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ON THE SIMPLICITY OF PERMUTATION GROUPS

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ABSTRACT

The aim of this research is to carry out investigation on the simplicity status of the Sylow subgroups of permutation groups. The standard program called Group Algorithms and Programming (GAP) is used to enhance and validate our result.

1.0 INTRODUCTION

Group is an algebraic structure consisting of a set of elements equipped with an operation that combines any two elements to form a third element. The operation satisfies four conditions called the group axioms, namely closure, associativity, identity and invertibility. One of the most familiar examples of a group is the set of integers together with the addition operation, but the abstract formalization of the group axioms, detached as it is from the concrete nature of any particular group and its operation, applies much more widely. It allows entities with highly diverse mathematical origins in abstract algebra and beyond to be handled in a flexible way while retaining their essential structural aspects. The ubiquity of groups in numerous areas within and outside mathematics makes them a central organizing principle of contemporary mathematics, Herstein (1975). Simple groups have been studied at least since early Galis theory, whereEvariste Galois realized the fact that alternating groups on five or more points are simple (and hence not solvable), which he proved in 1831, was the reason that one could not solve the quintic in radicals,*Galois Évariste (1846).*

It was later when Mathieu shown that a family of five groups, called the Mathie groups which found to be the first described group in 1861 and 1873, were also simple. Since these five groups were constructed by methods which did not yield infinitely many possibilities, they were called "sporadic" by Burnside in his 1897 textbook.

A finite group is simple when its only normal subgroups are the trivial subgroup and the whole group. A finite group of prime order is also simple since, it has no non-trivial proper sub groups at all while a finite abelian group not of prime order is not simple.

Simple group theory is the most active and glamorous area of research in the theory ofgroups and it seems certain that this will remain the case for many years to come. Roughly speaking,the central problem is to find some reasonable description of all finite simple groups.

Some important questions that need attention may arise for instance, What are simple groups and why are they important? Evariste Galois (1811—1832) called a groupsimple if its only normal subgroups were the identity subgroup and the group itself. The Abeliansimple groups are the group of order 1and the cyclic groups of prime order, while the nonabeliansimple groups generally have very complicated structures. These groups are important because theyplay a role in group theory somewhat analogous to that which the primes play in number theory

that is, they serve as the “building blocks” for all groups. These “buildingblocks” are called the composition factors of the group and may be determined in the following way.Given a finite group G,choose a maximal normal subgroup $G_1$of

$G = G_0$. Then the factor groupG_0/G_1 is simple, and we next choose a maximal normal subgroup $G_2\ of\ G_1$. Then G_1/G_2 is also simple,and we continue in this fashion until we arrive at$G_n = \{e\}$. The simple groupsG_0/G_1, G_0/G_1,.., G_{n-1}/G_n are the composition factors of G and by the Jordan-Holder theorem

these groups are independent of the choices of the normal subgroups made in the process described.

1.2 DEFINITION OF TERMS

Group: A group is a non-empty set G on which there is a binary operation '*' such that;

- if a and b belong to G then $a * b$ is also in G (closure),
- $a * (b * c) = (a * b) * c$ for all a,b,c in G (associativity),
- there is an element $1 \in G$ such that $a * 1 = 1 * a = a$ for all $a \in G$ (identity),
- if $a \in G$, then there is an element $-a \in G$ such that $a * -a = -a * a = 1$ (inverse).

Subgroup: given a group G under a binary operation $*$, a subset H of G is called a subgroup of G if H also forms a group under the operation $*$. More precisely, H is a subgroup of G if the restriction of $*$ to $H \times H$ is a group operation on H

Simple groups: A group $G \neq \{1\}$ is said to be simple if $\{1\}$ and G are the only normal subgroups of G

Dihedral group: A dihedral group is the group of symmetries of a regular polygon, which includes rotations and reflections.

Wreath Product: The Wreath product of C by D denoted by W = C wr D is the semidirect product of P by D, so that,$W = \{(f,d) \mid f \in P, d \in D\}$, with multiplication in W defined as $(f_1,d_1)(f_2,d_2) = ((f_1 {f_2}^{{d_1}^{-1}}), (d_1 d_2)$ for all $f_1, f_2 \in P and d_1, d_2 \in D$.

