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Non-Perishable Stochastic Inventory System with Non-Reneging of Customers in Service Facilities

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Abstract

This paper analysis an (s, S) perishable inventory system with postponed demands with service facilities. In this chapter, we assumed a non-perishable stochastic inventory system with non-reneging of customers in service facilities. We assume that customer arrive to the service according to Poisson process. It is assumed that initially the inventory level is in order level S. In this system, there is a finite buffer whose capacity varies according to the inventory level at any given time. When the maximum buffer size is reached, further demands join a pool which has finite capacity $P\,(<\infty)$ with probability γ and with probability $(1-\gamma)$ it is lost forever. The lead time is exponentially distributed with parameter β. When inventory level is larger than the number of customers in the buffer, an external demand can enter the buffer for service. The pool customer makes a transition to buffer leaves the pool size less by one as first come first service basis. If service occurred then a transition reducing the size of the buffer by one unit, as the same time reducing the size of the inventory by one unit. When inventory level reaches less or equal to re-order level s an order for replenishment is placed.

Key words: Non-perishable items, Non-reneging customers.

1.1 Introduction

Research on queuing systems with inventory control has captured much curiosity of researchers over the last decades. In this system, customers arrive at the service facility one by one and require service. In order to accomplish the customer service, an item from the inventory is needed. A served customer departs immediately from the system and the on-hand inventory decreases by one at the moment of service completion. This system is called queuing-inventory system of M. Schwarz et al. [1]. A. Krishnamoorthy and M. E. Islam(2004) advised an (s, S) inventory system with postponed demands where they assumed that customers arrive to the system according to a Poisson process[2]. When inventory level depletes to s due to demands or decay or service to pool customer, an order for replenishment is placed. The lead-time is exponentially distributed with parameter γ. When inventory level reaches zero, the incoming customers are sent to a pool of capacity M, from the pool customers will be picked up for fulfilling demands. A. Krishnamoorthy et al. (2003) examine retrial production inventory system with service time in which primary demands occur as per as Markovian Arrival Process (MAP), using matrix analytic method, they carry out the steady state analysis of the system and some performance measures and obtained[3]. When inventory level depletes to re-order level s due to demands or decay or service to pool customer, an order for replenishment is placed. The lead-time is exponentially distributed with parameter γ. When inventory level reaches zero, the incoming customers are sent to a pool of capacity M, from the pool customers will be picked up for fulfilling demands. O. Berman and K. P. Sapna (2000) studied queuing-inventory systems with Poisson arrivals, arbitrary distribution service times and zero lead times[4]. The optimal value of the maximum allowable inventory which minimizes the long-run expected cost rate has been obtained. O. Berman and E. Kim(2004) analyzed a queuing-inventory system with Poisson arrivals, exponential s service times and zero lead times[5]. The authors proved that the policy is never to order when the system is empty . J. R. Artalejo et al.(2006) made a numerical look over of (s, S) inventory system where arriving demands finding the system out of stock, leave the service area and repeat their request after some random time[6]. M. E. Islam and S. Khan (2012) considered a perishable inventory model at service facilities for system with postponed demands[7]. In that paper, they examined that non-reneging will be occurred to the pool as well as for the buffer customers.

In this model we consider a non-perishable stochastic inventory system with non-reneging of customers in service facilities with the maximum capacity for S units. In most of the inventory models it is assumed that the inventory deplete at a rate equal to demand rate. However, it becomes impractical for the service facilities where the stocked item is delivered to the customers after some service is performed. Arrival of demands form a Poisson process with parameter $\lambda\ (> 0)$ to a buffer of finite capacity equal to the inventory level is in order level S at any given time t. When the maximum buffer size is reached, further demands join a pool of finite capacity with probability γ or with probability $(1 - \gamma)$ itis lost forever. Pooled customers are taken for service at a service completion epoch if the inventory level is at least $s + 1$. The service time is assumed to be exponentially distributed with parameter μ. When inventory level reaches s due to service, an order for replenishment is placed. The lead time is exponentially distributed with parameter β.When $I(t) > B(t)$ and $I(t) \geq s + 1$and $B(t) = 0$, then a pool customer is sent to the buffer with probability p at the previous service epoch. Even when $I(t) > B(t)$ and $I(t) < s$ and $B(t) = 0$, then there is no pool customers are sent to the buffer.

