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Modified Adomian Decomposition Method for the Solution of Fourth Order Ordinary Differential Equations

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ABSTRACT

In this work, we introduced the concept of the Modified Adomian Decomposition (MADM) in solving fourth-order ordinary differential equations by first transforming the fourth order ordinary differential equations to an equivalent integral equation (Volterra integral equation of the second kind). Some test problems were considered and the results are compared with the existing methods. The convergence of the MADM was so rapid and involved only few terms of the series.

Keywords: Ordinary differential equations, Modified Adomian Decomposition method, Volterra integral equation, Initial value problems (IVP)

1.0 INTRODUCTION

Most phenomena in sciences, economics, management, engineering etc. can be modeled by differential and integral theories. Interestingly, solutions to most of the differential equations arising from such models do not have analytic solutions necessitating the development of numerical techniques. However, most of these methods require tedious analysis, large computational work, and lack of existing rule in the selection of initial value, scientifically unrealistic assumptions, perturbation, linearization and descritization of the variable which gives rise to rounding off errors causing loss of Physical nature of the problem. Some of the numerical methods used in literature include; variational Iteration method Kiymaz (2010), homotopy perturbation method Desai and Pradhan (2013) and Okuboye (2015) developed seven-step block method.

The modified Adomian decomposition method proposed in this work is easy to apply; it requires no linearization or discretization, it is free of trial and error method in the selection of initial value. The method was first introduced by Adomian (1976) to solve nonlinear stochastic differential equations. It has been widely applied to numerous problems, Biazar et al. (2008) for Riccati differential equations, Opanuga et al in (2014), Edeki et al in (2014). Gbadeyan and Agboola (2012) used DTM to solve vibration problem. The three examples solved in this present work gave the exact solution and the results are presented in tables in comparison with the Differential transform method and the exact solution.

2.0 METHODOLOGY

The Adomian decomposition method (ADM) provides the solution in an infinite series of components. The components $u_n(x), n \geq 0$ are easily computed if the inhomogeneous term $f(x)$ in the differential equation consists of a polynomial. However, if the function $f(x)$ consists of a combination of two or more of polynomials, trigonometric functions, hyperbolic functions, and others, the evaluation of the components $u_n(x), n \geq 0$ requires cumbersome work. Going by the work of Wazwaz (2011), a reliable modification of the Adomian decomposition method is presented. The modified decomposition method will facilitate the computational process and further accelerate the convergence of the series solution. The modified ADM depends mainly on splitting the function $f(x)$ into two parts. To give a clear description of the technique, we recall that the standard ADM admits the use of the recurrence relation. Where the solution $u(x)$ is expressed by an infinite sum of components defined before by

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \qquad \ldots (1)$$

In view of MADM, the components $u_n(x), n \geq 0$ can be easily evaluated.

The modified ADM introduces a slight variation to the recurrence relation that will lead to the determination of the components of $u(x)$ in an easier and faster manner. The function $f(x)$ can be set as the sum of two partial functions, namely $f_1(x)$ *and* $f_2(x)$. Wazwaz (2011)

In other words, it can be written as

$$f(x) = f_1(x) + f_2(x) \qquad \ldots (2)$$

An introduction is made to change the formation of the standard ADM, to minimize the size of calculations, we identified the zeroth component $u_0(x)$ by one part of $f(x)$, the other part of $f(x)$ can be added to the component $u_1(x)$ among other terms. The modified Adomian's Decomposition method (MADM) introduces the modified recurrence relation:

$u_0(x) = f_1(x),$

$u_1(x) = f_2(x) + \lambda \int_0^x k(x,t)u_0\, dt$

$\vdots$

$$u_{n+1}(x) = \lambda \int_0^x k(x,t)u_n(t)\, dt, \qquad n \geq 1 \qquad \ldots(3)$$

Therefore

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots = \sum_{n=0}^{\infty} u_n(x) \qquad \ldots(4)$$

3.0 NUMERICAL EXAMPLES

We illustrate the method by the following problems

Example 1:

Consider the fourth order ordinary differential equation

$u^{'v}(t) = u,$ $Pallavi$ (2017)

The initial conditions are:

$u(0) = 1, u^{'}(0) = -1, u^{''}(0) = 1, u^{'''}(0) = -1$

With exact solution

$u(t) = e^{-t}$

Integrating both sides four times and applying the n-fold integral formula yields

$u(t) = 1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{1}{3!}\int_0^t (t-x)^3\, u(x)dx$

Applying the (MADM) gives

$u_0(t) = 1$

$u_1(t) = -t + \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!}$

$u_2(t) = -\frac{t^5}{120} + \frac{t^6}{720} - \frac{t^7}{7!} + \frac{t^8}{8!}$

$\vdots$

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$$u(t) = 1 - t + \frac{t^2}{2} - \frac{t^3}{6} + \frac{t^4}{24} - \frac{t^5}{120} + \frac{t^6}{720} - \frac{t^7}{7!} + \frac{t^8}{8!} \ldots$$

