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Metrics of TOUGMA

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Abstract

In this paper, we present a TOUGMA's Field Equation solutions for a no electrical charge and non-rotating body in vacuum; and study theirs physical phenomena.

Key-Words: Ricci Tensor, Hermitian black hole,metric of TOUGMA, quantum relativity.

I. INTRODUCTION

TOUGMA's Quantum Relativity theory, first published in 2021, is a geometric quantum gravity theory. In this theory quantum and gravity are entangled and theorized to be a space-time curvature and it's dimensions interaction and manifestation caused by α-dimensions massive objects [1].

$$[(\alpha-3)R_{uv}-\frac{1}{2}g_{uv}R](1+\frac{2kL_m}{R})-2k(\alpha-3)g_{uv}L_m=T_{uv} \tag{1}$$

The solution imposed spherically symmetric metric, caused by no electrical charge and non-rotating n-dimensions massive object in vacuum,
to approch such a solution, the method used is calculating Ricci Ten- sor components for a metric general form for some conditions and give their equating to 0. To do this we are going to start by calculating Christoffel Symbols. After we are going to calculate the Ricci tensor components and solving fields equation and finish by study physics phenomena.

II. METHODS

Let's take [2 3 4 5 6 7 8]:

$$ds^2=-e^{2\gamma}c^2dt^2+e^{2\delta}dr^2+r^2(d\theta^2+r^2sin^2\theta d\varphi^2)+r^{\alpha-4}d\Omega_{\alpha-4} \tag{2}$$

the g^{uv} are:

$$g^{uv}=diag(-e^{-2\gamma},e^{-2\delta},\frac{1}{r^2},\frac{1}{r^2sin^2},\frac{1}{r^{\alpha-4}}) \tag{3}$$

then the Christoffel symbols are[9]:

$$\Gamma^u_{ij}=g^{uv}(\frac{\partial g_{vj}}{\partial x^i}+\frac{\partial g_{iv}}{\partial x^j}-\frac{\partial g_{ij}}{\partial x^v}) \tag{4}$$

$$\Gamma^0_{0r}=-e^{-2\gamma(r)}\gamma(r)e^{2\gamma(r)}=\gamma(r)=\Gamma^0_{r0} \tag{5}$$

$$\Gamma^r_{00}=\gamma' e^{-2(\gamma-\delta)} \tag{6}$$

$$\Gamma^r_{rr}=\delta' \tag{7}$$

$$\Gamma^r_{\theta\theta}=-re^{-2\delta} \tag{8}$$

$$\Gamma^r_{\varphi\varphi}=-rsin^2\delta e^{-2\delta} \tag{9}$$

$$\Gamma^r_{\Omega_{\alpha-4}\Omega_{\alpha-4}}=-(\alpha-4)r^{\alpha-7} \tag{10}$$

$$\Gamma^\theta_{r\theta}=\Gamma^\theta_{\theta r}=\frac{1}{r} \tag{11}$$

$$\Gamma^\theta_{\varphi\varphi}=-cos\theta sin\theta \tag{12}$$

$$\Gamma^\varphi_{r\varphi}=\Gamma^\varphi_{\varphi r}=\frac{1}{r} \tag{13}$$

$$\Gamma^\varphi_{\theta\varphi}=\Gamma^\varphi_{\varphi\theta}=\frac{1}{tan\theta} \tag{14}$$

$$\Gamma^{\Omega_{\alpha-4}}_{r\Omega_{\alpha-4}}=\Gamma^{\Omega_{\alpha-4}}_{\Omega_{\alpha-4}r}=\frac{1}{2r^{\alpha-4}}(-\frac{(\alpha-4)r^{\alpha-5}}{r^{2\alpha-8}})=-\frac{(\alpha-4)}{r^{2\alpha-7}} \tag{15}$$

and Ricci tensor componets are :

$$R_{00}=e^{2(\gamma-\delta)}[\gamma''+(\gamma')^2-\gamma'\delta'+\frac{2\gamma'}{r}-\frac{(\alpha-4)\gamma'}{r^{2\alpha-7}}] \tag{16}$$

and for r»0

$$R_{00}=e^{2(\gamma-\delta)}[\gamma''+(\gamma')^2-\gamma'\delta'+\frac{2\gamma'}{r}] \tag{17}$$

