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Modeling Negative Effect of Carrier of Meningitis Transmission Dynamics on Children In North-western Nigeria

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ABSTRACT

In this paper, mathematical model for the transmission dynamics of meningitis to determine the negative effect of carriers on meningitis transmission is developed. The model describes meningitis transmission into compartments which leads to a linear system of differential equations. The model used data on 2017 meningitis outbreaks on children in the northwestern Nigeria. From the analysis, all Eigen-values are found negatives and R_0 (the threshold parameter) is greater than 1, during and before the meningitis outbreak, by adjusting the rate of carrier from natural carrier rate it shows that the negative effect of carriers on meningitis transmission dynamics is more effective during outbreak than when there is no outbreak. Since R_0 is greater than 1 means that the endemic equilibrium is stable both locally and globally. The disease free equilibrium points are considered as well as the stability of a meningitis transmission dynamics model with all the eigenvalues of a Jacobian matrix are negatives. Thus, the disease free equilibrium points are locally asymptotically unstable.

Keywords: Meningitis, Carrier, Threshold value

INTRODUCTION

Carrier is a person who is capable of transmitting disease infection without displaying any of the symptoms and can pass it to others. When a susceptible individual come into contact with the bacteria or infected individual become carriage of a bacterium, such individual is called carrier. Such is equally a healthy person harboring a pathogenic organism, without having clinical manifestation and can transmit organism to others.

Meningitis infection infect humans hosts and the great majority of these infections result in asymptomatic colonization, which is extremely common as carriage in adult and the growth is very rare in infants (Trotter and Maiden, 2016). No matter the cause, the symptoms of meningitis are always similar and usually develop rapidly, often in as little as 12 hours, nearly all patients with meningitis experience vomiting, high fever, and a stiff neck. It also causes severe headache, back pain, muscle aches, sensitivity of the eyes to light, drowsiness, confusion, and even loss of consciousness, some children have convulsions (Samuel and Vivian, 2014).

Carrier can be asymptomatic or of no symptoms at all, it therefore play undoubtablerole in spreading the disease (Fresnadillo et al, 2013). It was believed that 10%to 20% of the people carry meningitis without their knowledge and the percentage may increase especially during epidemic outbreak (World HealthOrganization [WHO]). The bacterial meningitis carrier carried the bacteria in the throat and back of their nose and spread through cough and droplet and the incidence of the infection during outbreak mostly affect individuals under 15 years of age (WHO).

Materials and Methods

A mathematical model which will be formulated using differential equations based on the epidemiological compartment modeling used is the proposed S, C, I_F, I_M, R_D, R_N model for this paper, with carrier rate, which will be incorporated into the model. Data collected for the

2017 meningitis outbreaks in the north western Nigeria will be used and numerical simulations of the model will be conducted using Matlab application software.

Model Formulation and Analysis

In this paper, I consider the S, C, I_F, I_M, R_D, $R_N$epidemiological model with the assumption that, natural death and death rates due to infection being unequal. The deterministic, compartmental mathematical model is formulated to describe the transmission dynamics of meningitis infection in the northwestern Nigeria. It is also assumed that, the population is heterogeneous. That is no individuals that make up of population can be grouped into different compartment or groups according to their epidemiological class and is to be taken is constant, the population size in a compartment is differentiable with respect to time and deterministic. In order words that the changes in population of a compartment can be calculated using only using history to developed the model. Natural deaths in each compartment and death due to meningitis only will be considered not of any other cause. The proportions of the population of children are immunized against meningitis infection through vaccination. And the population mixed homogeneously. That is all susceptible individuals are equally likely to be infected by infectious individuals in case of contact.