1.2 Assumption

i) Arrival of demands follows Poisson process with parameter λ

ii) Lead time is exponentially distributed with parameter β

iii) Service time is exponentially distributed with parameter .

v) Initially the system is in order level S

vi) Replenishment take place when the inventory level depleted to re-order level s

vii) If $B(t) = I(t)$, the primary arrival is directed to the pool with probability γ and with probability $(1 - \gamma)$ it is lost forever

viii) When $B(t) = I(t)$, is the highest inventory level for that situation if perishability occurs immediate one buffer customer must be pull out from buffer

---

1.3 Notations

The notations used in the sequel are explained below

a) $\mathrm{I(t)}$ = Inventory level at time t

b) $\mathrm{B(t)}$ = Number of customers in the buffer at time t

c) $\mathrm{P(t)}$ = Number of customers in the pool at time t

d) ξ = Reneging rate

e) e = Denote the column vector of $\mathrm{I's}$

1.4 Model and Diagram Analysis

$$\{\mathrm{P(t), I(t), B(t) = (i,j,k)|\ 0 \le i \le P; 0 \le j \le S;\ 0 \le k \le B}\}$$

is formed a three dimensional Markov process with state space $\mathrm{E} = \mathrm{E_1 \times E_2 \times E_3}$ where

$$\mathrm{E_1} = \{0,1,\dots.\mathrm{P}\}; \mathrm{E_2} = \{0,1,\dots.\mathrm{S}\}; \mathrm{E_3} = \{0,1,\dots.\mathrm{B}\}$$

The infinitesimal generator of the process :

$\widetilde{A} = (\mathrm{a(i,j,k{:}\,l,m,n); (i,j,k),(l,m,n)} \in \mathrm{E}$ can be obtained using the following arguments:

. The arrival of the demand makes a transition from

$$\mathrm{(i,j,k)} \to \mathrm{(l = i+1, m = j, n = k)}\ \text{ if }\ 0 \le \mathrm{i} \le \mathrm{P}-1;\ 0 \le \mathrm{j} \le \mathrm{S};\ 0 \le \mathrm{k} \le \mathrm{B}$$

B. The pool customer makes a transition to buffer leaves the pool size less by one as first come first serve basis.

$$\mathrm{(i,j,k)} \to \mathrm{(l = i-1, m = j, n = k+1)}\ \text{if}\ 1 \le \mathrm{i} \le \mathrm{P};\ 1 \le \mathrm{j} \le \mathrm{S};\ 0 \le \mathrm{k} \le \mathrm{B}-1$$

C. If service occurred then a transition reducing the size of the buffer by one unit,as the same time reducing the size of the inventory by one unit.

$$\mathrm{(i,j,k)} \to \mathrm{(l = i, m = j-1; n = k-1)}\ \text{if}\ 0 \le \mathrm{i} \le \mathrm{P};\ \mathrm{s}+1 \le \mathrm{j} \le \mathrm{S};\ \mathrm{k} = 1$$

D. Replenishment take place only when inventory less or equal to re-order level s.

$$\mathrm{(i,j,k)} \to \mathrm{(l = i, m = j+\beta, n = k)}\text{if}\ 0 \le \mathrm{i} \le \mathrm{P};\ 0 \le \mathrm{j} \le \mathrm{s};\ 0 \le \mathrm{k} \le B\text{-}2$$

Diagram of Non-perishable stochastic inventory system with non-reneging of customers is shown in the following figure 1.1