Table 1: Numerical result for problem 1

t	MADM(n=2)	EXACT	MADM ERROR	DTM(n=8)	DTM ERROR
0.1	0.904837418	0.9048468	9.38177E-06	0.904837418	9.38179E-06
0.2	0.818730756	0.818747731	1.69756E-05	0.818730753	1.69781E-05
0.3	0.740818262	0.740841264	2.30019E-05	0.740818221	2.30437E-05
0.4	0.670320356	0.670347847	2.74917E-05	0.670320046	2.78012E-05
0.5	0.606532118	0.606562104	2.99863E-05	0.60653066	3.14447E-05
0.6	0.5488168	0.548845779	2.8979E-05	0.548811636	3.41429E-05
0.7	0.496600318	0.496621347	2.10287E-05	0.496585304	3.6043E-05
0.8	0.449366756	0.449366236	5.19252E-07	0.449328964	3.72722E-05
0.9	0.406654863	0.406607601	4.72617E-05	0.40656966	3.79412E-05
1	0.368055556	0.367917586	0.000137969	0.367879441	3.81453E-05

Problem 2:

Consider

$u^{'v}(t) = e^t, \;\; Pallavi \ (2017)$

With the initial conditions are:

$u(0) = 3, u^{'}(0) = 1, u^{''}(0) = 5, u^{'''}(0) = 1$

With exact solution

$u(t) = 2 + 2t^2 + e^t$

Integrating both sides four times and applying the n-fold integral formula yields

$$u(t) = 2 + 2t^2 + 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} \ldots$$

Applying the (MADM) gives

$u_0(t) = 3$

$u_1(t) = 2t^2 + t + \frac{t^2}{2!} + \frac{t^3}{3!} \ldots$

$u_2(t) = u_3(t) = u_4(t) = 0$

$\vdots$

$u(t) = 3 + t + \frac{5t^2}{2!} + \frac{t^3}{3!} \dots$

Table 2: Numerical result for problem 2

t	MADM(n=1)	EXACT	MADM ERROR	DTM(n=8)	DTM ERROR
0.1	3.125159459	3.125159459	0.000000000	3.125170918	2.53282E-05
0.2	3.30137743	3.30137743	0.000000000	3.301402758	4.19871E-05
0.3	3.52981682	3.52981682	0.000000000	3.529858808	6.18702E-05
0.4	3.811762827	3.811762827	0.000000000	3.811824698	8.5471E-05
0.5	4.1486358	4.1486358	0.000000000	4.148721271	0.000113352
0.6	4.542005449	4.542005449	0.000000000	4.5421188	0.000146151
0.7	4.993606557	4.993606557	0.000000000	4.993752707	0.000184595
0.8	5.505356333	5.505356333	0.000000000	5.505540928	0.000229509
0.9	6.079373602	6.079373602	0.000000000	6.079603111	0.000281828
1	6.718	6.718	0.000000000	6.718281828	0

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Example 3:

Consider the fourth order initial value problem

$u^{'v}(t) = 3\cos t$, $Pallavi$ (2017)

The initial conditions are:

$u(0) = 2, u^{'}(0) = 0, u^{''}(0) = -2,\ u^{'''}(0) = 0$

With exact solution

$$u(t) = 3\cos t + \frac{1}{2}t^2 - 1$$

Integrating both sides four times and applying the n-fold integral formula yields

$$u(t) = -1 + 3\cos t + \frac{1}{2}t^2$$

Applying the (MADM) gives

$u_0(t) = -1$

$$u_1(t) = 3\cos t + \tfrac{1}{2}t^2$$

$$u_2(t) = u_3(t) = u_4(t) = 0$$

$$\vdots$$

$$u(t) = -1 + 3\cos t + \frac{1}{2}t^2$$

Table 3: Numerical result for problem 3

t	MADM(n=2)	EXACT SOLUTION	Abs. Error
0.1	0.005008322	0.004987517	2.08E-05
0.2	0.020132622	0.019801064	0.000332
0.3	0.0456669	0.043999572	0.001667
0.4	0.082087822	0.076867492	0.00522
0.5	0.130034722	0.117443318	0.012591
0.6	0.1902816	0.164557921	0.025724
0.7	0.263701122	0.216881161	0.04682
0.8	0.351220622	0.27297491	0.078246
0.9	0.4537701	0.331350393	0.12242
1	0.572222222	0.390527532	0.181695

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4.0 CONCLUSION

The Modified Adomian Decomposition Method is a promising method for the solution of initial value problems in the treatment of higher-order differential equations. This was clearly seen in the test problems considered in this work. In all the problems the results obtained converges faster and in better agreement with the exact solutions than the Differential Transform method when compared, computational difficulties was minimized in this paper.

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