Global Scientific JOURNALS

GSJ: Volume 11, Issue 1, January 2023, Online: ISSN 2320-9186
www.globalscientificjournal.com

MODELLING NIGERIA EXCHANGE RATE USING THE GARCH MODEL

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ABSTRACT

The aim of the study was to modeled Nigeria exchange rate using the GARCH model with three distributions, the normal distribution, student-t distribution and the generalized error distribution with a view of comparing their ability to capture excess kurtosis in proximity to the theoretical value of kurtosis. The data employed in this study comprised of Nigeria exchange rate, Naira to Euro from 2000 to 2020, comprising of 252 observations. The result obtained from the analysis revealed that GARCH (1, 1)-std, was selected as the best models for naira to euro return series respectively. The selection was based on minimum information criteria and the ability to capture excess kurtosis in proximity to the theoretical value of kurtosis. The results of the fitted model indicate that the parameters were significant and that volatility was quite persistent. Diagnostic analysis of the selected model reviewed that the model was correctly specify with no ARCH effect and autocorrelation in the residuals of the models. Our study has established that the GARCH model with respect to student distribution perform better in terms of capturing excess kurtosis in proximity to the theoretical value of kurtosis.

Keywords: GARCH, ARCH, Kurtosis, Model, Diagnostic, Nigeria, Volatility

1.0 INTRODUCTION

In an era of globalization and of flexible exchange rate regimes in most economies, an analysis of foreign exchange rate volatility has become increasingly important among academics and policymakers in recent decades. Volatile exchange rates are likely to affect countries' international trade flow, capital flow, and overall economic welfare [1, 2 & 3]. It is also crucially important to understand exchange rate behavior to design proper monetary policy [4]. As a result, researchers, stakeholders, and policymakers are very interested in analyzing and learning about the nature of exchange rate volatility, which can help to design policies to alleviate the adverse effects of exchange rate volatility on important economic indicators.

Timely forecasting of the exchange rates is able to give important information to decision makers as well as partakers in the area of the internal finance, and policy making. For the giant multinational business units, an accurate forecasting of the exchange rate is crucial since it improves their overall profitability [5]. The importance of forecasting the exchange rates in practical aspect is that an accurate forecast can render valuable information to the investors, firms and central banks for use in allocation of assets, in hedging risk and in policy formulation. The significance of exchange rates forecasting stems from the reality that the findings of a given financial decision made today is conditional on the exchange rate which will be prevailed in the upcoming period. For this reason, forecasting exchange rate is essential for various international financial transactions, namely speculation, hedging as well as capital budgeting [6].

Despite the fact that policies are being employed to ensure exchange rate stability, the Nigerian currency has continued to depreciate against the major currency of the world. Previous modeling of the Nigeria exchange rate efforts had centered on the predictable component of the series [7]. Later attention shifted to the residual whereby it is assumed to be normally distributed. This is contrary to the argument that many financial series are non-normal. In modeling exchange rate volatility, several authors in developed nations have employed different specifications, ranging from the parametric standard Autoregressive Conditional Heteroscedastic (ARCH) model and its variants such as the GARCH, Exponential GARCH, Power ARCH, Threshold ARCH, Fractional Integrated GARCH, etc. to its nonparametric counterparts; Kernel's, Fourier Series and Least Squares Regression.

In an attempt to bridge the gap in the specification of models and estimation of parameters in modeling the exchange rate volatility of the Nigerian currency, the paper seeks to apply the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model which was developed as an extension to the Autoregressive Conditional Heteroskedasticity (ARCH) model. GARCH models are used to model high frequency time series data, hence often used to forecast volatility.

Moreover, other mathematical explanations have been suggested to illuminate the issue of fat tails in modeling financial series. Therefore, the study makes an effort to apply the GARCH model to appropriately model Nigeria Naira to the European Euro, taking into account the issue of Kurtosis and error distribution assumption by incorporating the normal, student t-distribution and the generalized error distribution in the model and comparing the ability of the GARCH models capture excess kurtosis with regards to normal, student-t and the generalized error distributions.