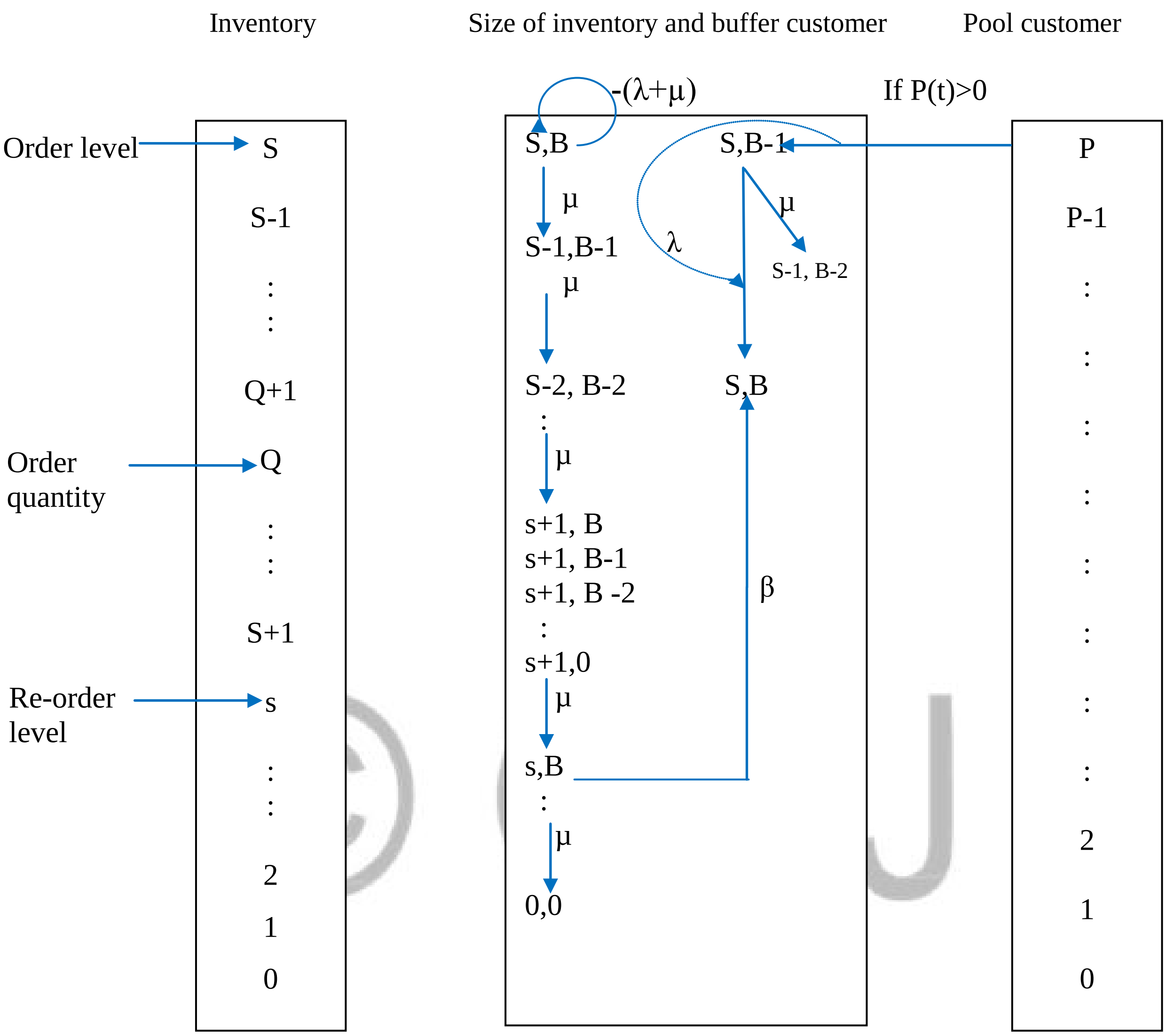

Diagram 1.1 of the Non-perishable stochastic inventory system with non-reneging of customers

Remark

If, $P(t) > 0$ and $I(t) > B(t)$, then

$$S, B-1 \xrightarrow{\delta\gamma} S-1, B-1$$

The Infinitesimal generator $\tilde{A}$ of the three dimensional Markov Process

$[P(t), I(t), B(t);\ t \geq 0]$ can be defined by the Markov chain $\{(P(t), I(t), B(t)), t \geq 0\}$ has the generator Q in partitioned from given by

$\tilde{A}$=$(a(i,j,k;\ l,m,n))$: $(i,j,k),(l,m,n) \in E, (i,l) \in E_1, (j,m) \in E_2, (k,n) \in E_3$

Now, the infinitesimal generator $\tilde{A}$ can be conveniently express as a partition matrix $\tilde{A} = A_{j,m}$

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$$
\left\{
\begin{array}{lcl}
 & & l = i + 1;\ i = 0 \ldots P - 1 \\
\lambda & : & m = j\ ;\ j = 0 \ldots S \\
 & & n = k\ ;\ k = 0 \ldots B \\
 & & l = i\ ;\ i = 0 \ldots P \\
\beta & : & m = j + \beta\ ;\ j = 0 \ldots s \\
 & & n = k\ ;\ k = 0{,}1 \\
 & & l = i\ ;\ i = 0 \ldots P \\
\mu & : & m = j - 1\ ;\ j = s + 1 \ldots S \\
 & & n = k - 1\ ;\ k = 1 \ldots B \\
 & & l = i - 1\ ;\ i = 1 \ldots P \\
\delta\gamma & : & m = j\ ;\ j = s \ldots S \\
 & & n = k + 1\ ;\ k = 0 \ldots B - 1 \\
 & & l = i;\ i = 0 \ldots P - 1 \\
-(\lambda + \beta) & : & m = j\ ;\ j = 0 \ldots s \\
 & & n = k\ ;\ k = 0 \\
 & & l = i;\ i = 0 \ldots P - 1 \\
-(\lambda + \mu + \beta) & : & m = j\ ;\ j = s \\
 & & n = k\ ;\ k = 1 \\
 & & l = i;\ i = 0 \ldots P - 1 \\
-(\lambda + \mu) & : & m = j\ ;\ j = s + 1 \ldots S \\
 & & n = k\ ;\ k = 1 \ldots B \\
 & & l = i\ ;\ i = 0 \\
-\lambda & : & m = j\ ;\ j = s + 1 \ldots S \\
 & & n = k\ ;\ k = 0 \\
 & & l = i\ ;\ i = 1 \ldots P - 1 \\
-(\lambda + \beta + \delta\gamma) & : & m = j\ ;\ j = s \\
 & & n = k\ ;\ k = 0 \\
 & & l = i\ ;\ i = P \\
-\beta & : & m = j\ ;\ j = 0 \\
 & & n = k\ ;\ k = 0 \\
 & & l = i\ ;\ i = 1 \ldots P - 1 \\
-(\lambda + \delta\gamma) & : & m = j\ ;\ j = s + 1 \ldots S \\
 & & n = k\ ;\ k = 0 \ldots B - 1 \\
 & & l = i\ ;\ i = P \\
-(\beta + \delta\gamma) & : & m = j\ ;\ j = s \\
 & & n = k\ ;\ k = 0
\end{array}
\right.
$$