Description of the Model Variables and Parameters

The following tables describe the variables and parameters used in this model:

Table 1: Variables used in the model

Model Variables	Description	Initial Value	Source
S(t)	Susceptible population	14, 624,414	NPC, 2017
C(t)	Carriers	3,656,104	NCDC, 2017
I_f (t)	Infected females	9,934	NCDC, 2017
$I_m(t)$	Infected males	11,661	NCDC, 2017
R_d (t)	Recovered with deficiency	2,160	NCDC, 2017
R_n (t)	Recovered without deficiency	4,319	NCDC, 2017

Table 2: Parameters used in the model used in the model

Model Parameters	Description	Initial Value	Source
Λ	Recruitment rate	0.41	Estimated
α	Death rate due to meningitis	0.01	NCDC, 2017
μ	Natural death rate	0.02	NCDC, 2017
β	Rate of Carrier contact	0.25	Coen, 2000
ϵ_1	Rate of female contact	0.035	Estimated
ϵ_2	Rate of male contact	0.040	Estimated
ψ_1	Rate of return to susceptible from R_d	0.045	Estimated
ψ_2	Rate of return to susceptible from R_n	0.055	Estimated
κ_1	Rate of female move to infectious from carrier	0.042	Estimated
κ_2	Rate of male move to infectious from carrier	0.032	Estimated

δ	Rate of moving from R_d to R_n	0.05	Estimated
ρ_1	Rate of female recovery from infectious class	0.045	Estimated
ρ_2	Rate of male recovery from infectious class	0.035	Estimated
ϖ	Rate of return from carrier to susceptible	0.125	Estimated

Compartmental Diagram

In this research, the population is divided into six disease-state compartments such as susceptible, carrier, infected female, infected male, recovered with deficiency and recovered without deficiency represented by S, C, I_F, I_M, R_D, $R_N$respectively. In which the model considered was S, C, I_F, I_M, R_D, $R_N$model : susceptible individuals (S), people who can catch the disease; carrier individuals (C), people whose body is a host for the infectious agent and are yet able to transmit the disease; infectious (infective) individuals I_F, and I_M, people who have the disease and can transmit the disease; recovered individual R_D, andR_N, proportion of people who have recovered from the disease with disability and without disability. it is however, assume that an individual can be infected only through contacts with infectious individuals and that immunity is permanent. The following tables describe the variables and parameters used in this model:
Thus, the compartmental diagram for the deterministic model is as follows;

