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Modeling Significance Impact of Vaccination of Meningitis Transmission Dynamics on Children in North-western Nigeria

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ABSTRACT

In this paper, mathematical model for the transmission dynamics of meningitis to determine the significance impact of vaccination by interchanging the rate of carriers and vaccination. The model describes meningitis transmission designed into compartments which leads to a linear system of differential equations; the model used 2017 data on meningitis outbreaks on children in the northwestern Nigeria. The model distinguished between female and male children population: between infected females and males, recovered with deficiency and without deficiency. All Eigen-values are found negatives and R_0 (the threshold parameter) is greater than 1 Equilibrium points are found and their stability based on a threshold value (R_0) are investigated, the result based on the analysis shows that the significance impact of vaccination during outbreak is negative or insignificant on meningitis transmission dynamics during outbreak. Since, since R_0 is greater than 1 the endemic equilibrium is stable both locally and globally.

Key words: Meningitis, Infected Carrier, Vaccination and Threshold value.

INTRODUCTION

Vaccination plays a vital role in preventing many infectious diseases by reducing carriage rate. It can be used to reduce the rate of infection. Chiyaka et al, (2015) Developed (Susceptible, Vaccinated, Exposed, Infected, Removed [SVEIR]) mathematical model and determined the impact of vaccination on malaria epidemics and carried out equilibria and stability analysis and concluded that vaccine reduces the secondary infection. Elmojtaba & Adam (2017) carried out (Susceptible, Vaccinated, Carrier, Infected, Removed, Survivors with dis- ability [SVCIRS]) model to study the dynamics of the meningitis diseases, modelling was used and studied the effect of vaccine and numerical simulation was conducted that the percentage of 10% of the infected population and 20% of the survivors will have disability.

Gumel et al, (2016) Formulate mathematical model which incorporates five compartment model such as (Susceptible, Vaccinated, Exposed, Infected, Removed [SVEIR]) model which described the transmission dynamics and control of severe acute respiratory syndrome (SARS) that spreads over 32 countries during 2003. The research analysis revealed that exposure to verilla vaccines will boost the immunity and protect against reactivation of the virus. Gray et al. (2009) developed model and investigated the impact of vaccines on chlamydia trichromatic (chlamydia) infection and concluded that vaccine will reduce the infection.

Broutin et al. (2010) conducted a study to determine the impact of vaccination on Bordello pertussis infection hence vaccination described as the best tool to improved strong herd immunity. Bolker & Grenfell (1996) Conducted stochastic models to study the impact of vaccination using temporal correlation of epidemic pattern between major cities in England and Wales in 1968.

Materials and Methods

A mathematical model which will be formulated using differential equations based on the epidemiological compartment modeling used is the proposed S, C, If, Im, Rd, Rn model for this paper, with carrier rate, which will be incorporated into the model. Data collected for the 2017 meningitis outbreaks in the north western Nigeria will be used.

Model Formulation and Analysis

In this model population is divided into seven categories, susceptible children, vaccinated female children, vaccinated male children, infected male children, infected female children, recovered with deficiency and recovered without deficiency as developed from the first model (S; C; V; I_F; I_M; R_D; R_N) in order to describe meningitis transmission dynamics with vaccination based on the data obtained on meningitis out breaks in the northern Nigeria, to determine the significant impact of Vaccination on meningitis transmission dynamics.

Description of the Model Variables and Parameters

The following tables describe the variables and parameters used in this model:

Table 1: Variables used in the model

Model Variable	Description	Value	Source
S(t)	Susceptible Population	14,624,404	NCDC, 2017
$C(t)$	Natural carrier	3,656,104	Estimated
$V(t)$	Vaccinated	7,312,207	Estimated
$I_F(t)$	Infected females	9,934	NCDC, 2017
$I_m(t)$	Infected males	11,661	NCDC, 2017
$R_d(t)$	Recovered without deficiency	2,160	NCDC, 2017
$R_n(t)$	Recovered with deficiency	4,319	NCDC, 2017

Table 2: Parameters used in the model used in the model

Model Parameters	Description	Value	Source
Λ	Recruitment rate	0.41	NPC
α	Death rate due to meningitis	0.01	NCDC, 2017
μ	Natural death rate	0.02	NCDC, 2017
β	Natural Carrier rate	0.25	Coen, 2000
ϖ	Rate of return to carrier	0.025	Estimated
ϵ_1	Rate of female contact	0.036	Estimated
ϵ_2	Rate of male contact	0.052	Estimated
ψ_1	Rate of return to susceptible from R_d	0.045	Estimated