$$
\left\{
\begin{array}{lcl}
 & & l = i\ ;\ i = P \\
-\delta\gamma & : & m = j\ ;\ j = s + 1 \ldots S \\
 & & n = k\ ;\ k = 0 \\
 & & l = i\ ;\ i = P \\
-\mu & : & m = j\ ;\ j = s + 1 \ldots S \\
 & & n = k\ ;\ k = 2 \ldots B \\
 & & l = i\ ;\ i = P \\
-(\mu + \beta) & : & m = j\ ;\ j = s \\
 & & n = k\ ;\ k = 1 \\
 & & l = i\ ;\ i = P
\end{array}
\right.
$$

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$$-(\mu+\delta\gamma): \quad m=j\,;\ j=s+1\ldots S \quad n=k\,;\ k=1\ldots B-1$$

$$\widetilde{A}_{[(P+1)(2S+B+1)\times(P+1)(2S+B+1)]} = \begin{matrix} & 0 & 1 & 2\ldots\ldots\ldots\ldots S \\ 0 & A & B & 0\ldots\ldots\ldots\ldots 0 \\ 1 & C & D & B\ldots\ldots\ldots\ldots 0 \\ 2 & 0 & C & D\ldots\ldots\ldots\ldots 0 \\ . & . & . & . \quad\quad . \\ . & . & . & . \quad\quad . \\ S & 0 & 0 & 0\ldots\ldots\ldots\ldots E \end{matrix}$$

Where $A_{j,m}$ is a $(S+1)(B+1)\times(S+1)(B+1)$ sub-matrix, which is given by

$$A_{j,m} = \begin{cases} A & : \text{ if } m=j\,;\ j=0 \\ B & : \text{ if } m=j+1\,;\ j\geq 0 \\ C & : \text{ if } m=j-1\,;\ j\geq 1 \\ D & : \text{ if } m=j\,;\ j\geq 1\ldots.P-1 \\ E & : \text{ if } m=j\,;\ j=P \end{cases}$$

With

$$A=(a_{cd})_{[(S+1)(B+1)\times(S+1)(B+1)]} =$$

$$\begin{cases} (c,d)\rightarrow(c,d) \text{ is } -(\lambda+\beta) \text{ if } c=0; d=o \\ (c,d)\rightarrow(c,d) \text{ is } -(\lambda+\beta) \text{ if } c=1; d=o \\ (c,d)\rightarrow(c,d) \text{ is } -(\lambda+\beta+\mu) \text{ if } c=1; d=1 \\ (c,d)\rightarrow(c,d) \text{ is } -\lambda \text{ if } c=s+1\ldots S\ ; d=0 \\ (c,d)\rightarrow(c,d) \text{ is } -(\lambda+\mu) \text{ if } c=s+1\ldots S; d=1\ldots B \\ (c,d)\rightarrow(c+\beta,d) \text{ is } \beta \text{ if } c=0\ldots s; d=0\ldots 1 \\ (c,d)\rightarrow(c-1,d-1) \text{ is } \mu \text{ if } c=s+1\ldots S; d=1\ldots B \\ \quad\textbf{All other elements are 0} \end{cases}$$

$$B=(a_{cd})_{[(S+1)(B+1)\times(S+1)(B+1)]} = \text{diag}(\lambda\lambda\ldots\ldots\lambda)$$

$$C=(a_{cd})_{[(S+1)(B+1)\times(S+1)(B+1)]=} (c,d)\rightarrow(c,d+1) \text{ is } \delta\gamma$$
$$\text{if } c=1\ldots S; d=0\ldots B-1$$