The rest of the paper is organized as follows, section two takes care of materials and methods, section three discussed data analysis while conclusion handled in section four

2.0 MATERIALS AND METHODS

2.1 Returns

The ARCH model, and its generalization (GARCH) is widely used methodology by researchers in modeling and forecasting financial time series. We used the following model to derived the returns series

$$z_t = log \frac{p_t}{p_{t-1}} \quad (1)$$

Where:

z_t, monthly return rates on exchange rate for time t

p_t, exchange rate at time t

p_{t-1}, exchange rate at time t-1

2.2 The ARCH Model

The model was developed by [8] and the specification is highlighted below.

Let x_t be a random variable from time series observations from a sequence of identically independent random variable and be N (0,1). The process $\{z_t\}$ is an Autoregressive conditional heteroscedastic process of order p, ARCH (p), if:

$$z_t = \sigma_t x_t \quad (2)$$

Where: x_t is the error term in a time series regression model. Therefore, we get;

$$z_t = \sigma_t \varepsilon_t$$

2.3 GARCH Model

In practice, p in the ARCH(p) model is often large. A move simpler representation is the GARCH model, GARCH(p,q) is often used to investigate volatility. Thus, the ARCH model has been generalized to allow liner dependent of the conditional variance (σ_t^2), on the past value of σ_t^2 as well as on past (squared) values of the series.

2.3.1 Properties

The GARCH model of order (p,q) assumes that the conditional variance depends on the squares of the last q values of $\varepsilon_t$in the series and on the last p value of σ_t^2. Hence, it is in line with the ARMA (p,q) model.

A GARCH (p,q) process $\varepsilon_t = \sigma_t z_t$ has a mean equation $z_t = \mu + \varepsilon_t$
Where:
μ, represents the mean
$\varepsilon_{t,}$ is the error term.
The conditional variance is represented as a linear function of its own lags

$$\sigma_t^2 = \omega + a\sigma_{t-1}^2 + \beta\varepsilon_{t-1}^2 \quad (3)$$

The unconditional variance is represented as,

$$var(\varepsilon) = \frac{\omega}{1-(\alpha+\beta)}$$

Where;
$\omega + \alpha + \beta = 1$ so as to follow the stationarity condition
$\omega > 0, \quad \alpha\beta \geq 0$
ω is a constant term
ε_{t-1}^2 is the information of the past errors
σ_{t-1}^2 is the last period forecast variance
$\alpha -$ Autoregressive parameter
$\beta -$ Moving Average parameter
The general specification for the GARCH(p,q) model is

$$\sigma_t^2 = \omega + \sum_{j=1}^{p} a_j\sigma_{t-1}^2 + \sum_{i=1}^{q} \beta_i\varepsilon_{t-1}^2 \quad (4)$$

$p -$ Number of lagged variance terms
$q -$ Number of lagged squared error terms
Moreover GACH (1,2) process implies an ARMA (1,1) representation in the ε_t^2

$$\varepsilon_t^2 = \omega + (\alpha+\beta)\varepsilon_{t-1}^2 - \beta v_{t-1} + v_t \quad (5)$$

For kurtosis,

$$k = \frac{E(z_t^4)}{\left(E(z_t^4)\right)^2} \quad (6)$$

2.3.2 Distributions

Time series data like volatility always experience shocks caused by the extreme events making them difficult to be represented by a normal distribution. Because of this reason, I addition to normal distribution, we used both Generalized Error Distribution (GED) and student –t distribution to represent our data.

Generalized Error Distribution (GED)

$$f(x) = \frac{\lambda.s}{2.\Gamma\left(\frac{1}{s}\right)}\exp(-\lambda^s I z_t - \mu I^s) \quad (7)$$

Where;