ψ_2	Rate of return to susceptible from R_n	0.055	Estimated
δ	Rate of moving from R_d to R_n	0.05	Estimated
ρ_1	Rate of female recovery from inf.	0.045	Estimated
ρ_2	Rate of male recovery from inf.	0.035	Estimated
χ	Rate of F return to S after vaccination	0.025	Estimated
ζ	Rate of M return to S after vaccination	0.035	Estimated
κ_1	Rate of F moving to I from Carrier	0.042	Estimated
κ_2	Rate of M moving to I from carrier	0.032	Estimated
τ_1	Rate of F moving to I from vaccine	0.045	Estimated
τ_2	Rate of M moving to I from vaccine	0.035	Estimated

Compartmental Diagram

The Meningitis transmission dynamics between compartments shall be described by a system of differential equations such shall be solved to find and obtain the disease free equilibrium, endemic equilibrium and stability analysis using a threshold parameter R_0, this termed as the main objective which will be achieved By developing simple S; C; V; I_F ; I_M; R_D; R_N Model. The compartments are: Susceptible, Carrier, vaccination, infected female, infected male, recovered with deficiency and recovered without deficiency. Other transaction such as the condition of death due to infection and natural death are modelled in linear terms with constant coefficients. Model is a deterministic compartmental S; C; V; I_F ; I_M; R_D; R_N, Model-type model.

Thus, the compartmental diagram for the deterministic model is as follows;

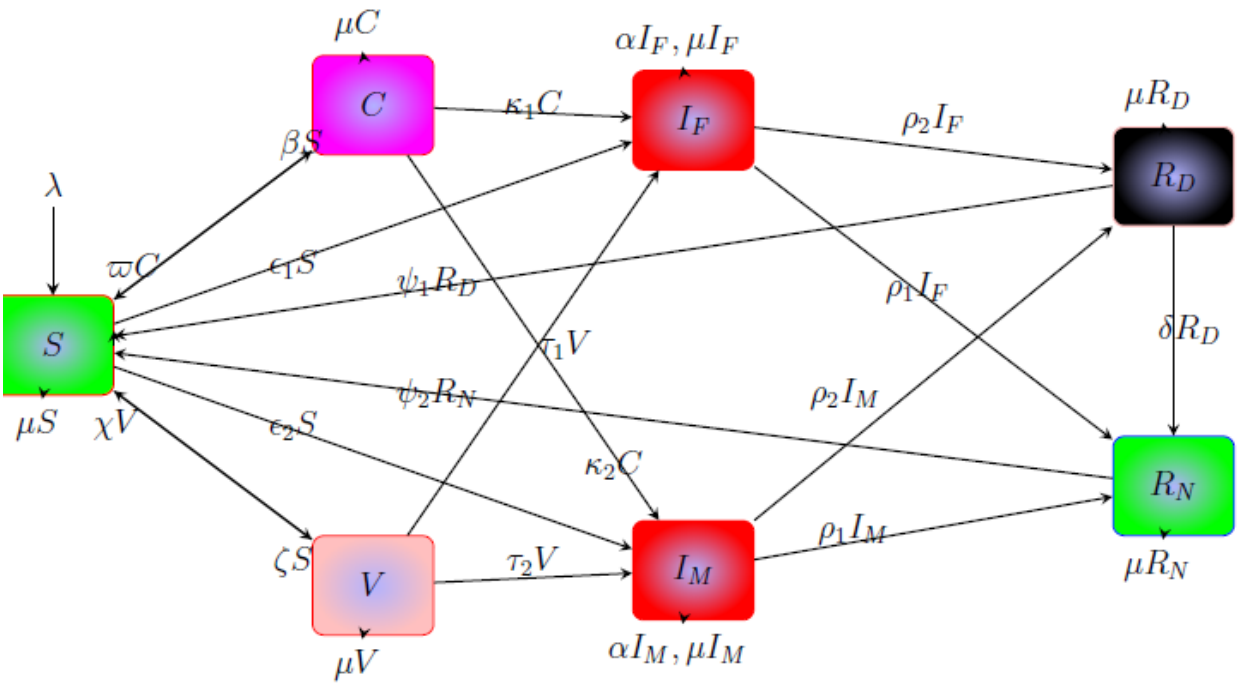

Figure 4.1: The flowchart diagram describing bacterial meningitis transmission dynamics within the population. The seven squares represent the seven compartments of individuals, the movement between the compartments is indicated by the continuous arrows, V is a control measure vaccination , with the consideration that both control measures lies in, $0 < V; V \leq 1$.