$$D = (a_{cd})_{[(S+1)(B+1)\times(S+1)(B+1)]} =$$

$$\begin{cases} (c,d) \to (c,d) \text{ is } -(\lambda+\beta) \text{ if } c=0; d=0 \\ (c,d) \to (c,d) \text{ is} -(\lambda+\beta+\delta\gamma) \text{ if } c=1; d=0 \\ (c,d) \to (c,d) \text{ is} -(\lambda+\beta+\mu) \text{ if } c=1; d=1 \\ (c,d) \to (c,d) \text{ is } -(\lambda+\delta\gamma) \text{ if } c=s+1\ldots S; d=0 \\ (c,d) \to (c,d) \text{ is } -(\lambda+\mu+\delta\gamma) \text{ if } c=s+1\ldots S; d=1\ldots B-1 \\ (c,d) \to (c,d) \text{ is } -(\lambda+\mu) \text{ if } c=s+1\ldots S; d=2\ldots B \\ (c,d) \to (c+\beta,d) \text{ is } \beta \text{ if } c=0\ldots s; d=0\ldots 1 \\ (c,d) \to (c-1,d-1) \text{ is } \mu \text{ if } c=1\ldots S; d=1\ldots B-1 \\ \textbf{All other elements are 0} \end{cases}$$

$$E = (a_{cd})_{[(S+1)(B+1)\times(S+1)(B+1)]} =$$

$$\begin{cases} (c,d) \to (c,d) \text{ is } -\beta \text{ if } c=0; d=0 \\ (c,d) \to (c,d) \text{ is} -(\beta+\delta\gamma) \text{ if } c=1; d=0 \\ (c,d) \to (c,d) \text{ is} -(\beta+\mu) \text{ if } c=1; d=1 \\ (c,d) \to (c,d) \text{ is } -\delta\gamma \text{ if } c=s+1\ldots S; d=0 \\ (c,d) \to (c,d) \text{ is } -(\mu+\delta\gamma) \text{ if } c=s+1\ldots S; d=1\ldots B-1 \\ (c,d) \to (c,d) \text{ is } -\mu \text{ if } c=s+1\ldots S; d=2\ldots B \\ (c,d) \to (c+\beta,d) \text{ is } \beta \text{ if } c=0\ldots s; d=0\ldots 1 \\ (c,d) \to (c-1,d-1) \text{ is } \mu \text{ if } c=1\ldots S; d=1\ldots B-1 \\ \textbf{All other elements are 0} \end{cases}$$

So, we can write the partitioned matrix as follows

$$\widetilde{A} = (a_c)_{[(P+1)(2S+B+1)\times(P+1)(2S+B+1)]} =$$

$$\begin{cases} c \to c & ; \text{ is } A & \text{if } c=0 \\ c \to c+1 & ; \text{ is } B & \text{if } c \geq 0 \\ c \to c-1 & ; \text{ is } C & \text{if } c \geq 1 \\ c \to c & ; \text{ is } D & \text{if } c \geq 1\ldots P \\ c \to c & ; \text{ is } E & \text{if } c = P \end{cases}$$

1.5 Steady-State Analysis

It can be seen from the structure of matrix $\tilde{A}$ that the state space E is irreducible. Let the limiting distribution be denoted by $x^{(i,j,k)}$

$$x^{(i,j,k)} = \lim_{t\to\infty} \Pr\left[P(t), I(t), B(t) = (i,j,k)\right], (i,j,k) \in E.$$

Let $x = (x^{(S)}, x^{(S-1)} \ldots\ldots x^{(1)}, x^{(0)}$ with,

$$x^{(j)} = [(x^{(P,j,0)}, \ldots., x^{(0,j,0)}), (x^{(P,j,0)}, \ldots., x^{(0,j,1)}) \ldots\ldots (x^{(P,j,B)}, \ldots., x^{(0,j,B)})]$$

for j=0,1,2,….S.

The limiting distribution exists, satisfies the following equation:

$$x\widetilde{A} = 0 \text{ and } \sum_{i=0}^{p}\sum_{j=1}^{S} j \sum_{k=0}^{j} x^{(i,j,k)} = 1 \quad (1.1)$$

The first equation of the above yields a set of equations, which can be represented as a general form in the following manner:

$$x^{(0)}A + x^{(1)}C = 0$$
$$x^{(0)}B + x^{(1)}D + x^{(2)}C = 0$$
$$x^{(1)}B + x^{(2)}D + x^{(3)}C = 0$$
$$x^{(2)}B + x^{(3)}D + x^{(4)}C = 0$$

……………………….