$\lambda -$ scale parameter
$\mu -$ Location parameter
$\Gamma(z) -$Euleer function
$s -$shape parameter
Student –t distribution;

$$f(z_t) = \frac{\Gamma\left(\frac{(n+1)}{2}\right)}{\sqrt{n\pi}\Gamma\left(\frac{n}{2}\right)}\left(1+\frac{z_t^2}{n}\right)^{\frac{(n+1)}{2}} \quad , z_t \in R \qquad (8)$$

where μ represents $E(z_t)$

2.4 Information Criteria

There are several information criteria available to determine the order, p, of an AR process and the order, q, of MA(q) process, all of them are likelihood based. The well-known Akaike information criterion (AIC), [9] is defined as

$$\text{AIC} = \frac{-2}{T} ln(\text{likelihood}) + \frac{2}{\text{T}} \text{x (number of parameters)} \qquad (9)$$

where the likelihood function is evaluated at the maximum likelihood estimates and T the sample size. For a Gaussian AR (p) model, AIC reduces to

$$AIC(P) = ln(\hat{\sigma}_P^2) + \frac{2P}{T} \qquad (10)$$

where $\hat{\sigma}_P^2$ is the maximum likelihood estimate of $\hat{\sigma}_a^2$, which is the variance of a_t, and T is the sample size. The first term of the AIC in equation (10) measures the goodness-of-fit of the AR(p) model to the data whereas the second term is called the penalty function of the criterion because it penalizes a chosen model by the number of parameters used. Different penalty functions result in different information criteria.

The next commonly used criterion function is the Schwarz information criterion [10] (Schwarz, 1978). For a Gaussian AR(p) model, the criterion is

$$SIC(P) = ln(\hat{\sigma}_P^2) + \left(\frac{Pln(T)}{T}\right). \qquad (11)$$

3.0 Data Analysis

Data analysis will be done using EViews and R package

3.1 **Trend of the exchange rate series**

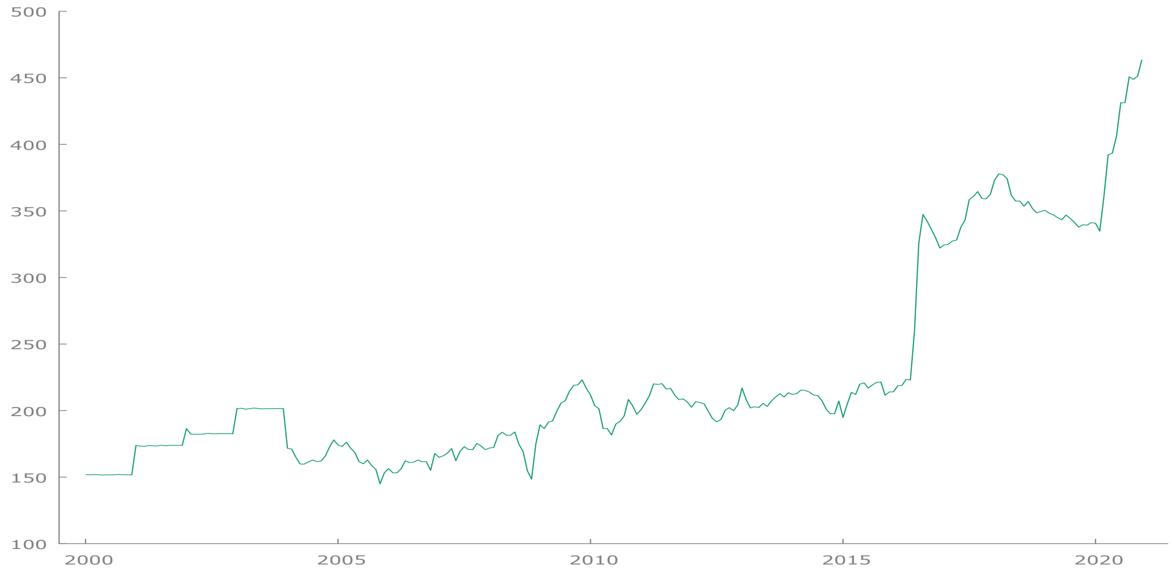


Figure 1: Time plot of Naira to Euro Exchange Rate

Figure 1 portrays the trend of the exchange rate for the period of the study. The path of Figure 1 presents a fitting picture of instability of exchange rate

3.2 Exchange Rate Returns

Exchange rate returns have been calculated from the exchange rate series of the Naira against the Euro. Returns over the period is graphically illustrated below;