Model Equation

The model equations are given below:-

$$\frac{\delta_S}{\delta_t} = \Lambda - \beta S + \varpi C + \psi_1 R_D + \psi_2 R_N + \chi V - \epsilon_1 S - \epsilon_2 S - \zeta S - \mu_S \ldots \text{(I)}$$

$$\frac{\delta_C}{\delta_t} = \beta S - \kappa_1 C - \kappa_2 C - \mu C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(II)}$$

$$\frac{\delta_V}{\delta_t} = \zeta S + \chi V + \tau_1 V + \tau_2 V - \mu V \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(III)}$$

$$\frac{\delta_{I_f}}{\delta_t} = \kappa_1 \mathrm{C} + \epsilon_1 \mathrm{S} + \tau_1 \mathrm{V} - \rho_2 I_f - \rho_1 I_f - \alpha I_f - \mu I_f \ldots\ldots\ldots\ldots\ldots\ldots \text{(IV)}$$

$$\frac{\delta_{I_m}}{\delta_t} = \kappa_2 \mathrm{C} + \epsilon_2 \mathrm{S} + \tau_2 \mathrm{V} - \rho_2 I_M - \rho_1 I_M - \alpha I_M - \mu I_M \ldots\ldots\ldots\ldots\ldots\ldots \text{(V)}$$

$$\frac{\delta R_d}{\delta_t} = \rho_2 I_f + \rho_2 I_m - \psi_1 R_D - \delta R_d - \mu R_d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(VI)}$$

$$\frac{\delta R_N}{\delta_t} = \rho_1 I_f + \rho_1 I_m + \delta R_d - \psi_2 R_N - \delta R_N \ldots\ldots\ldots\ldots\ldots\ldots \text{(VII)}$$

BASIC PROPERTIES OF THE MODEL

Existence and uniqueness of solution

For the mathematical model to predict the future of the system from its current state at time t_0, the initial value problem (IVP)

$x' = f(t,x), \qquad x(t_0) = x_0$.....(XII)

Must have a solution that exist and also unique.

In this section, the conditions for the existence and uniqueness of solution for the model system of equation shall be established. Let

$$f_1(t,x) = \lambda + \varpi C + \psi R_D + \psi R_N + \chi_V - \beta_S - \varepsilon 1_S - \varepsilon 2_S - \zeta_S - \mu S \ldots\ldots (XIII)$$

$$f_2(t,x) = \beta S - \kappa 1_C - \kappa 2_C - \mu C \ldots\ldots\ldots (XIV)$$

$$f_3(t,x) = \zeta_S - \tau_1 V - \tau_2 V - \mu V \ldots\ldots\ldots (XV)$$

$$f_4(t,x) = \kappa_1 C + \varepsilon_1 S + \tau_1 V - \rho_2 I_F - \rho_1 I_F - \alpha I_F - \mu I_F \ldots\ldots\ldots (XVI)$$

$$f_5(t,x) = \kappa_2 C + \varepsilon_2 S + \tau_2 V - \rho_2 I_M - \rho_1 I_M - \alpha I_M - \mu I_M \ldots\ldots\ldots (XVII)$$

$$f_6(t,x) = \rho_2 I_F + \rho_2 I_M - \delta R_D - \psi_1 R_D - \mu R_D \ldots\ldots\ldots (XVIII)$$

$$f_7(t,x) = \rho_1 I_F + \rho_1 I_M + \delta R_D - \psi_2 R_N - \mu R_N \ldots\ldots\ldots (XIX)$$

So that

$$x' = f(t,x), \qquad x(t_0) = f(x) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (XX)$$

Theorem 1, (Momoh et al, 2013) Let D' denotes the region

$|t - t_0| \leq a, \|x - x_0\| \leq b,$

$x = (x_1, \quad x_2, \quad \ldots\ldots x_n),$

$x_0 = (X_{10}, X_{20}, \ldots\ldots X_{n0})$.....(XXI)

And suppose that $f(t,x)$ satisfies the Lipchitz condition

$\|f(t,x_1) - f(t,x_2)\| \leq k\|x_1 - x_2\|$,.....(XXII)

Whenever the pairs $(t,x_1)\ and\ (t,x_2)$ belongs to D', where k is a positive constant. Then, there exist a constant $\delta > 0$ such that there exist a unique continuous vector solution $\bar{x}(t)$ of the system (XI) in the interval $|t - t_0| \leq \delta$.

It is important to note that condition (XXII) is satisfied by requirement that $\frac{\partial fi}{\partial xj}, i, j = 1, 2, \ldots . n$ be continuous and bounded in $D^{'}$.

Lemma 2. If $f(t, x)$ has continuous partial derivatives $\frac{\partial fi}{\partial xj}$ on a bounded closed covex domain R, then it satisfies a Lipchitz condition in R.

