$$\begin{bmatrix} A & \varpi & 0 & 0 & \psi_1 & \psi_2 \\ \beta & B & 0 & 0 & 0 & 0 \\ \epsilon_1 & \epsilon_1 & C & 0 & 0 & 0 \\ \epsilon_2 & \epsilon_2 & 0 & D & 0 & 0 \\ 0 & 0 & \rho_2 & \rho_2 & E & 0 \\ 0 & 0 & \rho_1 & \rho_1 & \delta & -\psi_2-\mu \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(XXX)}.$$

Let

$$A = -\beta - \epsilon_1 - \epsilon_1 - \mu$$

$$B = -\kappa_1 - \kappa_2 - \mu$$

$$C = -\rho_2 - \rho_1 - \alpha - \mu$$

$$D = -\rho_1 - \rho_2 - \alpha - \mu$$

$$E = -\delta - \psi 1 - \mu$$

Thus the characteristic equation is given as

$|J(P_0 - \lambda)|$

$$\begin{bmatrix} -\mu-\lambda & \varpi & 0 & 0 & 0 & 0 \\ 0 & -\mu-\lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha-\mu-\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha-\mu-\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & -\mu-\lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mu-\lambda \end{bmatrix} = 0\ldots\ldots\ldots\ldots\text{(XXXI)}$$

From here the eigenvalues of $J(P_0)$ can be obtained as

$$\lambda_1 = -\mu$$
$$\lambda_2 = -\mu$$
$$\lambda_3 = -\alpha - \mu$$
$$\lambda_4 = -\alpha - \mu$$
$$\lambda_5 = -\mu$$
$$\lambda_6 = -\mu$$

Since all eigen-values are negatives, it implies that the disease free equilibrium point is locally asymptotically stable if $R_0 < 1$. It is unstable if $R_0 > 1$.

The Threshold Parameter

The threshold parameter defined R_0=$\frac{1}{S^*} * \frac{\lambda}{\mu}$ according to (Momoh et al, 2013) approach as the parameter that is used to determine the equilibria.

DURING OUTBREAK

Theorem: if $R_0 > 1$, then P^* is globally asymptotically stable with respect to the interior of Ω. Considering natural carrier rate which is 25% .Therefore,

$$S^* = \frac{\lambda + \varpi C + \psi_1 R_D + \psi_2 R_N}{\beta + \varepsilon_1 + \varepsilon_2 + \mu}$$

$$= \frac{14{,}624{,}414 + 3{,}656{,}104 + 2{,}160 + 4{,}319}{0.25 + 0.35 + 0.40 + 0.02}$$

$$= \frac{18{,}286{,}997}{1.02}$$

$$= 17{,}928{,}428$$

$$R_0 = \frac{1}{S^*} * \frac{\lambda}{\mu}$$

$$= \frac{1}{17{,}928{,}428} * \frac{14{,}624{,}414}{0.02}$$

$$= \frac{14{,}624{,}414}{358{,}569}$$

$$= 40.78549456 \approx 41 > 1$$

When carrier rate raised to 60%

$$S^* = \frac{\lambda + \varpi C + \psi_1 R_D + \psi_2 R_N}{\beta + \varepsilon_1 + \varepsilon_2 + \mu}$$

$$\frac{14{,}624{,}414 + 8{,}774{,}648 + 2{,}160 + 4{,}319}{0.25 + 0.35 + 0.40 + 0.02}$$

$$= \frac{23{,}405{,}541}{1.02}$$

$$= 22{,}946{,}609$$

$$R_0 = \frac{1}{S^*} * \frac{\lambda}{\mu}$$

$$= \frac{1}{22,946,609} * \frac{14,624,414}{0.02}$$

$$= \frac{14,624,414}{458,932}$$

$$= 31.86618933 \approx 32 > 1$$

When carrier rate raised to 90%

$$S^* = \frac{\lambda + \varpi C + \psi_1 R_D + \psi_2 R_N}{\beta + \varepsilon_1 + \varepsilon_2 + \mu}$$

$$\frac{14,624,414 + 13,161,973 + 2,160 + 4,319}{0.25 + 0.35 + 0.40 + 0.02}$$

$$= \frac{27,792.866}{1.02}$$

$$= 27,247,908$$

$$R_0 = \frac{1}{S^*} * \frac{\lambda}{\mu}$$

$$= \frac{1}{27,247,908} * \frac{14,624,414}{0.02}$$

$$= \frac{14,624,414}{514,958}$$

$$= 26.83585524 \approx 27 > 1$$

BEFORE OUTBREAK

Theorem: if $R_0 > 1$, then P^* is globally asymptotically stable with respect to the interior of Ω. Considering natural carrier rate which is 25% .Therefore,