Figure 6: Returns series of Naira to Pound exchange rate

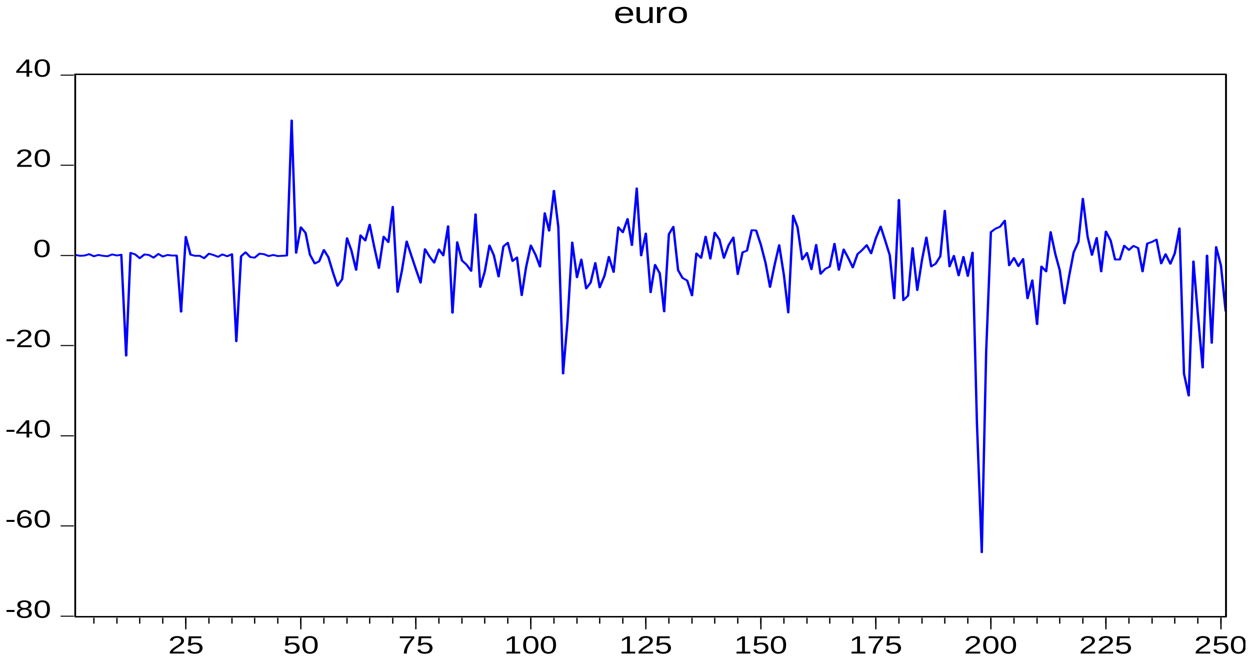
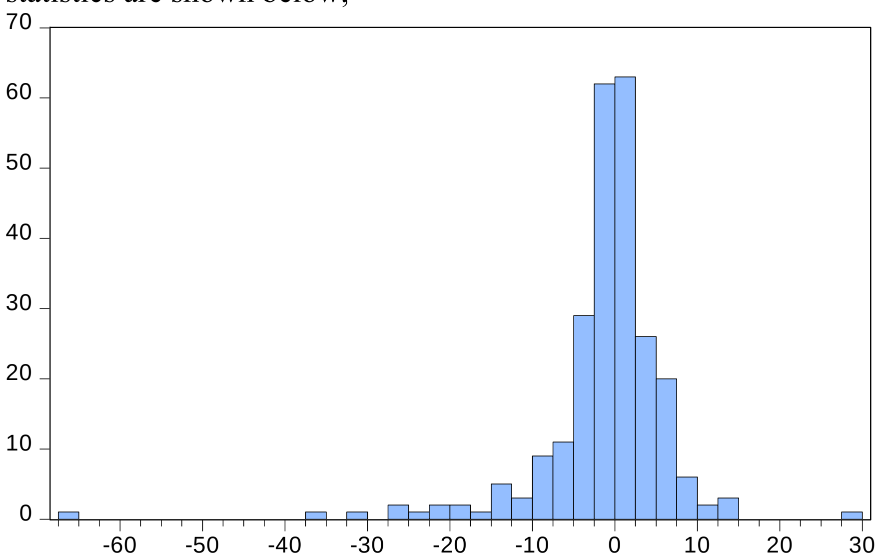


