_Y \rightarrow x \in X \subset R$

3.3.2. Linguistic variables determination

A linguistic variable of five linguistic terms namely very low, low, medium, high and very high which are shortly written by VL, L, M, H, VH respectively where each term has a underlying function of type-1 fuzzy set.

Let A be a linguistic variable and $x_i \in X \subset R^+$, i=1,2,…,n are real numbers. Then the linguistic variable A under 5 terms corresponding to 5 membership functions is expressed by

$$A(\mu_{x_1}, \mu_{x_2}, \mu_{x_3}, \mu_{x_4}, \mu_{x_5} \mid x_i \in X \subset R^+; i = 1, \cdots, 5).$$

For any input of real number $x \in X$, the linguistic variable will return a value say $\alpha \in [0,1]$ which is express by the function

$$A: (\mu_{x_1}, \mu_{x_2}, \mu_{x_3}, \mu_{x_4}, \mu_{x_5} \mid x \in X \subset R^+) \rightarrow]0, 1]$$

Note: Linguistic variable takes input of real number from the discussed domain and returns value(s) of membership grades.

Remark: In case of Type-2 fuzzy system: each term is a type-2 fuzzy sets consisting of two membership functions. For any input of real numbers the linguistic variable returns an interval whose values are in [0,1].

3.2 Type -2 Fuzzy Logic Systems

Type-2 fuzzy logic systems follow all steps of Type-1 fuzzy systems. Only Type Reducer step is an additional step for Type-2 fuzzy logic systems. Since Type-2 fuzzy sets consists with two membership functions stands on a foot print of uncertainty. In this section we define fuzzification. Implication operator, aggregation, type reducer and defuzzification process of Type-2 fuzzy systems.

Type -2 Fuzzy Rule Block: The structure of Type-2 fuzzy rules block is similar to the structure of type-1 fuzzy rules block. Let $\widetilde{A}$ and $\widetilde{B}$ be two Type-2 fuzzy sets which are considered as inputs and $\tilde{C}$ be the consequence of a Type-2 FLS. Rule wise operational overview of fuzzy inference system is shown in Table 3.

Antecedent (Facts)		$\min(x_i \in \widetilde{A}_i, y_i \in \widetilde{B}_i) \rightarrow \tilde{z} \in \widetilde{C}_i$ [if x is A and y is B then z is C]		**Inference/ Output** $\tilde{z} \in \widetilde{C}_i$
Rule 1	:	$x_1 \in \widetilde{A}_1$ and $y \in \widetilde{B_1}$	then $\tilde{z}^1 \in \widetilde{C}_i$	$\tilde{z}^1 = [z_{l\,l}^1, z_r^1] = \min(\min(\widetilde{A_2}, \widetilde{B_1}), \widetilde{C_1})$
Rule 2	:	$x_2 \in \widetilde{A}_2$ and $y \in \widetilde{B_2}$	then $\tilde{z}^2 \in \widetilde{C_2}$	$\tilde{z}^2 = [z_{l\,l}^2, z_r^2] = \min(\min(\widetilde{A_2}, \widetilde{B_2}), \widetilde{C_2})$
⋮		⋮	⋮	⋮
Rule i	:	$x_i \in \widetilde{A}_i$ and $y \in \widetilde{B}_i$	then $\tilde{z}^i \in \widetilde{C}_i$	$\tilde{z}^i = [z_{l_l}^i, z_r^i] = \min(\min(\widetilde{A}_i, \widetilde{B}_i), \widetilde{C}_i)$
⋮		⋮	⋮	⋮
Rule n	:	$x_n \in \widetilde{A}_n$ and $y \in \widetilde{B_n}$	then $\tilde{z}^n \in \widetilde{C_n}$	$\tilde{z}^n = [z_{l\,l}^n, z_r^n] = \min(\min(\widetilde{A_n}, \widetilde{B_n}), \widetilde{C_n})$
Conclusion		Aggregation of all outputs z_i		$\tilde{z}=\max(\tilde{z}^1, \tilde{z}^2 \cdots, \tilde{z}^n)$

Table 3: Type-2 Fuzzy Inference system operational overview

Type -2 Fuzzy inference systems consisting with n rules block and takes m inputs is expressed by

$$R^l: \widetilde{A}_1^l \times \cdots \times \widetilde{A}_m^l \rightarrow \widetilde{C}^l \text{ , } l = 1, \cdots, n$$

More implicitly we can write the above expression by

$$R^l: \widetilde{A}_m^l \rightarrow \widetilde{C}^l\ ,\ \ l = 1, \cdots, n, m = 1, \cdots m$$

This is the generalization of above tabular expression defined in Table 3.