Being interested in the region

$1 \leq \varepsilon \leq R$……..……..(XXIII)

By looking for a bounded solution of the form

$0 < R < \infty$……(XXIV),

Following the proving of existence theorem

Theorem 2. Let $D^{'}$ denote the region in (XX) such that (XXI) and (XXII) hold. Then there exist a solution of model system (XII) – (XVII) which is bounded in the region $D^{'}$.

Proof.

$$f_1 = \lambda + \varpi C + \psi R_D + \psi R_N + \chi_V - \beta_S - \varepsilon 1_S - \varepsilon 2_S - \zeta_S - \mu S$$

$$f_2 = \beta S - \kappa 1_C - \kappa 2_C - \mu C$$

$$f_3 = \zeta_S - \tau_1 V - \tau_2 V - \mu V$$

$$f_4 = \kappa_1 C + \varepsilon_1 S + \tau_1 V - \rho_2 I_F - \rho_1 I_F - \alpha I_F - \mu I_F$$

$$f_5 = \kappa_2 C + \varepsilon_2 S + \tau_2 V - \rho_2 I_M - \rho_1 I_M - \alpha I_M - \mu I_M$$

$$f_6 = \rho_2 I_F + \rho_2 I_M - \delta R_D - \psi_1 R_D - \mu R_D$$

$$f_7 = \rho_1 I_F + \rho_1 I_M + \delta R_D - \psi_2 R_N - \mu R_N$$

It suffices to show that $\frac{\delta fi}{\delta xj}, i, j = 1, 2, 3, 4$ are continuous

Consider the partial derivatives

$$\frac{\delta f1}{\delta s} = -\beta - \varepsilon_1 - \varepsilon_2 - \zeta - \mu, \left|\frac{\delta f1}{\delta s}\right| = |-\beta - \varepsilon_1 - \varepsilon_2 - \zeta - \mu| < \infty$$

$$\frac{\delta f1}{\delta C} = \varpi, \left|\frac{\delta f1}{\delta C}\right| = |\varpi| < \infty$$

$$\frac{\delta f1}{\delta V} = \chi, \left|\frac{\delta f1}{\delta V}\right| = |\chi| < \infty$$

$$\frac{\delta f1}{\delta I_F} = 0, \left|\frac{\delta f1}{\delta I_F}\right| = |0| < \infty$$

$$\frac{\delta f1}{\delta I_M} = 0, \left|\frac{\delta f1}{\delta I_M}\right| = |0| < \infty$$

$$\frac{\delta f1}{\delta R_D} = \psi_1, \left|\frac{\delta f1}{\delta R_D}\right| = |\psi_1| < \infty$$

$$\frac{\delta f1}{\delta R_N} = \psi_2, \left|\frac{\delta f1}{\delta R_N}\right| = |\psi_2| < \infty$$

Similarly

$$\frac{\delta f2}{\delta s} = \beta, \left|\frac{\delta f2}{\delta s}\right| = |\beta| < \infty$$

$$\frac{\delta f2}{\delta C} = -\kappa_1 - \kappa_2 - \mu, \left|\frac{\delta f1}{\delta s}\right| = |-\kappa_1 - \kappa_2 - \mu| < \infty$$

$$\frac{\delta f1}{\delta V} = 0, \left|\frac{\delta f1}{\delta V}\right| = |0| < \infty$$

$$\frac{\delta f2}{\delta I_F} = 0, \left|\frac{\delta f2}{\delta I_F}\right| = |0| < \infty$$

$$\frac{\delta f2}{\delta I_M} = 0, \left|\frac{\delta f2}{\delta I_M}\right| = |0| < \infty$$

$$\frac{\delta f2}{\delta R_D} = 0, \left|\frac{\delta f2}{\delta R_D}\right| = |0| < \infty$$

$$\frac{\delta f2}{\delta R_N} = 0, \left|\frac{\delta f2}{\delta R_N}\right| = |0| < \infty$$

The same way

$$\frac{\delta f3}{\delta s} = \zeta, \left|\frac{\delta 31}{\delta s}\right| = |\zeta| < \infty$$

$$\frac{\delta f3}{\delta C} = 0, \left|\frac{\delta f3}{\delta C}\right| = |0| < \infty$$

$$\frac{\delta f1}{\delta V} = -\tau_1 - \tau_2 - \mu, \left|\frac{\delta f1}{\delta V}\right| = |-\tau_1 - \tau_2 - \mu| < \infty$$

$$\frac{\delta f3}{\delta I_F} = 0, \left|\frac{\delta f3}{\delta I_F}\right| = |0| < \infty$$

$$\frac{\delta f3}{\delta I_M} = 0, \left|\frac{\delta f3}{\delta I_M}\right| = |0| < \infty$$

$$\frac{\delta f3}{\delta R_D} = 0, \left|\frac{\delta f3}{\delta R_D}\right| = |0| < \infty$$

$$\frac{\delta f3}{\delta R_N} = 0, \left|\frac{\delta f3}{\delta R_N}\right| = |0| < \infty$$