Figure 2: Returns series of Naira to Pound exchange rate

As revealed in figure 2 above the returns over the period has been volatile, taking positive and negative values with different scales. The ups and downs in returns over the period shows volatility in the stock market. Though, by merely looking at the trend robust decisions may not be drawn until a full statistical analysis is done. The result in figure 2 also revealed the return series is stationary because the series fluctuate around a common mean.

3.3 Summary Statistics

The summary statistics of exchange rate returns provide critical information about volatility. The statistics are shown below;



Series: EURO	
Sample 1 251	
Observations 251	
Mean	-1.241793
Median	-0.050000
Maximum	29.92000
Minimum	-65.87000
Std. Dev.	8.117610
Skewness	-2.809619
Kurtosis	21.34833
Jarque-Bera	3851.145
Probability	0.000000

Figure 3: Summary Statistics of the Returns Series of Naira to Euro Exchange Rate

Figure 3 summarizes the descriptive statistics of the returns series of Naira to Euro exchange rate which have negative skewness and high positive kurtosis. The negative skewness for the returns series is an indication that the lower tail of the distribution is thicker than the upper tail which implies that the returns drops more often than it rises, reflecting the lack confidence in the exchange rate market. These values also signifies that the distributions of the series has a long-left tail and leptokurtic form. The kurtosis (21.34833) of the return series is greater than 3, which implies that the return series is fat tailed and does not follow a normal distribution and is further established by Jarque-Bera test statistics, which is significant at 5% level and hence the null hypothesis of normality is rejected.

3.4 ARCH Effects Test

The ARCH-LM test is applied to find out the existence of arch effect in the residuals of the return series. From the ARCH-LM test above it is revealed that the test statistics is highly significant. Since p value is less than 0.05, the null hypothesis of 'no arch effect' is rejected at 5% level, confirming the presence of ARCH effects in the returns and hence the results permit the estimation of GARCH family models. This result was also confirmed by [11].

Table 1: Testing for Arch Effect

Currency	ARCH LM	
	Statistic	P-value
Euro	50.9249	0.0001

3.5 Estimation of GARCH Model

Nine GARCH models were applied to the returns series as shown in table 2, above, based on the information selection criteria GARCH (1, 1) with student-t was found to be the most efficient model with minimum information criteria. The parameters estimation of GARCH (1, 1)-st in table 3 revealed that all the parameters were statistically significant. From the estimates of GARCH (1,1)-st, in table 3 the findings clearly established the presence of time varying conditional volatility of the returns of exchange rate. Result from table 3 also satisfy the covariance stationary condition that the sum of the ARCH term and the GARCH term is less than one. The ARCH term 0.177453 is significance at 5% while the GARCH term 0.483662is also significant at 5% level. The implication of the positive and significant values of the coefficient of ARCH and GARCH term is that previous month's exchange information (ARCH) can influence the present month's naira exchange rate volatility with respect to the euro. On the other hand, the significant GARCH term also means that the previous month's exchange rate volatility can influence the present month volatility.

Table 2: GARCH Model Selection for Return Series of Naira to Euro Exchange Rate

Model	Probability	Best fit		
		AIC	BIC	
GARCH (1,1)	Normal	6.738420	6.808849	
	Std	6.068261	6.152777	
	Ged	6.153670	6.238185	
GARCH (1,2)	Normal	6.742777	6.827292	GARCH(1,1)std
	Std	6.185045	6.283645	
	Ged	6.156725	6.255326	
GARCH (2, 1)	Normal	6.742777	6.827292	
	Std	6.185045	6.283645	
	Ged	6.156725	6.557525	

Table 3: Parameter Estimates of GARCH (1,1)-std Model of Naira to Euro Returns Series

Model	Equation	Parameter	Coefficient	p-value
	Mean	Intercept	0.019786	0.0075
		Lag value	0.019219	0.0136
GARCH(1,1)std		Constant	0.442124	0.0092