Fuzzification of type-2 fuzzy set: Let $x \in X \subset R_+$ be nonzero real numbers. Again let A be the type -2 fuzzy sets. Fuzzification of an crisp input is expressed by the function $Fuz: X \rightarrow [f_l, f_u] \subseteq [0, 1]$ and the fuzzification process is defined by $Fuz(x) = [a_l, a_u]$, where $f_l \leq f_u$ and $f_l, f_u \in [0,1]$. The fuzzification of type-2 fuzzy logic system takes crisp input and returns an interval.

Fuzzification of type-2 fuzzy rule: Let $x_i \in X_i \subset R_+$ be nonzero real numbers and $\widetilde{A}_i$ be linguistic variables of type-2 fuzzy sets. Again let $\tilde{y}^j$ be the membership functions of B the linguistic variable of decision.

A Type-2 fuzzy logic system of n rules can express by the following expression:

$$x_i \circ \widetilde{A}_i^j \rightarrow \tilde{y}^j = [\, y_l^j, y_u^j] \,;\, i, j = 1,2, \cdots\ n$$

The first term is computed by $x_i \circ \widetilde{A}_i^j = \min\left(\min\left(x_i, \widetilde{A}_i^j\right)\right) = \left[\, \alpha_l^j\,, \alpha_u^j\,\right] \subset [0, 1]$ which is the fuzzification steps of type -2 fuzzy rule system.

Implication operator: Implication process of type -2 fuzzy logics **system is**

$$\left[\, \alpha_l^j\,, \alpha_u^j\,\right] \circ\ \tilde{y}^j = \min\left(\left[\, \alpha_l^j\,, \alpha_u^j\,\right], \left[\, y_l^j, y_u^j\right]\right) = [Y_l^j, Y_u^j]\ , j = 1,2, \cdots\ n$$

Aggregation: This process operated by $\max\left(\widetilde{Y}^j\right) = \max\left(\left[Y_l^j, Y_u^j\right]\right) = f_{\widetilde{Y}} = [f_l, f_u]$ **,** $j = 1,2, \cdots, n$. Here f_l, f_u are lower and upper membership functions obtained in aggregation process.

Type reducer: [3] In this process Type -2 fuzzy set reduces to a Type-1 fuzzy set. Let $f_{\widetilde{Y}}$ be a type-2 fuzzy set which is obtained by applying aggregation process. The Type reducer can be written by the mapping

$$g: f_{\widetilde{Y}} \rightarrow f_Y \text{ , here } f_Y \text{ is a type-1 fuzzy set.}$$

A type-2 fuzzy set $f_{\widetilde{Y}}$ under discourse $X \in R$. Again let the upper and lower membership function of

$f_{\widetilde{Y}} = [f_l(x), f_u(x)]$, where $f_l(x) \leq\ f_u(x)\ \forall\ x \in X$; and $f_l(x), f_u(x) \subseteq [0,1]$

$$f_Y(x) =\ 0.5 * (f_l(x) + f_u(x))\ |\ \forall\ x \in X$$

Other way one can reduce type of a 'Type 2 fuzzy'set by defuzzifying individual membership functions of $f_l(x)$ and $f_u(x)$. Let c_l and c_u are the defuzzified value of the membership functions $f_l(x), f_u(x) > 0;\ \forall x \in X$ obtained by applying centre of area method. Then c_l and c_u are obtained by the expression

$$c_l = \frac{\int_{x \in X}\ f_l(x) x dx}{\int_{x \in X} f_l(x) dx} \quad \text{and} \quad c_r = \frac{\int_{x \in X}\ f_u(x) x dx}{\int_{x \in X} f_u(x) dx}$$

Finally we will take the average of c_l and c_u to obtained the crisp value of the fuzzy logic system.

$$c = 0.5(c_l + c_r)$$

In [26] Dongrui and Mendel (2007) investigate the type reducer algorithm and Taskin and Kumbasar (2015) apply the eight type reducer methods [27].