Also

$$\frac{\delta f4}{\delta s} = \epsilon_1, \left|\frac{\delta f4}{\delta s}\right| = |\epsilon_1| < \infty$$

$$\frac{\delta f4}{\delta C} = \kappa_1, \left|\frac{\delta f4}{\delta s}\right| = |\kappa_1| < \infty$$

$$\frac{\delta f4}{\delta V} = \tau_1, \left|\frac{\delta f4}{\delta s}\right| = |\tau_1| < \infty$$

$$\frac{\delta f4}{\delta I_F} = -\rho_1 - \rho_2 - \alpha - \mu, , \left|\frac{\delta f4}{\delta I_F}\right| = |-\rho_1 - \rho_2 - \alpha - \mu, | < \infty$$

$$\frac{\delta f4}{\delta I_M} = 0, \left|\frac{\delta f4}{\delta I_M}\right| = |0| < \infty$$

$$\frac{\delta f4}{\delta R_D} = 0, \left|\frac{\delta f4}{\delta R_D}\right| = |0| < \infty$$

$$\frac{\delta f4}{\delta R_N} = 0, \left|\frac{\delta f4}{\delta R_N}\right| = |0| < \infty$$

Similarly

$$\frac{\delta f5}{\delta s} = \epsilon_2, \left|\frac{\delta f5}{\delta s}\right| = |\epsilon_2| < \infty$$

$$\frac{\delta f5}{\delta C} = \kappa_2, \left|\frac{\delta f5}{\delta s}\right| = |\kappa_2| < \infty$$

$$\frac{\delta f5}{\delta V} = \tau_2, \left|\frac{\delta f5}{\delta s}\right| = |\tau_2| < \infty$$

$$\frac{\delta f5}{\delta I_F} = 0, \left|\frac{\delta f5}{\delta I_F}\right| = |0| < \infty$$

$$\frac{\delta f5}{\delta I_M} = -\rho_2 - \rho_1 - \alpha - \mu \left|\frac{\delta f5}{\delta I_M}\right| = |-\rho_2 - \rho_1 - \alpha - \mu| < \infty$$

$$\frac{\delta f5}{\delta R_D} = 0, \left|\frac{\delta f5}{\delta R_D}\right| = |0| < \infty$$

$$\frac{\delta f5}{\delta R_N} = 0, \left|\frac{\delta f5}{\delta R_N}\right| = |0| < \infty$$

Also

$$\frac{\delta f6}{\delta s} = 0, \left|\frac{\delta f6}{\delta s}\right| = |0| < \infty$$

$$\frac{\delta f6}{\delta C} = 0, \left|\frac{\delta f6}{\delta C}\right| = |0| < \infty$$

$$\frac{\delta f6}{\delta V} = 0, \left|\frac{\delta f6}{\delta V}\right| = |0| < \infty$$

$$\frac{\delta f6}{\delta I_F} = \rho_2, \left|\frac{\delta f6}{\delta I_F}\right| = |\rho_2| < \infty$$

$$\frac{\delta f6}{\delta I_M} = \rho_2, \left|\frac{\delta f6}{\delta I_M}\right| = |\rho_2| < \infty$$

$$\frac{\delta f6}{\delta R_D} = -\psi_1 - \delta - \mu, \left|\frac{\delta f6}{\delta R_D}\right| = |-\psi_1 - \delta - \mu| < \infty$$

$$\frac{\delta f6}{\delta R_N} = 0, \left|\frac{\delta f6}{\delta R_N}\right| = |0| < \infty$$

Finally becomes

$$\frac{\delta f7}{\delta s} = 0, \left|\frac{\delta f6}{\delta s}\right| = |0| < \infty$$

$$\frac{\delta f7}{\delta C} = 0, \left|\frac{\delta f6}{\delta C}\right| = |0| < \infty$$

$$\frac{\delta f7}{\delta V} = 0, \left|\frac{\delta f6}{\delta V}\right| = |0| < \infty$$

$$\frac{\delta f7}{\delta I_F} = \rho_1, \left|\frac{\delta f6}{\delta I_F}\right| = |\rho_1| < \infty$$

$$\frac{\delta f7}{\delta I_M} = \rho_1, \left|\frac{\delta f6}{\delta I_M}\right| = |\rho_1| < \infty$$

$$\frac{\delta f7}{\delta R_D} = \delta, \left|\frac{\delta f6}{\delta R_D}\right| = |\delta| < \infty$$

$$\frac{\delta f7}{\delta R_N} = -\psi_2 - \mu, \left|\frac{\delta f6}{\delta R_N}\right| = |-\psi_2 - \mu| < \infty$$

Clearly, all these partial derivatives are continuous and bounded, hence, by theorem (2), there exist a unique solution of (XII) – (XVIII) in the region D'.