GROUP GENERATED BY WREATH PRODUCT

Recently, wreath product of groups has been used to explore some useful characteristics of finite groups in connection with permutation design and construction of lattices Praeger and Scheider(2002), as well as in the study of interconnection networks, for instance. Further, Audu(2001) used wreath product to study the structure of some finite permutation groups. Wreath product constructions has been used to obtain for any positive integer n, solvable groups of derived length n, and commutator length at most equal to 2.

The Wreath product of C by D denoted by W = C wr D is the semidirect product of P by D, so that,$W = \{(f,d) \mid f \in P, d \in D\}$, with multiplication in W defined as $(f_1,d_1)(f_2,d_2) = ((f_1 {f_2}^{{d_1}^{-1}}), (d_1 d_2)$ for all $f_1, f_2 \in P and d_1, d_2 \in D$. Henceforth, we write f d instead of (f, d) for elements of W.

Theorem 1.1

Let D act on P as $f^d(\delta) = f(\delta d^{-1})$ where $f \in P, d \in D and \delta \in \Delta$. Let W be the group of all juxtaposed symbols f d, with $f \in P, d \in D$ and multiplication given by $(f_1, d_1)(f_2, d_2) = f_1 {f_2}^{{d_1}^{-1}}, (d_1 d_2)$. Then W is a group called the semi-direct product of P by D with the defined action.

Based on the forgoing we note the following:

- If C and D are finite groups, then the wreath product W determined by an action of D on a finite set is a finite group of order $|W| = |C|^{|\Delta|}.|D|$.
- P is a normal subgroup of W and D is a subgroup of W.
- The action of W on Γ × Δ is given by $(\alpha, \beta)fd = (\alpha f(\beta), \beta d) where \alpha \in \Gamma and \beta \in \Delta$.

2.0 RESULTS

2.1 Consider the permutation groups $C_1 = \{(1), (123), (132)\}$ and $D_1 = \{(1), (12)\}$ acting on $X = \{1,2,3\}$ *and* $\Delta = \{1,2\}$ respectively. Let $P_1 = {C_1}^{\Delta} = \{f: \Delta_1 \rightarrow C_1\}$. Then $|P_1| = |C_1|^{|\Delta_1|} = 3^2 = 9$. The order of the wreath product is given by $|W_1| = |C_1|^{|\Delta_1|} \times |D_1| = 3^2 \times 2$.

The mappings are as follows

$$f_1 : 1 \rightarrow (1), 2 \rightarrow (1)$$
$$f_2 : 1 \rightarrow (123), 2 \rightarrow (123)$$
$$f_3 : 1 \rightarrow (132), 2 \rightarrow (132)$$
$$f_4 : 1 \rightarrow (1), 2 \rightarrow (123)$$
$$f_5 : 1 \rightarrow (1), 2 \rightarrow (132)$$
$$f_6 : 1 \rightarrow (123), 2 \rightarrow (1)$$
$$f_7 : 1 \rightarrow (132), 2 \rightarrow (1)$$
$$f_8 : 1 \rightarrow (132), 2 \rightarrow (123)$$
$$f_9 : 1 \rightarrow (123), 2 \rightarrow (132)$$

The elements of W are

$$(f_1, d_1), (f_1, d_2), (f_2, d_1), (f_2, d_2), (f_3, d_1), (f_3, d_2), (f_4, d_1), (f_4, d_2), (f_5, d_1),$$
$$(f_5, d_2), (f_6, d_1), (f_6, d_2), (f_7, d_1), (f_7, d_2), (f_8, d_1), (f_8, d_2), (f_9, d_1), (f_9, d_2),$$
$$(\alpha, \delta)^{fd} = (\alpha f(\delta), \delta d).$$

Further, $\Gamma \times \Delta = \{(1,1), (1,2), (2,1), (2,2), (3,1), (3,2)\}$

We obtain the following permutations by the action of W on $\Gamma \times \Delta$

$$(\alpha, \beta) f_1 d_1 = (\alpha f_i(\delta), \beta d_i)$$
$$(1,1) f_1 d_1 = (1 f_1(1), 1 d_1) = (1,1)$$
$$(1,2) f_1 d_1 = (1 f_1(2), 2 d_1) = (1,2)$$
$$(2,1) f_1 d_1 = (2 f_1(1), 1 d_1) = (2,1)$$
$$(2,2) f_1 d_1 = (2 f_1(2), 2 d_1) = (2,2)$$
$$(3,1) f_1 d_1 = (3 f_1(1), 1 d_1) = (3,1)$$
$$(3,2) f_1 d_1 = (3 f_1(2), 2 d_1) = (3,2)$$