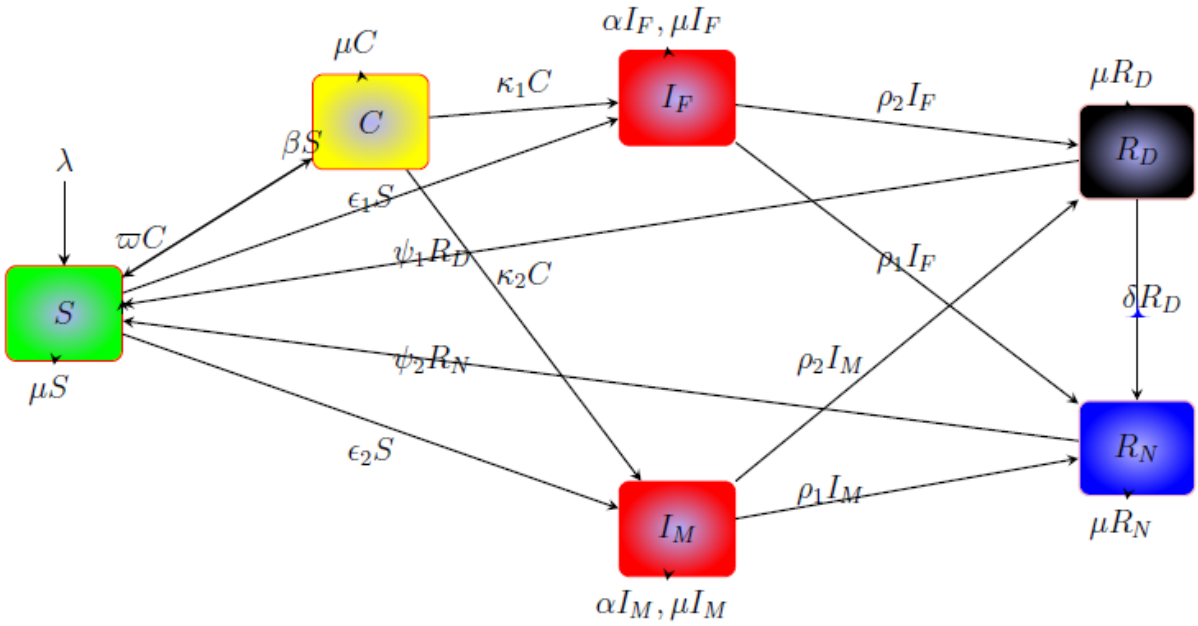

Figure 1: The compartmental diagram describing bacterial meningitis transmission dynamics within the population. The six squares represent the six compartments of individuals, such as, susceptible, carrier, infected male, infected female, recovered with deficiency and recovered without deficiency. The movement between the compartments is indicated by the continuous arrows, with the consideration that carrier measures lies in, $0 < C; C \leq 1$.

Model Equation

The transitions between model classes can now be expressed by the following system of first order differential equations:-

$$\frac{\partial S}{\partial t} = \lambda + \varpi C + \psi R_D + \psi R_N - \beta S - \epsilon_1 S - \epsilon_2 S - \mu S \ldots\ldots\ldots (I)$$

$$\frac{\partial C}{\partial t} = \beta S - \kappa_1 C - \kappa_2 C - \mu C \ldots\ldots\ldots (II)$$

$$\frac{\partial I_F}{\partial t} = \kappa_1 C + \epsilon_1 S - \rho_2 I_F - \rho_1 I_F - \alpha I_F - \mu I_F \ldots\ldots\ldots (III)$$

$$\frac{\partial I_M}{\partial t} = \kappa_2 C + \epsilon_2 S - \rho_2 I_M - \rho_1 I_M - \alpha I_M - \mu I_M \ldots\ldots\ldots (IV)$$

$$\frac{\partial R_D}{\partial t} = \rho_2 I_F + \rho_2 I_M - \delta R_D - \psi_1 R_D - \mu R_D \ldots\ldots\ldots (V)$$

$$\frac{\partial R_N}{\partial t} = \rho_1 I_F + \rho_1 I_M + \delta R_D - \psi_2 R_N - \mu R_N \ldots\ldots\ldots (VI)$$

BASIC PROPERTIES OF THE MODEL

Existence and uniqueness of solution

For the mathematical model to predict the future of the system from its current state at time t_0, the initial value problem (IVP)

$x' = f(t, x), \qquad x(t_0) = x_0$……..……(XI)

Must have a solution that exist and also unique.

In this section, the conditions for the existence and uniqueness of solution for the model system of equation shall be established. Let

$$f_1(t,x) = \lambda + \varpi C + \psi R_D + \psi R_N - \beta S - \epsilon_1 S - \epsilon_2 S - \mu S \ldots\ldots (XII)$$

$$f_2(t,x) = \beta S - \kappa_1 C - \kappa_2 C - \mu C \ldots\ldots\ldots (XIII)$$

$$f_3(t,x) = \kappa_1 C + \epsilon_1 S - \rho_2 I_F - \rho_1 I_F - \alpha I_F - \mu I_F \ldots\ldots\ldots (XIV)$$

$$f_4(t,x) = \kappa_2 C + \epsilon_2 S - \rho_2 I_M - \rho_1 I_M - \alpha I_M - \mu I_M \ldots\ldots\ldots (XV)$$

$$f_5(t,x) = \rho_2 I_F + \rho_2 I_M - \delta R_D - \psi_1 R_D - \mu R_D \ldots\ldots\ldots (XVI)$$

$$f_6(t,x) = \rho_1 I_F + \rho_1 I_M + \delta R_D - \psi_2 R_N - \mu R_N \ldots\ldots\ldots (XVII)$$

So that

$x' = f(t,x), \quad x(t_0) = f(x)$…….(XVIII)

Theorem 1, (Momoh et al, 2013) Let D' denotes the region

$|t - t_0| \leq a, \|x - x_0\| \leq b,$

$x = (x_1, \quad x_2, \quad \ldots\ldots x_n),$

$x_0 = (X_{10}, X_{20}, \ldots . . X_{n0})$……..(XIX)

And suppose that $f(t,x)$ satisfies the Lipchitz condition

$\|f(t,x_1) - f(t,x_2)\| \leq k\|x_1 - x_2\|$,……..(XX)

Whenever the pairs $(t,x_1)\ and\ (t,x_2)$ belongs to D', where k is a positive constant. Then, there exist a constant $\delta > 0$ such that there exist a unique continuous vector solution $\bar{x}(t)$ of the system (XI) in the interval $|t - t_0| \leq \delta$.