Feasible region

All parameters of the model are assumed to be non-negative. On the other hand, model system above, monitors' human population, so the state variables are non-negative for all time t ≥ 0. Total population can be written as:

$N(t) = S(t) + C(t) + V(t) + I_F(t) + I_M(t) + R_D(t) + R_N(t)$……(VIII)

Here the eq. (VIII) is changing at a rate

$$\frac{\delta_N}{\delta_t} = \frac{\delta_S}{\delta_t} + \frac{\delta C}{\delta_t} + \frac{\delta V}{\delta_t} + \frac{\delta I_f}{\delta_t} + \frac{\delta I_m}{\delta_t} + \frac{\delta R_D}{\delta_t} + \frac{\delta R_N}{\delta_t} = \Lambda - \mu N \text{ ……..} \text{ ……(IX)}$$

In the absence of the disease ie, for $I_F = I_M = R_D = R_N = 0$ which become

$$\frac{\delta_N}{\delta_t} \leq \Lambda - \mu N \text{……..(X)}$$

By the separation of variables of differentials inequality eq. (X) become

$$\frac{\delta_N}{\Lambda - \mu N} \leq \delta_t \text{……(XI)}$$

Integrating the above equation we have:

$$\Lambda - \mu N \geq Ce^{-\mu t} \text{……..……..(XII)}$$

Where C is a constant which is to be determined,

Let at t = 0, N = N_0, so this become

$C = \Lambda - \mu N_0$……………………………………………………………………………(XIII)

from equation eq. (XIII) we have

$$\Lambda - \mu N \geq (\Lambda - \mu N_0)\, e^{-\mu t}$$

$N_{(t)} \leq \left[\frac{\Lambda - \mu N_0}{\mu}\right] e^{-\mu t}$ ………………………………………………………………….(XIV)

$$t \to \infty, 0 \leq N_{(t)} \leq \frac{\Lambda}{\mu}$$

Therefore, the feasible solutions set of the equation system eq. (I-VII) enters the region.

$$\Omega = (S, C, V, I_F, I_M, R_D, R_N) \epsilon \mathbb{R}^7 \to \frac{\Lambda}{\mu}$$

MODEL ANALYSIS

Disease free equilibrium

The disease free equilibrium points are steady solution where there is no disease. Hence the disease free equilibrium exists when V, C, I_F, I_M, R_D, R_N are set to 0, that is S= C= V= I_F= I_M= R_D= $R_N = 0$. Let $N(t)$ represent the human population during the disease free equilibrium which can be written as $\frac{dN}{dt} = \lambda - \mu S$.

Stability of a Disease Free Equilibrium

To understand how the parameters affect the meningitis model, the stability nature of the Disease Free Equilibrium is analyzed by finding the Jacobian matrix for the $S, V, C, I_F, I_M, R_d, R_n$ system. Jacobian matrix is used in order to determine the local stability of the disease free equilibrium $p_0 = (\frac{\lambda}{\mu}, 0, 0, 0,0,0,0)$.

Evaluation of the stability of the disease-free equilibrium P_0, by jacobian matrix

The evaluation follows

Jacobian matrix at $p_0 = (\frac{\lambda}{\mu}, 0, 0,0, 0,0,0)$

$J(P_0) =$

$$J(P_0) = \begin{bmatrix} -\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\alpha-\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha-\mu & 0 & 0 & \epsilon-\theta & 0 \\ 0 & 0 & 0 & -\alpha-\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\alpha-\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\psi & -\delta \\ 0 & 0 & 0 & 0 & 0 & \delta & 0 \end{bmatrix} \text{.....................................(XXX)}$$

Thus the characteristic equation is given as

$$|J(P_0 - \lambda)|$$

$$J(P_0) = \begin{bmatrix} -\mu-\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\alpha-\mu-\lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha-\mu-\lambda & 0 & 0 & \epsilon-\theta & 0 \\ 0 & 0 & 0 & -\alpha-\mu-\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\alpha-\mu-\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\psi-\lambda & -\delta \\ 0 & 0 & 0 & 0 & 0 & \delta & -\lambda \end{bmatrix} = 0$$

.(XXX1)

From here the eigenvalues of $J(P_0)$ can be obtained as

$$\lambda_1 = -\mu - \lambda$$
$$\lambda_2 = -\alpha - \mu - \lambda$$
$$\lambda_3 = -\alpha - \mu - \lambda$$
$$\lambda_4 = -\alpha - \mu - \lambda$$
$$\lambda_5 = -\alpha - \mu - \lambda$$
$$\lambda_6 = -\psi - \lambda$$
$$\lambda_7 = -\lambda$$

Since all eigen-values are negatives, it implies that the disease free equilibrium point is locally asymptotically stable if $R_0 < 1$. It is unstable if $R_0 > 1$.