Rename the symbols as

$(1,1) \rightarrow 1$	$(2,1) \rightarrow 2$	$(3,1) \rightarrow 3$
$(1,2) \rightarrow 4$	$(2,2) \rightarrow 5$	$(3,2) \rightarrow 6$

And in summary,

$$(\Gamma \times \Delta)^{(f_1, d_1)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \end{Bmatrix}$$
$$(\Gamma \times \Delta)^{(f_1, d_2)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (1,2)(1,1)(2,2)(2,1)(3,2)(3,1) \end{Bmatrix}$$
$$(\Gamma \times \Delta)^{(f_2, d_1)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (2,1)(2,2)(3,1)(3,2)(1,1)(1,2) \end{Bmatrix}$$
$$(\Gamma \times \Delta)^{(f_2, d_2)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (2,2)(2,1)(3,2)(3,1)(1,2)(1,1) \end{Bmatrix}$$

$$(\Gamma \times \Delta)^{(f_3,d_1)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (3,1)(3,2)(1,1)(1,2)(2,1)(2,2) \end{Bmatrix}$$
$$(\Gamma \times \Delta)^{(f_3,d_2)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (3,2)(3,1)(1,2)(1,1)(2,2)(2,1) \end{Bmatrix}$$
$$(\Gamma \times \Delta)^{(f_4,d_1)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (1,1)(2,2)(2,1)(3,2)(3,1)(1,2) \end{Bmatrix}$$
$$(\Gamma \times \Delta)^{(f_4,d_2)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (1,2)(2,1)(2,2)(3,1)(3,2)(1,1) \end{Bmatrix}$$
$$(\Gamma \times \Delta)^{(f_5,d_1)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (1,1)(3,2)(2,1)(1,2)(3,1)(2,2) \end{Bmatrix}$$
$$(\Gamma \times \Delta)^{(f_5,d_2)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (1,2)(3,1)(2,2)(1,1)(3,2)(2,1) \end{Bmatrix}$$
$$(\Gamma \times \Delta)^{(f_6,d_1)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (2,1)(1,2)(3,1)(2,2)(1,1)(3,2) \end{Bmatrix}$$
$$(\Gamma \times \Delta)^{(f_6,d_2)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (2,2)(1,1)(3,2)(2,1)(1,2)(3,1) \end{Bmatrix}$$
$$(\Gamma \times \Delta)^{(f_7,d_1)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (3,1)(1,2)(1,1)(2,2)(2,1)(3,2) \end{Bmatrix}$$
$$(\Gamma \times \Delta)^{(f_7,d_2)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (3,2)(1,1)(1,2)(2,1)(2,2)(3,1) \end{Bmatrix}$$
$$(\Gamma \times \Delta)^{(f_8,d_1)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (3,1)(2,2)(1,1)(3,2)(2,1)(1,2) \end{Bmatrix}$$
$$(\Gamma \times \Delta)^{(f_8,d_2)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (3,2)(2,1)(1,2)(3,1)(2,2)(1,1) \end{Bmatrix}$$
$$(\Gamma \times \Delta)^{(f_9,d_1)} = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (2,1)(3,2)(3,1)(1,2)(1,1)(2,2) \end{Bmatrix}$$
$$(\Gamma \times \Delta)(f_9,d_2) = \begin{Bmatrix} (1,1)(1,2)(2,1)(2,2)(3,1)(3,2) \\ (2,2)(3,1)(3,2)(1,1)(1,2)(2,1) \end{Bmatrix}$$

Then the permutations in cyclic form $3^2 \times 2$ are

$$W_1 = \begin{Bmatrix} (1),(14)(25)(36),(123)(456),(153426),(132)(465), \\ (162435),(456),(142536),(465),(143625),(123), \\ (152634),(132),(163524),(134)(456),(35), \\ (123)(465),(15)(26)(34). \end{Bmatrix}$$