Defuzzification: This defuzzification process is analogous to that of a Type-1 fuzzy logic system. This step produces a crisp output, which is written as dFuz: $f_Y \rightarrow\ x \in\ X \subset\ \ R$.

4. An application of 'Sugeno' Interval Type 2 Fuzzy Logic System

4.1 Application description

In a restaurant, after receiving service and food, customers usually leave tips that reflect the restaurant's service and food quality. Customer loyalty is essential for the sustainable success

of a restaurant. Since receiving meals, a client pays the bill and, on occasion, sends a monetary token known as a tip to the service providers. The willingness to offer tips, as well as the amount given, can be a measure of customer loyalty [28, 29].
Measure of food quality depend appearance, aroma, temperature, texture, taste and other subjective and objective attributes. On the other hand measure of service depends on following dimensions including tangibles, customization, access, communication, knowing the customer or customer understanding, security, courtesy, competence, credibility, reliability, responsiveness and cost [29]. Food quality and service quality both are very important for restaurant business. Consumer satisfaction brings a long term reflection to the organization in such a way that consumer may refer to other to visit the organization or he may revisit for the service. Furthermore, consumers who is highly satisfied with the service may left tips a little amount or consumer who is somehow satisfied with the service may left tips a considerably big amount [30]. Amount of tips sometimes may measure the customer satisfaction.

In this application customer satisfaction will be infer in the form or amount of customer tips which he left after having food and service. So overall satisfaction here we call tip which is implied by the satisfaction function of service and food quality of a restaurant entertained by a customer. Therefore satisfaction can be expressed in the form of tips by

$$\text{Satisfaction} : (\text{satisfaction from service quality} \times \text{satisfaction from food quality}) \\ \rightarrow \text{Tips (chip, average, generous)}$$

For example: $\text{Satisfaction(good, medium)} \rightarrow \text{ average}$

The service quality of a restaurant is determined by a variety of dimensions and characteristics, and in this study, we considered three states of satisfaction associated with service quality, namely poor, good, and excellent. The state of service quality can be measured as follows:

$$\text{Service quality} : (\text{Service dimensions }) \rightarrow \text{Satisfaction (poor, good, excellent)}$$

For example $\text{Service quality (eg. Good tangible decoration, comfortable place)} \rightarrow \text{ good}$

The food quality depends on different dimensions and attributes of food in the study three state of food satisfaction are considered which are called rancid, medium and delicious.
We expressed the state of food quality as follows:

$$\text{Food quality} : (\text{Food dimensions }) \rightarrow \text{Satisfaction (rancid, medium , delicious)}$$

For example $\text{Food quality (good aroma, good in test and affordable cost)} \rightarrow \text{ medium}$

For sake of simplicity we are considering only two inputs service quality and food quality and an inference output called Tip which will be produced by the fuzzy logic system. In case of output variable (Tips/ customer satisfaction) value of output is taken from the unit interval [0,1].

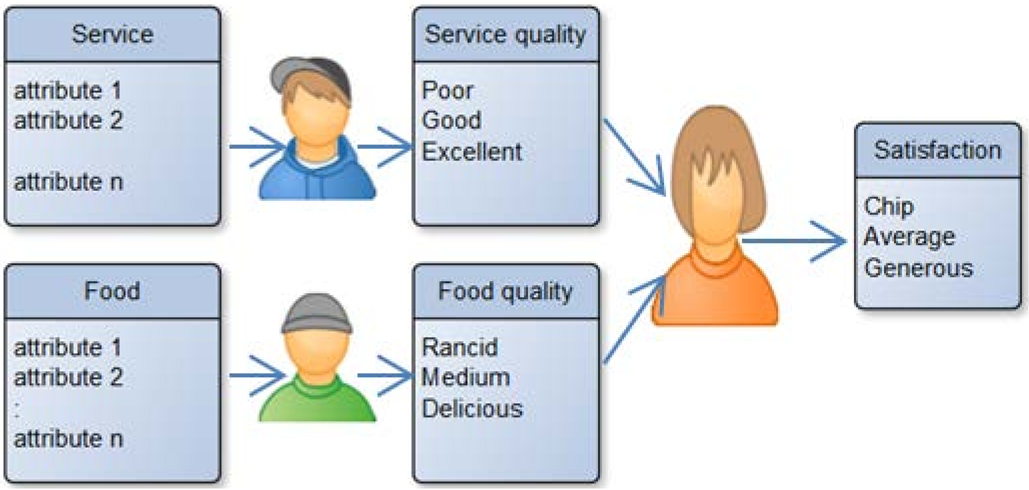


Figure 3: Process of satisfaction

Figure 3 illustrates the measure of satisfaction process based on the service and food dimensions of attributes. The rule producers and generators are represented by the men-like faces.