The Threshold Parameter

The threshold parameter defined $R_0 = \frac{1}{S^*} * \frac{\lambda}{\mu}$ according to (Momoh et al, 2013) approach as the parameter that is used to determine the equilibria.

DURING OUTBREAK

Theorem: if $R_0 > 1$, then P^* is globally asymptotically stable with respect to the interior of Ω. Considering normal carrier rate which is 25% and the proportion of 25% is vaccinated to eliminate meningitis infection. Therefore,

$$R_0 = \frac{1}{S^*} * \frac{\lambda}{\mu}$$
$$S^* = \frac{\Lambda + \varpi C + \psi_1 R_D + \psi_2 R_N + \chi V}{\xi + \beta + \epsilon_1 + \epsilon_2 + \mu}$$
$$= \frac{14{,}624{,}414 + 3.656{,}104 + 7{,}312{,}207 + 2{,}160 + 4{,}319}{0.5 + 0.25 + 0.35 + 0.4 + 0.02}$$
$$= \frac{25{,}599{,}204}{1.52}$$

$$\boldsymbol{S}^* = 16{,}841{,}582$$

$$R_0 = \frac{1}{S^*} * \frac{\lambda}{\mu}$$

$$\frac{1}{16{,}841{,}582} * \frac{14{,}624{,}414}{0.02}$$

$$= \frac{14{,}624{,}414}{336{,}832}$$

$$= 43.41753159 \approx 43 > 1$$

Since $R_0 > 1$, therefore the disease will persist

Considering normal carrier rate which is 60% and the proportion of 60% is vaccinated to eliminate meningitis infection. Therefore,

$$R_0 = \frac{1}{S^*} * \frac{\lambda}{\mu}$$

$$S^* = \frac{\Lambda + \varpi C + \psi_1 R_D + \psi_2 R_N + \chi V}{\xi + \beta + \epsilon_1 + \epsilon_2 + \mu}$$

$$= \frac{14{,}624{,}414 + 8.774{,}648 + 11{,}699{,}531 + 2{,}160 + 4{,}319}{0.5 + 0.25 + 0.35 + 0.4 + 0.02}$$

$$= \frac{19{,}288{,}501}{1.52}$$

$$\boldsymbol{S}^* = 19{,}288{,}501$$

$$R_0 = \frac{1}{S^*} * \frac{\lambda}{\mu}$$

$$\frac{1}{19{,}288{,}501} * \frac{14{,}624{,}414}{0.02}$$

$$= \frac{14{,}624{,}414}{385{,}770}$$

$$= 37.90967157 \approx 38 > 1$$

Since $R_0 > 1$, therefore the disease will persist

Considering normal carrier rate which is 90% and the proportion of 100% is vaccinated to eliminate meningitis infection. Therefore,

$$R_0 = \frac{1}{S^*} * \frac{\lambda}{\mu}$$

$$S^* = \frac{\Lambda + \varpi C + \psi_1 R_D + \psi_2 R_N + \chi V}{\xi + \beta + \epsilon_1 + \epsilon_2 + \mu}$$

$$\frac{14{,}624{,}414 + 13{,}161{,}973 + 14{,}624{,}414 + 2{,}160 + 4{,}319}{1 + 0.9 + 0.05 + 0.05 + 0.02}$$

$$= \frac{42{,}417{,}280}{2.02}$$

$$\boldsymbol{S}^* = 20{,}998{,}653$$

$$R_0 = \frac{1}{S^*} * \frac{\lambda}{\mu}$$

$$\frac{1}{20{,}998{,}653} * \frac{14{,}624{,}414}{0.02}$$

$$= \frac{14{,}624{,}414}{419{,}973}$$

$$= 34.82227101 \approx 34 > 1$$

= Since $R_0 > 1$, therefore the disease will persist

BEFORE OUTBREAK

Theorem: if $R_0 > 1$, then P^* is globally asymptotically stable with respect to the interior of Ω. Considering natural carrier rate which is 25% .Therefore,