……………………….

$$x^{(n)}B + x^{(n+1)}D + x^{(n)}C = 0$$
$$x^{(n-1)}B + x^{(n)}E = 0$$

The solution of the above equations (except the last one) can conveniently be expressed as:

$$x^{(i)} = x^{(0)}\beta_i \quad \text{where } i = 0,1,\dots s$$
$$\beta_i = 1 \quad , i = 0$$
$$\beta_i = -AC^{-1} \quad , i = 1$$
$$\beta_i = -\beta_{i-1}DC^{-1} - \beta_{i-2}BC^{-1} \quad , i = 2,3,\dots S$$

To compute $\mathbf{x^{(0)}}$ we can use the following equation:

Theorem: If $x = \{x_j \ , j \geq 0\}$ is stationary distribution, then $x\widetilde{A} = 0$.

Proof: We have from Kolmogorov forward differential equation

$$x_t' = x_t\widetilde{A} \quad (1.2)$$

$$x_{jm}'(t) = -P_{jm}(t)a(m,m) + \sum_{k\neq j} x_{mk}(t)\, a(k,m) \quad (1.3)$$

Since x is stationary then $t \to \infty$, if limit exists, it is independent of time parameter and hence

$$x_{jm}'(t) \to 0$$

From equation (a) we get, $-x_{jm}(t)a(m,m) + \sum_{k\neq j} x_{jk}(t)a(k,m) = 0$

In matrix notation which can be written as, $x\widetilde{A} = 0$ (1.4)

and Normalizing condition hold; then $\sum\sum\sum x^{(i,j,k)}$ can be completely evaluated.

By using the equation (1.4) with normalizing condition, we calculate all the steady state probability vector by using Mathematica software can be measured (see **1.12** Estimated values of statistics probability vector).

1.6 System Performance Measures

In this section we derive some of the system performance measures of the system under consideration :

1. Expected inventory level in the system is given by-

$$Q_1 = \sum_{i=0}^{p} \sum_{j=1}^{S} j \sum_{k=0}^{j} x^{(i,j,k)}$$

2. Expected number of customers in the buffer is

$$Q_2 = \sum_{i=0}^{P} \sum_{j=1}^{S} \sum_{k=1}^{j} kx^{(i,j,k)}$$

3. Average customers lost to the system is

$$Q_3 = \delta(1-\gamma) \sum_{i=0}^{P} \sum_{k=J-0}^{S} x^{(i,j,k)}$$

4.Expected number of customers in the pool

$$Q_4 = \sum_{i=1}^{P} \sum_{j=0}^{S} \sum_{k=0}^{j} x^{(i,j,k)}$$

5. Expected re-order level of the system

$$Q_5 = \mu \sum_{i=0}^{P} \sum_{k=0}^{B} x^{(i,s+1,k)}$$

6. Expected rate that a customer will enter the pool is

$$Q_6 = \lambda \sum_{i=0}^{P-1} \sum_{j=0}^{S} \sum_{k=0}^{j} x^{(i,j,k)}$$

7. The average rate at which the pooling customers will enter the buffer is given by

$$Q_7 = \delta\gamma \sum_{i=0}^{P} \sum_{j=s+1}^{S} \sum_{k=1}^{j} x^{(i,j,k)}$$

1.7 Cost Function

Define

c_1= Inventory holding cost per unit

c_2=Cost of buffer customers in the system

c_3= Cost of lost customers in the system

c_4= Cost of pool customer in the system

c_5=Cost of re-order of the system

So the total expected cost of the system is

$$E(TC) = c_1Q_1 + c_2Q_2 + c_3Q_3 + c_4Q_4 + c_5Q_5$$

$$E(TC) = C_1\sum_{i=0}^{P}\sum_{j=1}^{S} j\sum_{k=0}^{j} x^{(i,j,k)} + C_2\sum_{i=0}^{P}\sum_{j=1}^{S}\sum_{k=1}^{j} kx^{(i,j,k)} + C_3\delta(1-\gamma)\sum_{i=0}^{P}\sum_{k=J=0}^{S} x^{(i,j,k)} + C_4\sum_{i=1}^{P}\sum_{j=0}^{S}\sum_{k=0}^{j} x^{(i,j,k)}$$

$$+C_5\mu\sum_{i=0}^{P}\sum_{k=0}^{B} x^{(i,j,k)}$$

By using above cost function, we can exploit a lot of interesting feature and can make a sensitivity analysis.

1.8 Numerical Illustration

By giving values to the underlying parameters we provide some numerical illustrations. Take $S = 3, s = 1, Q = 2, \lambda = 0.4, \beta = 0.6, \delta = 0.7, \mu = 0.5, \gamma = 0.8,$

$$c_1 = 1,\ c_2 = 2,\ c_3 = 3,\ c_4 = 1,\ c_5 = 2$$

Then we get the numerical values of different system characteristics described in the following table.