4.2. System Architecture

Input variables: The numbers of input variables are service quality and food quality. In system 1, two input variables are taken as triangular shaped membership functions and In system 2, inputs variables are taken as Gaussian shaped membership functions.

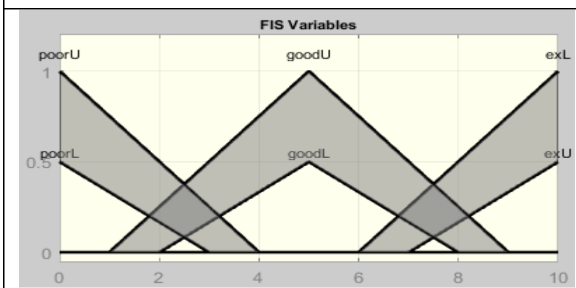
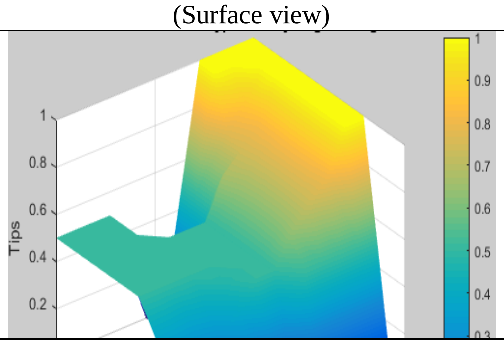
Output variables: Output variable for both systems is considered as a number.

Fuzzy Rule base: The matrix of fuzzy rule base for the system 1 and 2 is as given follows

R_i =

R1: If service is poor and food is rancid then tip chip
R2:If service is good and food is delicious then tip is average
R3: If service is excellent and food is medium then tip is generous
R4:If service is excellent and food is delicious then tip is generous
R5: If service is poor and food is delicious then tip is average
R6:If service is poor and food is medium then tip is average

System 1: A 'Sugeno' Type IT2FLS is constructed by taking Triangular type membership functions. In case of output variable no membership function are applied.

Inputs variables	Fuzzy Rules	Outputs (Tip) (Surface view)
 Input variable service quality	R_i	

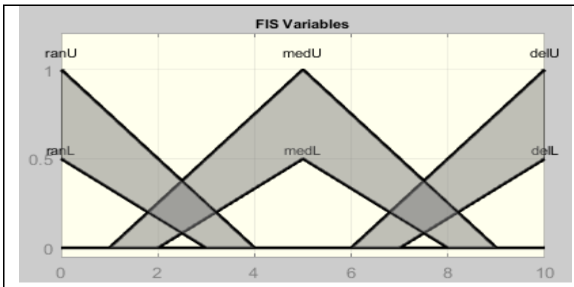
<table>
<tr><td>Input variable food quality</td><td></td><td></td></tr>
</table>

Table 4: Input-Rules and Output of System1

System 2: A 'Sugeno' Type IT2FLS is constructed by taking a combination of Triangular and Gaussian type membership functions.