The Sylow subgroups of W_1 are;

$H_1 = \{(1),(14)(25)(36)\}$ which is simple

$H_2 = \{(1),(465),(456),(123)(456),(123),(123)(465),(132)(465),(132)(456),(132)\}$ which not is simple

2.2 Consider the permutation groups $C_2 = \{(1),(12)\}$ and $D_2 = \{(1),(34)\}$ acting on $X = \{1,2\}$ *and* $\Delta = \{3,4\}$ respectively. Let $P = C^{\Delta} = \{f: \Delta \to C\}$. Then $|P| = |C|^{|\Delta|} = 2^2 = 4$. The order of the wreath product is given by $|W_2| = |C_2|^{|\Delta|} \times |D_2|$

Then the permutations in cyclic form are

$W_2 = \{(1),(34),(12),(12)(34),(13)(24),(1324),(1423),(14)(23)\}$

$|W_2| = 2^2 \times 2 = 8$

The Sylow subgroup of W_2 is;

$H_3 = \{(1),(34),(13)(24),(1324),(14)(23),(1423),(12)(34),(12)\}$ which not is simple

2.3 Consider the permutation groups $C_3 = \{(1),(12)\}$ and $D_3 = \{(1),(123),(132)\}$ acting on $X = \{1,2,3\}$ *and* $\Delta = \{1,2\}$ respectively. Let $P = C^{\Delta} = \{f: \Delta \to C\}$. Then $|P| = |C|^{|\Delta|} = 2^3 = 8$. The order of the wreath product is given by $|W_3| = |C_3|^{|\Delta|} \times |D_3| = 2^3 \times 3$

Then the permutations in cyclic form are

$$W_3 = \begin{Bmatrix} (1),(56)(34),(34),(56),(12),(12)(56),(12)(34),(12)(34)(56), \\ (153)(264),(154263),(153264),(154)(263),(164253),(163)(254),(164) \\ (253),(163254),(135)(246),(135246),(136245),(136)(245),(146235), \\ (146)(235),(145)(236),(145236) \end{Bmatrix}$$

$$W_3 = 2^3 \times 3 = 24$$

The Sylow subgroups of W_3 are;

$H_4 = \{(1),(56),(34),(34)(56),(12)(34)(56),(12)(34),(12)(56),(12)\}$ which not is simple
$H_5 = \{(1),(153)(264),(135)(246)\}$ which is simple

2.4 Consider the permutation groups $C_4 = \{(1),(123),(132)\}$ and $D_4 = \{(1),(456),(465)\}$ acting on $X = \{1,2,3\}$ *and* $\Delta = \{4,5,6\}$ respectively. Let $P = C^{\Delta} = \{f: \Delta \to C\}$. Then $|P| = |C|^{|\Delta|} = 3^3 = 27$. The order of the wreath product is given by $|W_4| = |C_4|^{|\Delta|} \times |D_4|$
Then the permutations in cyclic form are

$$W_4 = \begin{Bmatrix} (1),(798),(789),(465),(465)(798),(465)(789),(456),(456)(798),(456)(789),(132),(132) \\ (798),(132)(789),(132)(465),(132)(465)(798),(132)(465)(789),(132)(456),(132)(456) \\ (798),(132)(456)(789),(123),(123)(798),(123)(789),(123)(465),(123)(465)(798),(123) \\ (465)(789),(123)(456),(123)(456)(798),(123)(456)(789),(174)(285)(396),(176395284), \\ (175286394),(174396285),(176284395),(175)(286)(394),(174285396),(176)(284)(395), \\ (175394286),(196385274),(195276384),(194)(275)(386),(196274385),(195)(276)(384), \\ (194386275),(196)(274)(385),(195384276),(194275386),(185296374),(184)(295)(376), \\ (186375294),(185)(296)(374),(184376295),(186294375),(185374296),(184295376), \\ (186)(294)(375),(147)(258)(369),(147369258),(147258369),(149368257),(149257368), \\ (149)(257)(368),(148259367),(148)(259)(367),(148367259),(169358247),(169247358), \\ (169)(247)(358),(168249357),(168)(249)(357),(168357249),(167)(248)(359), \\ (167359248),(167248359),(158269347),(158)(269)(347),(158347269),(157)(268) \\ (349),(157349268),(157268349),(159348267),(159267348),(159)(267)(348) \end{Bmatrix}$$

$$|W_4| = 3^3 \times 3$$

The Sylow subgroups of W_4 is;