ISSN 2320-9186

Var.	ARCH	0.177453	0.0012
	GARCH	0.483662	0.0000

3.6 Model Diagnostics of GARCH (1,1)-std for Naira to Euro Return Series

Goodness of fit of the ARCH-GARCH model is built on residuals. The residuals are presumed to be independently and identically distributed following a normal distribution [12] and [13]). If the model fits the data well the residuals should be approximately symmetric. The Lagrange Multiplier's test and the ACF and the PACF of the standardized residuals are used for checking the adequacy of the conditional variance model in our work. Having established that the model fits the data well, the fitted model can be used for forecasting.
The results of these tests are given in Table 4. Using the ARCH-LM test, the residuals show no heterocedasticity left in the residual with all p-values greater than 0.01 and 0.05 for most of the models. The ACF and PACF from table 5 also show no serial correlation in the residual up to the 15th lag. These results confirmed the adequacy of the fitted model.

Table 4: Heteroskedasticity Test: ARCH for GARCH (1,1)-std for Naira to Euro Return Series

F-statistic	0.007007	Prob. F(1,247)	0.9334
Obs*R-squared	0.007064	Prob. Chi-Square(1)	0.9330

Table 5: Autocorrelation in the Standardized Residual Squared of GARCH (1,1) model of Naira to Dollar Return Series

	AC	PAC	Q-Stat	Prob
1	0.012	0.012	0.0352	0.851
2	-0.050	-0.050	0.6589	0.719
3	0.050	0.052	1.3023	0.729
4	-0.035	-0.039	1.6102	0.807
5	-0.017	-0.011	1.6876	0.890
6	0.045	0.039	2.2039	0.900
7	-0.072	-0.072	3.5660	0.828
8	-0.045	-0.039	4.1039	0.848
9	0.027	0.016	4.2914	0.891

10 0.014	0.019	4.3437	0.931
11 -0.005	-0.003	4.3507	0.959
12 0.003	-0.004	4.3533	0.976
13 -0.009	-0.004	4.3730	0.987
14 0.021	0.022	4.4908	0.992
15 -0.026	-0.036	4.6781	0.995

3.7 Comparative Analysis of the Value of Excess Kurtosis Captured by the GARCH Models in Propinquity to Theoretical Value of kurtosis of Naira to Euro Returns

Table 6: The kurtosis captured by the GARCH models for Naira to Euro in Comparison with theoretical value of Kurtosis

Model	Theoretical value of Kurtosis	Value of kurtosis captured by the GARCH Model
GARCH (1,1)-norm	3	24.8745.
GARCH (1,1)-std	3	11. 0876
GARCH (1,1)-ged	3	21.9840

From table 6 above, GARCH (1,1)-std perform better than the other models by capturing the value of excess kurtoses in closer proximity to the theoretical value than the other competing models

4 Conclusion

Three different GARCH models (GARCH (1,1), GARCH (1,2) and GARCH (2,1)) with normal distribution, student t distribution and the generalized error distribution were selected for the naira to euro returns through a search algorithm that tries a number of different coefficients before converging on the optimum values. Empirical analysis of the models from table 2, revealed that GARCH (1,1)-std was the best model based on minimum information criteria. diagnostic analysis of the model revealed that the model was correctly specified. The result of the ARCH test and the autocorrelation test in table 4 and table 5 respectively confirmed that the residuals of the model are white noise. The closeness of kurtosis value captured by the GARCH (1,1) with different distributions (normal distribution, student t distribution and the generalized error distribution) model for naira to euro return presents in table 6 revealed that GARCH (1,1)-std was more effective in capturing the excess kurtosis in closeness to the theoretical value of kurtosis for naira to dollar return.

ISSN 2320-9186

The GARCH-std models provided a better fit than GARCH with other distributions because it was able capture excess kurtosis in close proximity to the theoretical value of kurtosis. The fitted models were able to account serial correlation and heteroscedasticity in the residual. Also the results showed that the conditional variance is a persistence process for the currency considered. This result is in consonance with those of other developing markets where significant persistence of volatility is observed for the exchange rate.

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