It is important to note that condition (XX) is satisfied by requirement that $\frac{\partial fi}{\partial xj}, i,j = 1,2, \ldots . . n$ be continuous and bounded inD'.

Lemma 2. If $f(t,x)$ has continuous partial derivatives $\frac{\partial fi}{\partial xj}$ on a bounded closed covex domain R, then it satisfies a Lipchitz condition in R.

Being interested in the region

$1 \leq \varepsilon \leq R$……..(XXI)

By looking for a bounded solution of the form

$0 < R < \infty$……(XXII),

Following the proving of existence theorem

Theorem 2. Let $D^{'}$ denote the region in (XX) such that (XXI) and (XXII) hold. Then there exist a solution of model system (XII) – (XVII) which is bounded in the region $D^{'}$.

Proof.

$$f_1 = \lambda + \varpi C + \psi R_D + \psi R_N - \beta S - \epsilon_1 S - \epsilon_2 S - \mu S$$

$$f_2 = \beta S - \kappa_1 C - \kappa_2 C - \mu C$$

$$f_3 = \kappa_1 C + \epsilon_1 S - \rho_2 I_F - \rho_1 I_F - \alpha I_F - \mu I_F$$

$$f_4 = \kappa_2 C + \epsilon_2 S - \rho_2 I_M - \rho_1 I_M - \alpha I_M - \mu I_M$$

$$f_5 = \rho_2 I_F + \rho_2 I_M - \delta R_D - \psi_1 R_D - \mu R_D$$

$$f_6 = \rho_1 I_F + \rho_1 I_M + \delta R_D - \psi_2 R_N - \mu R_N$$

It suffices to show that $\frac{\delta fi}{\delta xj}, i, j = 1, 2, 3, 4$ are continuous

Consider the partial derivatives

$$\frac{\delta f1}{\delta s} = -\beta - \varepsilon_1 - \varepsilon_2 - \mu, \left|\frac{\delta f1}{\delta s}\right| = |-\beta - \varepsilon_1 - \varepsilon_2 - \mu| < \infty$$

$$\frac{\delta f1}{\delta C} = \varpi, \left|\frac{\delta f1}{\delta s}\right| = |\varpi| < \infty$$

$$\frac{\delta f1}{\delta I_F} = 0, \left|\frac{\delta f1}{\delta I_F}\right| = |0| < \infty$$

$$\frac{\delta f1}{\delta I_M} = 0, \left|\frac{\delta f1}{\delta I_M}\right| = |0| < \infty$$

$$\frac{\delta f1}{\delta R_D} = \psi_1, \left|\frac{\delta f1}{\delta R_D}\right| = |\psi_1| < \infty$$

$$\frac{\delta f1}{\delta R_N} = \psi_2, \left|\frac{\delta f1}{\delta R_N}\right| = |\psi_2| < \infty$$

Similarly

$$\frac{\delta f2}{\delta s} = \beta, \left|\frac{\delta f2}{\delta s}\right| = |\beta| < \infty$$

$$\frac{\delta f2}{\delta C} = -\kappa_1 - \kappa_2 - \varpi - \mu, \left|\frac{\delta f1}{\delta s}\right| = |-\kappa_1 - \kappa_2 - \varpi - \mu| < \infty$$

$$\frac{\delta f2}{\delta I_F} = 0, \left|\frac{\delta f2}{\delta I_F}\right| = |0| < \infty$$

$$\frac{\delta f2}{\delta I_M} = 0, \left|\frac{\delta f2}{\delta I_M}\right| = |0| < \infty$$

$$\frac{\delta f2}{\delta R_D} = 0, \left|\frac{\delta f2}{\delta R_D}\right| = |0| < \infty$$

$$\frac{\delta f2}{\delta R_N} = 0, \left|\frac{\delta f2}{\delta R_N}\right| = |0| < \infty$$