$H_6 = \{(1),(798),(789),(465)(789),(465),(465)(798),(456)(798),(456)(789),(456),(123)(456)(789),$
$(123)(456),(123)(456)(798),(123)(798),(123)(789),(123),(123)(465),(123)(465)(798),(123)$
$(465)(789),(132)(465)(798),(132)(465)(789),(132)(465),(132)(456),(132)(456)(798),(132)$
$(456)(789),(132)(789)(132),(132)(798),(174)(285)(396),(176395284),(175286394),$
$(175)(286)(394),(174396285),(176284395),(176)(284)(395),(175394286),(174285396),$
$(186)(294)(375),(185374296),(184295376),(184)(295)(376),(186375294),(185296374),$
$(185)(296)(374),(184376295),(186294375),(195)(276)(384),(194386275),(196274385),$
$(196)(274)(385),(195384276),(194275386),(194)(275)(386),(196385274),(195276384),$
$(147)(258)(369),(147369258),(147258369),(149)(257)(368),(149368257),(149257368),$
$(148)(259)(367),(148367259),(148259367),(159)(267)(348),(159348267),(159267348),$
$(158)(269)(347),(158347269),(158269347),(157)(268)(349),(157349268),(157268349),$
$(168)(249)(357),(168357249),(168249357),(167)(248)(359),(167359248),(167248359),$
$(169)(247)(358),(169358247),(169247358)\}$ which is not simple

2.6 DIHEDRAL GROUPS

2.6.1 For $n = 3$

$D_6 = \{(1),(23),(132),(13),(123),(12)\}$
The Sylow subgroups of D_6 are;
$N_1 = \{(1),(23)\}$ which is simple
$N_2 = \{(1),(123),(132)\}$ which is simple

2.6.2 For $n = 4$

$D_8 = \{(1),(24),(13)(24),(13),(1432),(14)(23),(1234),(12)(34)\}$
The Sylow subgroup of D_8 is;
$N_3 = (1),(24),(13)(24),(13),(1432),(14)(23),(1234),(12)(34)\}$ which is not simple

2.6.3 For $n = 12$

D_{24} = {(1), (2 12)(311)(410)(5 9)(6 8), (1 9 5)(210 6)(311 7)(412 8),
(1 9)(2 8)(3 7)(4 6)(1012), (1 5 9)(2 610)(3 711)(4 812), (1 5)(2 4)(612)(711)(810),
(111 9 7 5 3)(21210 8 6 4), (111)(210)(3 9)(4 8)(5 7), (1 7)(2 8)(3 9)(410)(511)(612),
(1 7)(2 6)(3 5)(812)(911), (1 3 5 7 911)(2 4 6 81012), (1 3)(412)(511)(610)(7 9),
(1121110 9 8 7 6 5 4 3 2), (112)(211)(310)(4 9)(5 8)(6 7), (1 8 310 512 7 2 9 411 6),
(1 8)(2 7)(3 6)(4 5)(912)(1011), (1 4 710)(2 5 811)(3 6 912)
, (1 4)(2 3)(512)(611)(710)(8 9), (110 7 4)(211 8 5)
(312 9 6), (110)(2 9)(3 8)(4 7)(5 6)(1112), (1 611 4 9 2 712 510 3 8), (1 6)(2 5)(3 4)
(712)(811)(910), (1 2 3 4 5 6 7 8 9101112), (1 2)(312)(411)(510)(6 9)(7 8)}

The Sylow subgroups of D_8 are;

N_4 = {(1), (2 12)(311)(4 10)(5 9)(6 8), (1 7)(2 6)(35)(8 12)(9 11), (1 7)(2 8)(3 9)(410)(5 11)(612),
(1 4 710)(2 5 811)(3 6 912), (1,4)(2 3)(512)(6 11)(710)(8 9), (110)(29)(38)(4 7)(56)
(11 12), (1 10 74)(2 11 85)(3 12 96)} which is not simple