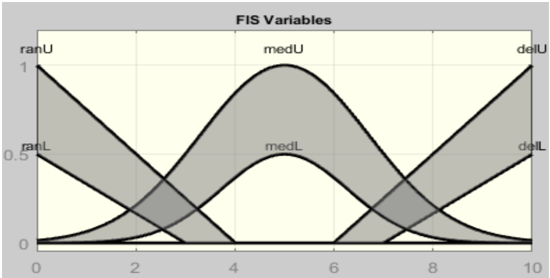
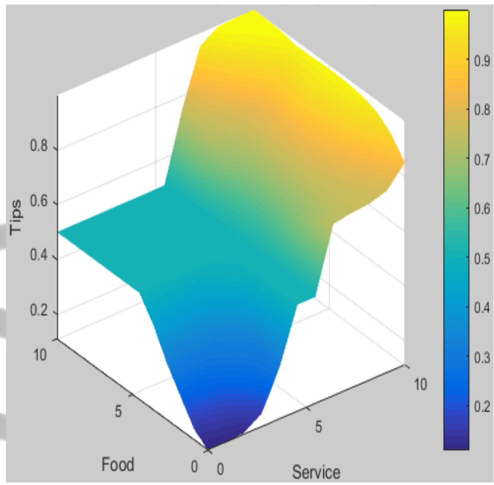
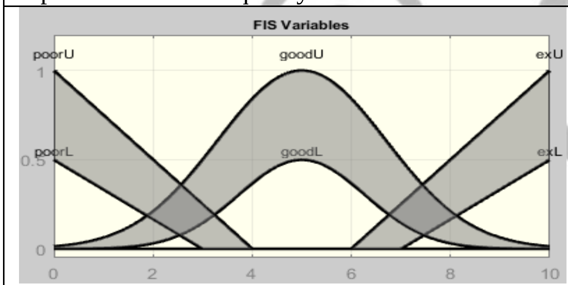
<table>
<tr><td>Input variables</td><td>Fuzzy Rules</td><td>Outputs
(Surface view)</td></tr>
<tr><td>Input variable Food quality</td><td rowspan="2">R_i</td><td rowspan="2"></td></tr>
<tr><td>Input variable service quality</td></tr>
</table>

Table 5: Input-Rules and Output of System 2

4.3 Discussion

Fuzzy logic system are found in apply for measuring the service quality of different service providing enterprise. Abdullah & Khadiah (2011) suggested a fuzzy linguistic method for determining customer satisfaction based on perceived service efficiency, and the degree of customer satisfaction was calculated by considering the low value of the difference between the overall integral value and the integral value for each linguistic expression. The results indicate that linguistic assessment is a feasible and meaningful method for determining consumer satisfaction [31]. Stefano et al., (2015) used the fuzzy SERVQUAL and fuzzy AHP to assess the service level of a large hotel. The findings indicated that the services of the hotel had several deficiencies that need to be addressed [29]. Djamal and Woduamto (2017)study determined the service quality using Fuzzy SERVQUAL by determining the difference between customer expectations and consumer perceptions, determining the degree of importance between consumer expectations and consumer perceptions, determining the level of service quality, and recommending improvements dependent on priority by examining the gap of each quality parameter [32].

In the study, two systems are designed for measuring overall service quality of a restaurant

from two input dimensions of service quality and food quality. We have seen that a Type-2 fuzzy set consists of upper and lower membership functions. The membership function of a fuzzy number can be in many forms such as (Triangular, Trapezoidal, Sigmoidal, Gaussian, S-shaped, Z-shaped and etc.). In the study System 1 is designed by taking triangular membership functions and system 2 is designed by taking a combination of triangular and Gaussian membership functions. In the case of output variables rather applying any membership functions only crisp values of unit interval are applied. In these two systems, we need to use any defuzzified and type reducing technique. The system generating code is given in the section of MATLAB IT2FLS Generated Code.
The architecture of both systems is shown in Table 4 and 5. In the output surface, a small difference of appearance in figure is noticed. It is also found that the numerical values of Tip are found almost the same in both systems. While generating outputs we observed that system 2 performs faster than system 1.

5. Conclusion

Considerable software tools for applying type-1 fuzzy logic are (1) Matlab Fuzzy Logic Toolbox a GUI of fuzzy logic suitable for designing type-1 FL inference system, (2) fuzzyTECH is fuzzy logic and neural-fuzzy solutions developed by GmbH and Inform Software Corporation and (3) FixPro is a fuzzy inference system professional. Other open source fuzzy logic software, tools and library for designing fuzzy inference systems are Fuzzy Light and jFuzzyLogic and Fuzzy Python Library [33]. All fuzzy logic tools have some specific GUI facilities for designing 'type-1 fuzzy logic' systems. 'Type -2 fuzzy logic' tools did not get popular among our known researchers as 'type -1 fuzzy logic' systems. To develop T2FLS only Interval T2FLs is an open source tool which can apply in the MATLAB environment. The tool comes with limited options but it contains an extra feature of easily transferring to a Simulink environment for further analysis. The present study reviews the 'type-2 fuzzy logic' system and tries to define the underlying steps and processes. The review may help interested researchers to develop tools for 'type2 fuzzy logic' inference systems.

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