The same way

$$\frac{\delta f3}{\delta s} = \varepsilon_1, \left|\frac{\delta 31}{\delta s}\right| = |\varepsilon_1| < \infty$$

$$\frac{\delta f3}{\delta C} = \kappa_1, \left|\frac{\delta f3}{\delta C}\right| = |\kappa_1| < \infty$$

$$\frac{\delta f3}{\delta I_F} = -\rho_2 - \rho_1 - \alpha - \mu, \left|\frac{\delta f3}{\delta I_F}\right| = |-\rho_2 - \rho_1 - \alpha - \mu| < \infty$$

$$\frac{\delta f3}{\delta I_M} = 0, \left|\frac{\delta f3}{\delta I_M}\right| = |0| < \infty$$

$$\frac{\delta f3}{\delta R_D} = 0, \left|\frac{\delta f3}{\delta R_D}\right| = |0| < \infty$$

$$\frac{\delta f3}{\delta R_N} = 0, \left|\frac{\delta f3}{\delta R_N}\right| = |0| < \infty$$

Also

$$\frac{\delta f4}{\delta s} = \epsilon_2, \left|\frac{\delta f4}{\delta s}\right| = |\epsilon_2| < \infty$$

$$\frac{\delta f4}{\delta C} = \kappa_2, \left|\frac{\delta f4}{\delta C}\right| = |\kappa_2| < \infty$$

$$\frac{\delta f4}{\delta I_F} = 0, \left|\frac{\delta f4}{\delta I_F}\right| = |0| < \infty$$

$$\frac{\delta f4}{\delta I_M} = -\rho_1 - \rho_2 - \alpha - \mu, \left|\frac{\delta f4}{\delta I_M}\right| = |-\rho_1 - \rho_2 - \alpha - \mu| < \infty$$

$$\frac{\delta f4}{\delta R_D} = 0, \left|\frac{\delta f4}{\delta R_D}\right| = |0| < \infty$$

$$\frac{\delta f4}{\delta R_N} = 0, \left|\frac{\delta f4}{\delta R_N}\right| = |0| < \infty$$

Similarly

$$\frac{\delta f5}{\delta s} = 0, \left|\frac{\delta f5}{\delta s}\right| = |0| < \infty$$

$$\frac{\delta f5}{\delta C} = 0, \left|\frac{\delta f5}{\delta C}\right| = |0| < \infty$$

$$\frac{\delta f5}{\delta I_F} = \rho_2, \left|\frac{\delta f5}{\delta I_F}\right| = |\rho_2| < \infty$$

$$\frac{\delta f5}{\delta I_M} = \rho_1, \left|\frac{\delta f5}{\delta I_M}\right| = |\rho_1| < \infty$$

$$\frac{\delta f5}{\delta R_D} = -\delta - \psi_1 - \mu, \left|\frac{\delta f5}{\delta R_D}\right| = |-\delta - \psi_1 - \mu| < \infty$$

$$\frac{\delta f5}{\delta R_N} = 0, \left|\frac{\delta f5}{\delta R_N}\right| = |0| < \infty$$

Finally becomes

$$\frac{\delta f6}{\delta s} = 0, \left|\frac{\delta f6}{\delta s}\right| = |0| < \infty$$

$$\frac{\delta f6}{\delta C} = 0, \left|\frac{\delta f6}{\delta s}\right| = |0| < \infty$$

$$\frac{\delta f6}{\delta I_F} = \rho_1, \left|\frac{\delta f6}{\delta I_F}\right| = |\rho_1| < \infty$$

$$\frac{\delta f6}{\delta I_M} = \rho_1, \left|\frac{\delta f6}{\delta I_M}\right| = |\rho_1| < \infty$$

$$\frac{\delta f6}{\delta R_D} = \delta, \left|\frac{\delta f6}{\delta R_D}\right| = |\delta| < \infty$$

$$\frac{\delta f6}{\delta R_N} = -\psi_2 - \mu, \left|\frac{\delta f6}{\delta R_N}\right| = |-\psi_2 - \mu| < \infty$$

Clearly, all these partial derivatives are continuous and bounded, hence, by theorem (2), there exist a unique solution of (XII) – (XVIII) in the region D'.