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$$R_{rr} = -\gamma'' - (\gamma')^2 + \gamma'\delta' + \frac{2\delta'}{r} - \frac{(\alpha-4)(2n-7)}{r^{2\alpha-6}} - \frac{(\alpha-4)\delta'}{r^{2\alpha-7}} + \frac{(\alpha-4)^2}{r^{4\alpha-14}} \quad (18)$$

and for r»0

$$R_{rr} = -\gamma'' - (\gamma')^2 + \gamma'\delta' + \frac{2\delta'}{r} \quad (19)$$

for r very large the others components are:

$$R_{\theta\theta} = e^{-2\delta}[r(\delta-\gamma)-1]+1 \quad (20)$$

$$R_{\varphi\varphi} = sin^2\theta e^{-2\delta}[r(\delta-\gamma)-1]+1 \quad (21)$$

$$R_{\Omega_{n-4}\Omega_{n-4}} \simeq 0 \quad (22)$$

the others components are zero.. Ricci scalar is given by $R = g^{ij}R_{ij}$:

$$R = -e^{-2\gamma(r)}R_{00} + e^{-2\delta(r)}R_{rr} + \frac{1}{r^2}R_{\theta\theta} + \frac{1}{r^2 sin^2\theta} \quad (23)$$

and the calculated :

$$R = 2e^{-2\alpha}[\nu' + (\nu')^2 - \nu'\alpha' + \frac{2}{r}\nu' + \frac{2}{r}(\alpha'-\nu') + \frac{2}{r^2}(e^{2\alpha}-1)] \quad (24)$$

III. RESULTS

Now we are going to resolve TOUGMA's field equation reduce:

$$(\alpha-3)R_{uv} - \frac{1}{2}g_{uv}R - kg_{uv}L_m = T_{uv} \quad (25)$$

with R_{ij} and R, we have :

$$W_{00} = (\alpha-3)R_{00} - \frac{1}{2}R(-e^{2\gamma}) - kL_m(-e^{2\gamma}) \quad (26)$$

$$W_{rr} = (\alpha-3)R_{rr} - \frac{1}{2}R(-e^{2\delta}) - kL_m(e^{2\delta}) \quad (27)$$

$$W_{\theta\theta} = (\alpha-3)R_{\theta\theta} - \frac{1}{2}Rr^2 - kL_m r^2 \quad (28)$$

$$W_{\varphi\varphi} = (\alpha-3)R_{\varphi\varphi} - \frac{1}{2}Rr^2 sin^2\theta - kL_m r^2 sin^2\theta \quad (29)$$

If we replace R_{ij} and R by their terms, we have :

$$W_{00} = (\alpha-4)[\gamma'' + \gamma'^2 - \gamma'\delta' + \frac{2}{r}\gamma'] - \frac{1}{r^2}(e^{2\delta} - 1 + 2r\delta') + kL_m e^{2\gamma} \quad (30)$$

$$W_{rr} = (\alpha-4)[\gamma'' + \gamma'^2 - \gamma'\delta' + \frac{2(\alpha-2)}{(\alpha-4)r}\gamma'] - \frac{1}{r^2}(-e^{2\delta} + 1 + 2r\gamma') - kL_m e^{\delta} \quad (31)$$

$$W_{\theta\theta} = r^2 e^{-2\delta}[\gamma'' + \gamma'^2 - \gamma'\delta' + \frac{\alpha-5}{r}(\delta'-\gamma')] + e^{-2\delta} + (\alpha-4) - 2kl_m r^2 \quad (32)$$