Table 1.1 Non-Perishable Stochastic Inventory System with Non-Reneging of Customers

Expected inventory leel in the system	Expected no.of customer in the buffer	Average customer lost in the system	Expected no. of customer in the pool	Expected re-order level	Expected rate that a customer will enter the pool	Average rate at which the pooling customer will enter the buffer	Expected total cost of the system
2.41635954	1.21692624	0.049563004	0.80671204	0.2433739	0.24446624	0.27346966	6.292360872

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1.9 Graphical presentation

We have plotted the figures based on ODE's and performance measures. MATHEMATICA software is used to develop the computational program.

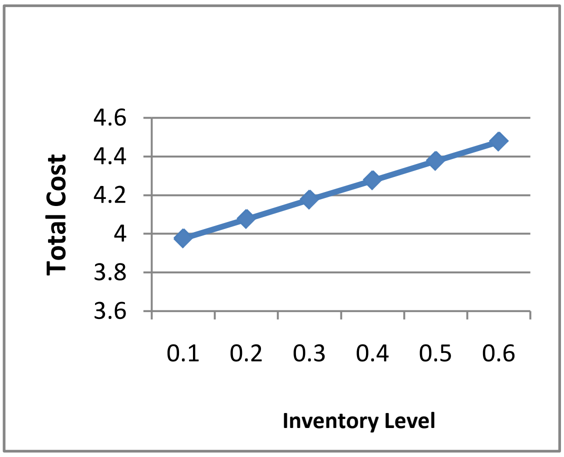

Figure 1.1 Inventory level vs Total cost

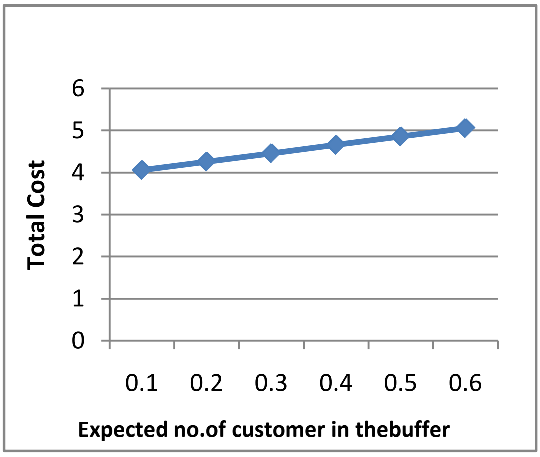

Figure 1.2 Expected no. of customer in the buffer vs Total cost

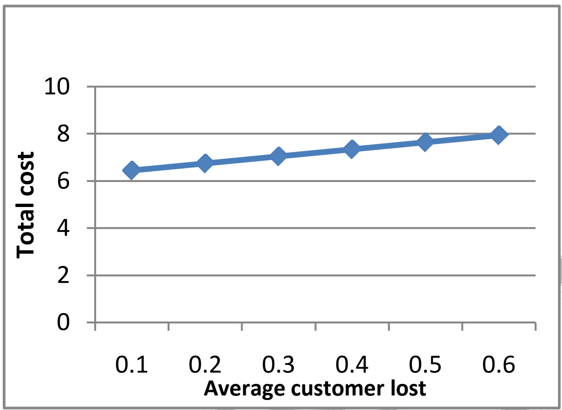

Figure 1.3 Average Customer lost vs Total cost

Graphical presentation

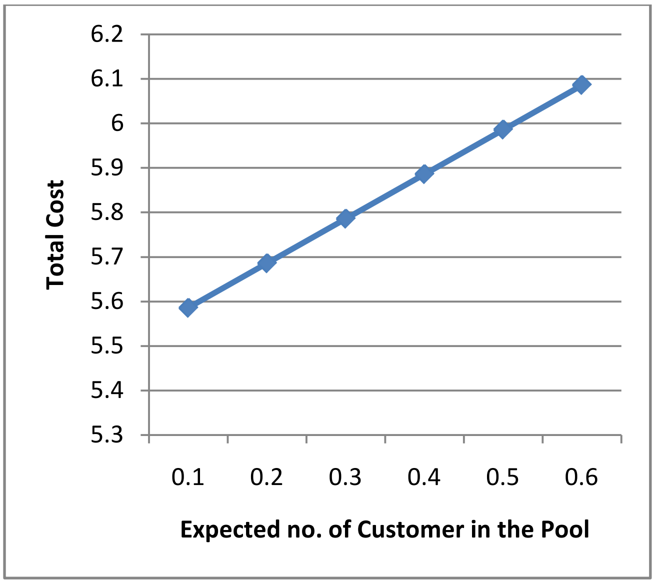

Figure 1.4 Expected no. of customer in the

Pool vs. Total cost

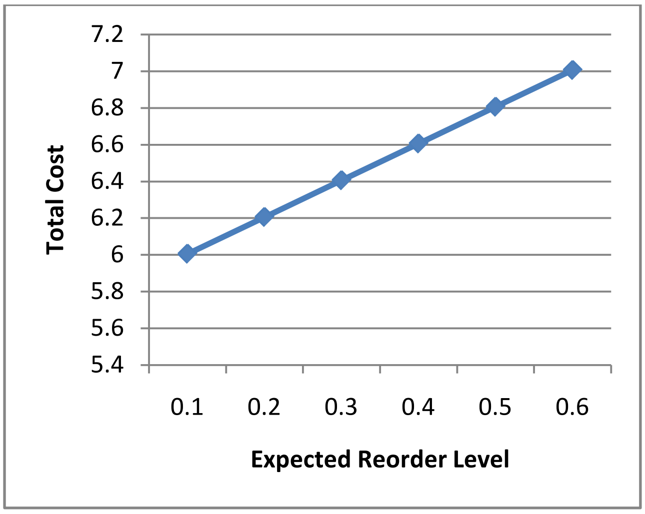

Figure 1.5 Expected re-order level the pool vs. total cost

1.10 Sensitivity analysis

From the figure 1.1, it is observed that if the inventory level increased then the total cost is increased in noteworthy amount. The result is obvious as the inventory level is increase it has impact on higher re-ordering , lost sales and also increase the cost of carrying pool and buffer customers. Hence the inventory level is vital to this system. From the figure 1.2, it is observed that expected number of customer in the buffer has a mini impact as it increase total cost is increase .From the figure 1.3, it is observed that average customer lost has a little impact as it increase, total cost is increase. From the figure 1.4, it is observed that if expected number of customers in the pool is increased than the total cost is measured in a noteworthy amount. From the figure 1.5, it is observed that if expected the re-order level has a tiny impact, as it increase total cost is increased. Hence it must be inspect accurately for selection per unit cost for the item.