Feasible region

Since the model monitors human population, all associated parameters of the model and state variables are assumed to be non-negative t ≥ 0. It is quite simple to show that the state variables of the model remain non- negative for all non-negative initial conditions.

Lemma 1. The closed Ω is positively invariant and attracting.

Proof: Adding (1) – (VI) gives rate of change of the total population. The total population can be written as:

N(t) = S(t) + C(t) + I_F (t) + I_M(t) + R_D(t) + R_N(t)……..……(VII) therefore the eq. (VII) is changing at a rate

$\frac{\delta_N}{\delta_t} = \frac{\delta_S}{\delta_t} + \frac{\delta_C}{\delta_t} + \frac{\delta I_f}{\delta_t} + \frac{\delta I_m}{\delta_t} + \frac{\delta R_D}{\delta_t} + \frac{\delta R_N}{\delta_t} = \Lambda - \mu N$ ……..(VIII)

In the absence of the disease ie, for $I_F = I_M = R_D = R_N = 0$

Which become

$\frac{\delta_N}{\delta_t} \leq \Lambda - \mu N$……..……..(IX)

By the separation of variables of differentials inequality eq. (IX) become

$\frac{\delta_N}{\Lambda - \mu N} \leq \delta_t$……(X)

Thus, the human population (N) is bounded by$\frac{\lambda}{\mu}$, so that $\frac{\delta N}{\delta t} = 0$ whenever $N(t) = \frac{\lambda}{\mu}$,

It can be shown that $N(t) = \frac{\lambda}{\mu} + \left(N_0 - \frac{\lambda}{\mu}\right) e^{-\mu t}$. In particular $N(t) = \frac{\lambda}{\mu}$,if $N(0) = \frac{\lambda}{\mu}$.

Hence, the region Ω is positively invariant and attract all solutions in R_+^6

Therefore, the feasible solutions set of the equation system eq. (I-VI) enters the region.

$$\Omega = \{ (S, C, I_F, I_M, R_D, R_N) \epsilon \mathbb{R}^6 \rightarrow \frac{\Lambda}{\mu} \}$$

MODEL ANALYSIS

Disease free equilibrium

The disease free equilibrium points are steady solution where there is no disease. Hence the disease free equilibrium exists when C, I_F, I_M, R_D, R_N are set to 0, that is S= C= I_F=I_M=R_D=R_N = 0. Let $N(t)$ represent the human population during the disease free equilibrium which can be written as $\frac{dN}{dt} = \lambda - \mu S$.

Stability of a Disease Free Equilibrium

To understand how the parameters affect the meningitis model, the stability nature of the Disease Free Equilibrium is analyzed by finding the Jacobian matrix for the S, C, I_F, I_M, R_d, R_n system. Jacobian matrix is used in order to determine the local stability of the disease free equilibrium $p_0 = (\frac{\lambda}{\mu}, 0, 0, 0, 0, 0)$.

Evaluation of the stability of the disease-free equilibrium P_0, by jacobian matrix

The evaluation follows

Jacobian matrix at $p_0 = (\frac{\lambda}{\mu}, 0, 0, 0, 0, 0)$

$$J(P_0) =$$

$$\begin{bmatrix} A & \varpi & 0 & 0 & \psi_1 & \psi_2 \\ \beta & B & 0 & 0 & 0 & 0 \\ \epsilon_1 & \epsilon_1 & C & 0 & 0 & 0 \\ \epsilon_2 & \epsilon_2 & 0 & D & 0 & 0 \\ 0 & 0 & \rho_2 & \rho_2 & E & 0 \\ 0 & 0 & \rho_1 & \rho_1 & \delta & -\psi_2-\mu \end{bmatrix} \ldots\ldots\ldots\ldots(XXX).$$