$$W_{\varphi\varphi} = sin^2(\theta)[r^2 e^{-2\delta}[\gamma'' + \gamma'^2 - \gamma'\delta' + \frac{\alpha-5}{r}(\delta'-\gamma')] + e^{-2\delta} + (\alpha-4) - 2kl^m r^2] \quad (33)$$

if $T_{uv} = 0$. TOUGMA' equation become:

$$(\alpha-4)[\gamma'' + \gamma'^2 - \gamma'\delta' + \frac{2}{r}\gamma'] - \frac{1}{r^2}(e^{2\delta} - 1 + 2r\delta') + kL_m e^{2\gamma} = 0 \quad (34)$$

$$(\alpha-4)[\gamma'' + \gamma'^2 - \gamma'\delta' + \frac{2(\alpha-2)}{(\alpha-4)r}\gamma'] - \frac{1}{r^2}(-e^{2\delta} + 1 + 2r\gamma') - kL_m e^{2\delta} = 0 \quad (35)$$

$$r^2 e^{-2\delta}[\gamma'' + \gamma^2 - \gamma'\delta' + \frac{\alpha-5}{r}(\delta'-\gamma')] + e^{-2\delta} + (\alpha-4) - 2kl_m r^2 = 0 \quad (36)$$

$$sin^2\theta[r^2 e^{-2\delta}[\gamma'' + \gamma^2 - \gamma'\delta' + \frac{\alpha-5}{r}(\delta'-\gamma')] + e^{-2\delta} + (\alpha-4) - 2kl^m r^2] = 0 \quad (37)$$

by taken equation 43-45:

$$[(2\alpha-6)r - 2(\alpha-2)r^2]\gamma' - 2r\delta' + kL_m r^2(e^{2\gamma} + e^{\delta}) + 2 = 0 \quad (38)$$

the term $(e^{2\gamma} + e^{\delta})$ sugests that $\delta = -\gamma$ and $\gamma = iN(r)$;then it happens:

$$i[(2\alpha-6)r - 2(\alpha-2)r^2 + 2r]N' + 2kL_m r^2 cos(2N) = -2 \quad (39)$$

if we take this equation wihtout second member, we have:

$$i[(2\alpha-6)r-2(\alpha-2)r^2+2r]N'+2kL_m r^2 cos(2N)=0 \quad (40)$$

then

$$i[(2\alpha-6)r-2(\alpha-2)r^2+2r]N'=-2kL_m r^2 cos(2N) \quad (41)$$

and

$$i\frac{N'}{cos(2N)} = -2kL_m\frac{r^2}{[(2\alpha-6)r-2(\alpha-2)r^2+2r]} \quad (42)$$

$$i\int\frac{dN}{cos(2N)}=-2kL_m\int\frac{r^2}{[(2\alpha-6)r-2(\alpha-2)r^2+2r]}dr \quad (43)$$

$$i2ln[tan(N+\frac{\pi}{4})] = -\frac{kL_m}{2(\alpha-2)}\int\frac{r}{[1-r]}dr \quad (44)$$

$$i2ln[tan(N+\frac{\pi}{4})] = -\frac{kL_m}{2(\alpha-2)}\int[1+\frac{1}{[1-r]}]dr \quad (45)$$

$$i2ln[tan(N+\frac{\pi}{4})] = -\frac{kL_m}{2(\alpha-2)}[1-ln(1-r)] \quad (46)$$

$$[tan(N+\frac{\pi}{4})]^{2i} = e^{\frac{-kL_m}{2(\alpha-2)}}\frac{e^1}{(1-r)} \quad (47)$$

$$[tan(N+\frac{\pi}{4})] = \frac{e^{\frac{2(\alpha-2)-kL_m}{4i(\alpha-2)}}}{\sqrt{(1-r)}} \quad (48)$$

then the general solution is:

$$N = tan^{-1}[\frac{e^{\frac{2(\alpha-2)-kL_m}{4i(\alpha-2)}}}{\sqrt{(1-r)}}] - \frac{\pi}{4} + C \quad (49)$$