1.11 Conclusion

In this model we consider a stochastic inventory system without perishable items and reneging of customers in a service facility with the maximum capacity for inventory S units. In this paper, we assume that external customers can directly met their demands at a negligible service time. We considered that customer arrive to the service according to Poisson process. Here we considered a Pool and Buffer system. In this system, there is a finite buffer whose capacity varies according to the inventory level at any given time. When the maximum buffer size is reached, further demands join a pool which has finite capacity. When inventory level is larger than the number of customers in the buffer, an external demand can enter the buffer for service. If service occurred then a transition reducing the size of the buffer by one unit, as the same time reducing the size of the inventory by one unit.

1.12 Estimated values of statistics probability vector

$x^{(0,0,0)}$	0.00654206	$x^{(0,1,0)}$	0.0156276
$x^{(0,1,1)}$	0.0130841	$x^{(0,2,0)}$	0.0349485
$x^{(0,2,1)}$	0.0312553	$x^{(0,2,2)}$	0.0239648
$x^{(0,3,0)}$	0.0234414	$x^{(0,3,1)}$	0.0201083
$x^{(0,3,2)}$	0.0148743	$x^{(0,3,3)}$	0.00944174
$x^{(1,0,0)}$	0.0159017	$x^{(1,1,0)}$	0.0136496
$x^{(1,1,1)}$	0.0265697	$x^{(1,2,0)}$	0.036951
$x^{(1,2,1)}$	0.0300847	$x^{(1,2,2)}$	0.0479719
$x^{(1,3,0)}$	0.0182983	$x^{(1,3,1)}$	0.0239052
$x^{(1,3,2)}$	0.0151742	$x^{(1,3,3)}$	0.0130459
$x^{(2,0,0)}$	0.0193503	$x^{(2,1,0)}$	0.0189909
$x^{(2,1,1)}$	0.0259793	$x^{(2,2,0)}$	0.0425615
$x^{(2,2,1)}$	0.0483319	$x^{(2,2,2)}$	0.0393209
$x^{(2,3,0)}$	0.0194936	$x^{(2,3,1)}$	0.0289368
$x^{(2,3,2)}$	0.0142225	$x^{(2,3,3)}$	0.0091376
$x^{(3,0,0)}$	0.0354571	$x^{(3,1,0)}$	0.0155012
$x^{(3,1,1)}$	0.0270683	$x^{(3,2,0)}$	0.0918205
$x^{(3,2,1)}$	0.02077	$x^{(3,2,2)}$	0.0387668
$x^{(3,3,0)}$	0.0305324	$x^{(3,3,1)}$	0.0262412

$X^{(3,3,2)}$	0.00536696	$X^{(3,3,3)}$	0.00731008

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