$$S^* = \frac{\lambda + \varpi C}{\beta + \mu}$$

$$= \frac{14,624,414 + 3,656,104}{0.25 + 0.02}$$

$$= 67,705,622$$

$$R_0 = \frac{1}{S^*} * \frac{\lambda}{\mu}$$

$$= \frac{1}{67,705,622} * \frac{14,624,414}{0.02}$$

$$= 10.800000325 \approx 11 > 1$$

Since R0 > 1, therefore the disease will persist as observed from the threshold that during the outbreaks that R_0 is obtained when natural carrier rate is 25% with $R_0 = 41$ when carrier rate is increased to 60%, and $R_0 = 32$ when carrier rate is finally increased to 90% $R_0 = 26$ bur when there is no outbreak,$R_0 = 11$ therefore, these indicate that carrier negative effect is mostly observed during outbreak.

Stability of Disease Free Equilibrium

The disease free equilibrium point is locally stable if $R_0 < 1$ and unstable if $R_0 > 1$. Therefore base on this research, the disease free equilibrium is unstable.

NUMERICAL SIMULATION

The key parameters were used to investigate the meningitis transmission dynamics as well as to investigate the negative effect of carrier on disease transmission by interchanging rate of carrier from the data collected during 2017 epidemic outbreak. Based on the graph, the carrier rates possess increasing approaching the susceptible population during outbreak.

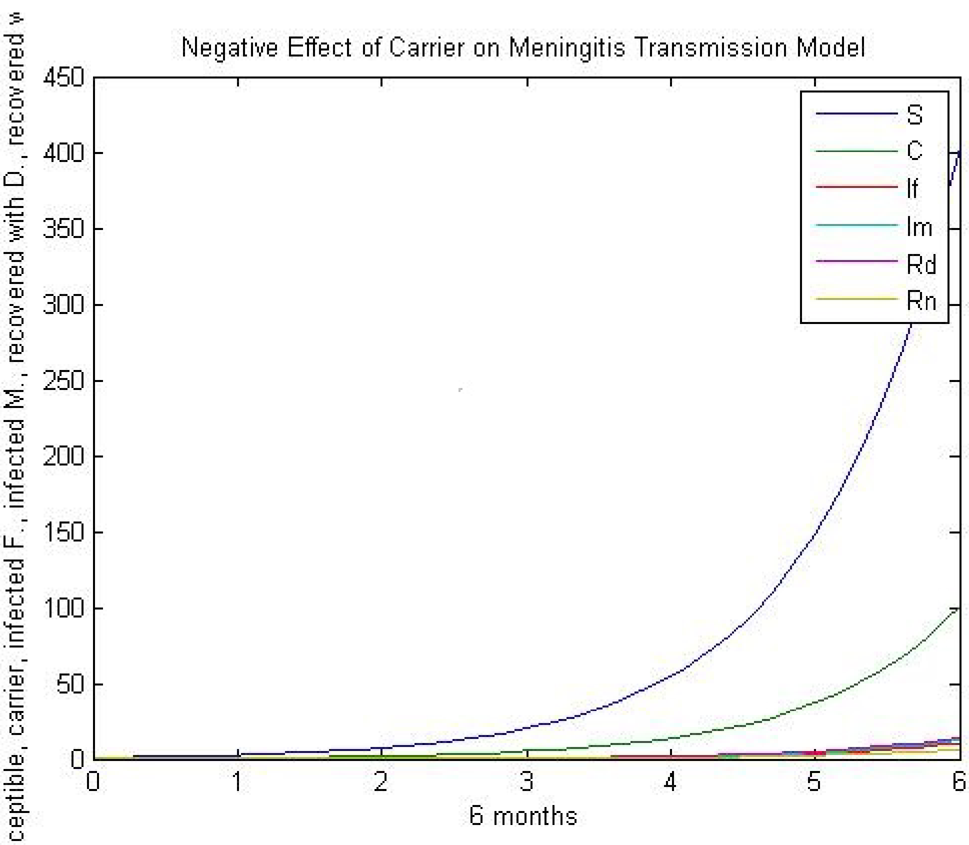

Figure 2: Graphical representation of Meningitis Transmission with carrier rate at 25 percent

CONCLUSION

Since all the Eigen- values are negatives it implies that the disease free equilibrium point is locally asymptotically unstable with $R_0 > 1$. Means that each infected individual infect more than one individual such that there is expectation of the disease spread out.

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