Let

$$A = -\beta - \epsilon_1 - \epsilon_1 - \mu$$

$$B = -\kappa_1 - \kappa_2 - \mu$$

$$C = -\rho_2 - \rho_1 - \alpha - \mu$$

$$D = -\rho_1 - \rho_2 - \alpha - \mu$$

$$E = -\delta - \psi 1 - \mu$$

Thus the characteristic equation is given as

$|J(P_0 - \lambda)|$

$$\begin{bmatrix} -\mu-\lambda & \varpi & 0 & 0 & 0 & 0 \\ 0 & -\mu-\lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha-\mu-\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha-\mu-\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & -\mu-\lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mu-\lambda \end{bmatrix} = 0\ldots\ldots\ldots\ldots(XXXI)$$

From here the eigenvalues of $J(P_0)$can be obtained as

$$\lambda_1 = -\mu$$
$$\lambda_2 = -\mu$$
$$\lambda_3 = -\alpha - \mu$$
$$\lambda_4 = -\alpha - \mu$$
$$\lambda_5 = -\mu$$
$$\lambda_6 = -\mu$$

Since all eigen-values are negatives, it implies that the disease free equilibrium point is locally asymptotically stable if $R_0 < 1$. It is unstable if $R_0 > 1$.

The Threshold Parameter

The threshold parameter defined $R_0=\frac{1}{S^*}*\frac{\lambda}{\mu}$ according to (Momoh et al, 2013) approach as the parameter that is used to determine the equilibria.

DURING OUTBREAK

Theorem: if $R_0 > 1$, then P^* is globally asymptotically stable with respect to the interior of Ω. Considering natural carrier rate which is 25% .Therefore,

$$S^* = \frac{\lambda + \varpi C + \psi_1 R_D + \psi_2 R_N}{\beta + \varepsilon_1 + \varepsilon_2 + \mu}$$

$$= \frac{14{,}624{,}414 + 3{,}656{,}104 + 2{,}160 + 4{,}319}{0.25 + 0.35 + 0.40 + 0.02}$$

$$= \frac{18{,}286{,}997}{1.02}$$

$$= 17{,}928{,}428$$

$$R_0 = \frac{1}{S^*} * \frac{\lambda}{\mu}$$

$$= \frac{1}{17{,}928{,}428} * \frac{14{,}624{,}414}{0.02}$$

$$= \frac{14{,}624{,}414}{358{,}569}$$

$$= 40.78549456 \approx 41 > 1$$

When carrier rate raised to 60%

$$S^* = \frac{\lambda + \varpi C + \psi_1 R_D + \psi_2 R_N}{\beta + \varepsilon_1 + \varepsilon_2 + \mu}$$

$$\frac{14{,}624{,}414 + 8{,}774{,}648 + 2{,}160 + 4{,}319}{0.25 + 0.35 + 0.40 + 0.02}$$

$$= \frac{23{,}405{,}541}{1.02}$$

$$= 22{,}946{,}609$$

$$R_0 = \frac{1}{S^*} * \frac{\lambda}{\mu}$$

$$= \frac{1}{22,946,609} * \frac{14,624,414}{0.02}$$

$$= \frac{14,624,414}{458,932}$$

$$= 31.86618933 \approx 32 > 1$$

When carrier rate raised to 90%

$$S^* = \frac{\lambda + \varpi C + \psi_1 R_D + \psi_2 R_N}{\beta + \varepsilon_1 + \varepsilon_2 + \mu}$$

$$\frac{14,624,414 + 13,161,973 + 2,160 + 4,319}{0.25 + 0.35 + 0.40 + 0.02}$$

$$= \frac{27,792.866}{1.02}$$

$$= 27,247,908$$

$$R_0 = \frac{1}{S^*} * \frac{\lambda}{\mu}$$

$$= \frac{1}{27,247,908} * \frac{14,624,414}{0.02}$$

$$= \frac{14,624,414}{514,958}$$

$$= 26.83585524 \approx 27 > 1$$

BEFORE OUTBREAK

Theorem: if $R_0 > 1$, then P^* is globally asymptotically stable with respect to the interior of Ω. Considering natural carrier rate which is 25% .Therefore,

$$S^* = \frac{\lambda + \varpi C}{\beta + \mu}$$

$$= \frac{14,624,414 + 3,656,104}{0.25 + 0.02}$$

$$= 67,705,622$$

$$R_0 = \frac{1}{S^*} * \frac{\lambda}{\mu}$$

$$= \frac{1}{67,705,622} * \frac{14,624,414}{0.02}$$

$$= 10.800000325 \approx 11 > 1$$

Since R0 > 1, therefore the disease will persist as observed from the threshold that during the outbreaks that R_0 is obtained when natural carrier rate is 25% with $R_0 = 41$ when carrier rate is increased to 60%, and $R_0 = 32$ when carrier rate is finally increased to 90% $R_0 = 26$ bur when there is no outbreak,$R_0 = 11$ therefore, these indicate that carrier negative effect is mostly observed during outbreak.