we have the metric:

$$ds^2=-\exp(2i[tan^{-1}(\frac{e^{\frac{2(\alpha-2)-kL_m}{4i(\alpha-2)}}}{\sqrt{(1-r)}})-\frac{\pi}{4}])c^2dt^2 +\exp(-2i[tan^{-1}(\frac{e^{\frac{2(\alpha-2)-kL_m}{4i(\alpha-2)}}}{\sqrt{(1-r)}})-\frac{\pi}{4}])dr^2 +r^2(d\theta^2+r^2sin^2\theta d\varphi^2)+r^{\alpha-4}d\Omega_{\alpha-4} \quad (50)$$

$$ds^2=-\exp(2i[tan^{-1}(\frac{e^{i\frac{[2(\alpha-2)-kL_m]}{4(\alpha-2)}}}{\sqrt{(1-r)}})-\frac{\pi}{4}])c^2dt^2 +\exp(-2i[tan^{-1}(\frac{e^{i\frac{[2(\alpha-2)-kL_m]}{4(\alpha-2)}}}{\sqrt{(1-r)}})-\frac{\pi}{4}])dr^2 +r^2(d\theta^2+r^2sin^2\theta d\varphi^2)+r^{\alpha-4}d\Omega_{\alpha-4} \quad (51)$$

with dim(r)=α for non rotating space-time.
if the term $(e^{2\gamma}+e^{\delta})$ sugests that $\delta=-\gamma$ and $\gamma=N$;then it happens:

$$[(2\alpha-6)r-2(\alpha-2)r^2+2r]N'+2kL_m r^2 cosh(2N)=-2 \quad (52)$$

if we take this equation wihtout second member, we have:

$$[(2\alpha-6)r-2(\alpha-2)r^2+2r]N'+2kL_m r^2 cosh(2N)=0 \quad (53)$$

then

$$[(2\alpha-6)r-2(\alpha-2)r^2+2r]N'=-2kL_m r^2 cosh(2N) \quad (54)$$

and

$$\frac{N'}{cosh(2N)} = -2kL_m\frac{r^2}{[(2\alpha-6)r-2(\alpha-2)r^2+2r]} \quad (55)$$

$$\int\frac{dN}{cosh(2N)}=-2kL_m\int\frac{r^2}{[(2\alpha-6)r-2(\alpha-2)r^2+2r]}dr \quad (56)$$

$$2artan(e^N) = -\frac{kL_m}{2(\alpha-2)}\int\frac{r}{[1-r]}dr \quad (57)$$

$$2artan(e^N) = -\frac{kL_m}{2(\alpha-2)}\int[1+\frac{1}{[1-r]}]dr \quad (58)$$

$$2artan(e^N) = -\frac{kL_m}{2(\alpha-2)}[1-ln(1-r)] \quad (59)$$

$$\exp(N) = \tan(-\frac{kL_m}{4(\alpha-2)}[1-ln(1-r)]) \quad (60)$$

$$N = \ln(\tan(-\frac{kL_m}{4(\alpha-2)}[1-ln(1-r)])) \quad (61)$$

then the general solution is:

$$N = \ln(\tan(-\frac{kL_m}{4(\alpha-2)}[1-ln(1-r)])) + C \quad (62)$$

and

$$\gamma = \ln(\tan(-\frac{kL_m}{4(\alpha-2)}[1-ln(1-r)])) + C \quad (63)$$

we have the metric:

$$ds^2=-[\tan(-\frac{kL_m}{4(\alpha-2)}[1-ln(1-r)])]^2c^2dt^2 +\frac{dr^2}{[\tan(-\frac{kL_m}{4(\alpha-2)}[1-ln(1-r)])]^2} +r^2(d\theta^2+r^2sin^2\theta d\varphi^2)+r^{\alpha-4}d\Omega_{\alpha-4} \quad (64)$$

with dim(r)=α for non rotating space-time.

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