$$S^* = \frac{\lambda + \varpi C}{\beta + \mu}$$

$$= \frac{14{,}624{,}414 + 3{,}656{,}104}{0.25 + 0.02}$$

$$= 67{,}705{,}622$$

$$R_0 = \frac{1}{S^*} * \frac{\lambda}{\mu}$$

$$= \frac{1}{67{,}705{,}622} * \frac{14{,}624{,}414}{0.02}$$

$$= 10.800000325 \approx 11 > 1$$

Since R0 > 1, therefore the disease will persist therefore; it is observed from the threshold that during the outbreaks that $R_0 = 44$ when natural carrier and vaccination rate are 25%. Also, when carrier and vaccination rate are increased to 60%, and $R_0 = 38$ when carrier rate is finally increased to 90%, $R_0 = 35$ when there is no outbreak, $R_0 = 11$ therefore, these indicate that carrier negative effect is mostly observed during outbreak.

NUMERICAL SIMULATION

Numerical simulation of this model is carried out. The key parameters were used to investigate the meningitis transmission dynamics as well as the insignificance impact of vaccination on the meningitis transmission from the data collected during 2017 epidemic outbreak. Based on the graph obtained, it can be observed that the vaccination rates keep increasing while the susceptible population decreases.

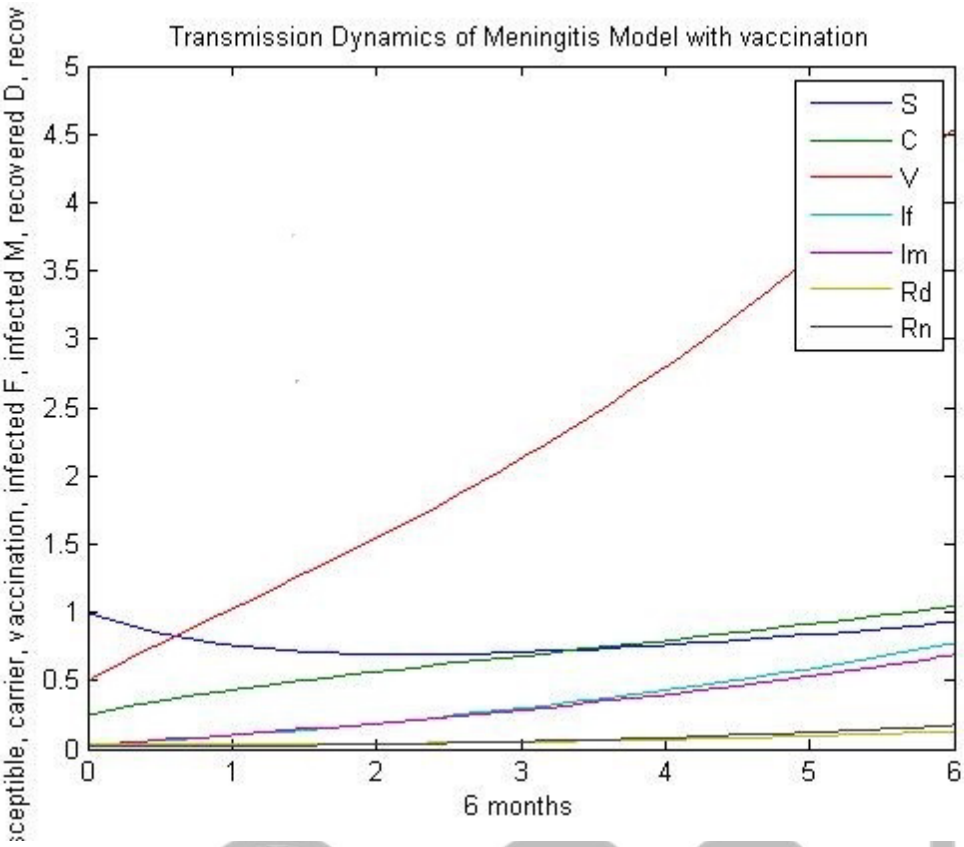

Figure .2: Graphical representation of Meningitis Transmission Dynamics with 50 percent Vaccination

CONCLUSION

Since all the Eigen- values are negatives it implies that the disease free equilibrium point is locally asymptotically unstable with $R_0 > 1$. Means that each infected individual infect more than one individual such that there is expectation of the disease spread out. Also the significance impact of vaccination is negative during outbreak than when no outbreak. In this study, the significance impact of vaccination is modelled on the transmission dynamics of meningitis infections and found negative. The analysis of the Endemic equilibrium state of the model, using the threshold value, R_0 threshold value (R_0) is investigated as 44, 39and 35 by adjusting the rate of carrier from natural carrier rate of $25\%, 60\%$ and 90% and vaccination respectively during outbreak. But obtained value of $R_0 = 11$, whose are far greater than 1 shows the significance impact of vaccination is negative on meningitis transmission dynamics.

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