Stability of Disease Free Equilibrium

The disease free equilibrium point is locally stable if $R_0 < 1$ and unstable if $R_0 > 1$. Therefore base on this research, the disease free equilibrium is unstable.

NUMERICAL SIMULATION

The key parameters were used to investigate the meningitis transmission dynamics as well as to investigate the negative effect of carrier on disease transmission by interchanging rate of carrier from the data collected during 2017 epidemic outbreak. Based on the graph, the carrier rates possess increasing approaching the susceptible population during outbreak.

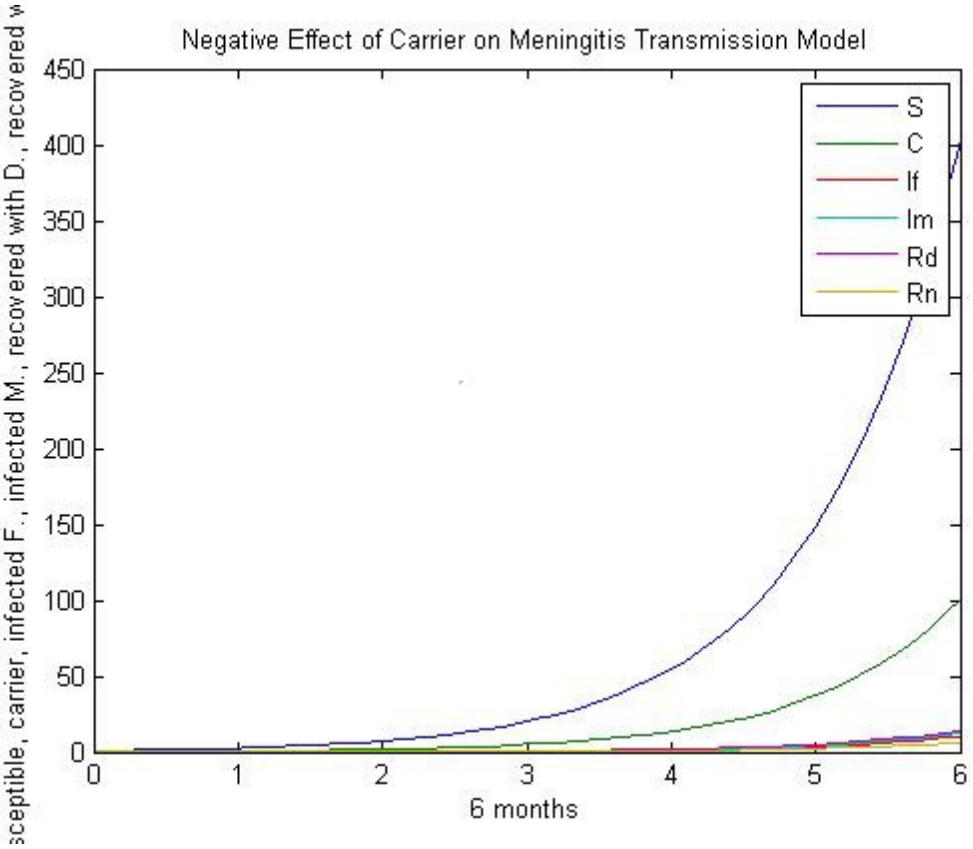

Figure 2: Graphical representation of Meningitis Transmission with carrier rate at 25 percent

CONCLUSION

Since all the Eigen- values are negatives it implies that the disease free equilibrium point is locally asymptotically unstable with $R_0 > 1$. Means that each infected individual infect more than one individual such that there is expectation of the disease spread out.

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