N_5 = {(1), (1 9 5)(210 6)(311 7)(412 8), (1 5 9)(2 610)(3 711)(4 812)} which is simple

TABLE 4.1: RESULT SUMMARY

Method/Type	Order of Group	Sylow Subgroup	Simplicity
Wreath Product	$3^2 \times 2$	2 3	T F
	$2^2 \times 2$	2	F
	$2^3 \times 3$	2 3	F T
	$3^3 \times 3$	3	F
Dihedral Group	2×3	2 3	T T
	2^3	2	F
	$2^3 \times 3$ 2 3	 2 3	 F T

Key: ***True = T*** ***False = F***

3.0 VALIDATION OF RESULTS

3.1 Algorithms for the result in 2.1

```
gap> C1:=Group((1,2,3));
Group([ (1,2,3) ])
gap> D1:=Group((4,5));
Group([ (4,5) ])
gap> W1:=WreathProduct(C1,D1);
Group([ (1,2,3), (4,5,6),
(1,4)(2,5)(3,6) ])
gap> H1:=SylowSubgroup(W1,2);
Group([ (1,4)(2,5)(3,6) ])
gap> H2:=SylowSubgroup(W1,3);
Group([ (1,3,2)(4,6,5), (1,2,3)(4,6,5)
])
gap> for i in H1 do;
> Print(i,"");
> od;
()(1,4)(2,5)(3,6)
gap> for i in H2 do;
> Print(i,"");
> od;
()(4,6,5)(4,5,6)(1,2,3)(4,5,6)(1,2,3)(1
,2,3)(4,6,5)(1,3,2)(4,6,5)(1,3,2)(4,5,6
)(1,3,2)
gap> IsSimple(H1);
true
gap> IsSimple(H2);
false
gap> quit;
```

3.2 Algorithms for the result in 2.2

```
gap> C2:=Group((1,2));
Group([ (1,2) ])
gap> D2:=Group((3,4));
Group([ (3,4) ])
gap> W2:=WreathProduct(C2,D2);
Group([ (1,2), (3,4), (1,3)(2,4) ])
gap> H3:=SylowSubgroup(W2,2);
Group([ (1,3)(2,4), (1,2), (1,2)(3,4)
])
gap> for i in H3 do;
> Print(i,"");
> od;
()(3,4)(1,3)(2,4)(1,3,2,4)(1,4)(2,3)(1,
4,2,3)(1,2)(3,4)(1,2)
gap> IsSimple(H3);
false
gap> quit;
```

3.3 Algorithms for the result in 2.3

```
gap> C3:=Group((1,2));
Group([ (1,2) ])
gap> D3:=Group((3,4,5));
Group([ (3,4,5) ])
gap> W3:=WreathProduct(C3,D3);
Group([ (1,2), (3,4), (5,6),
(1,3,5)(2,4,6) ])
gap> H4:=SylowSubgroup(W3,2);
Group([ (1,2)(3,4)(5,6), (1,2)(5,6),
(1,2)(3,4) ])
gap> for i in H4 do;
> Print(i,"");
> od;
()(5,6)(3,4)(3,4)(5,6)(1,2)(3,4)(5,6)(1
,2)(3,4)(1,2)(5,6)(1,2)
gap> H5:=SylowSubgroup(W3,3);
Group([ (1,3,5)(2,4,6) ])
gap> for i in H5 do;
> Print(i,"");
> od;
()(1,5,3)(2,6,4)(1,3,5)(2,4,6)
gap> IsSimple(H4);
false
gap> IsSimple(H5);
true
gap> quit;
```

3.8 Algorithms for the result in 2.4

```
gap> C4:=Group((1,2,3));
Group([ (1,2,3) ])
gap> D4:=Group((4,5,6));
Group([ (4,5,6) ])
gap> W4:=WreathProduct(C4,D4);
Group([ (1,2,3), (4,5,6), (7,8,9),
(1,4,7)(2,5,8)(3,6,9) ])
gap> H6:=SylowSubgroup(W4,3);
Group([ (1,4,7)(2,5,8)(3,6,9), (1,2,3),
(1,3,2)(4,5,6), (1,2,3)(4,5,6)(7,8,9)
])
gap> for i in H6 do;
> Print(i,"");
> od;
()(7,9,8)(7,8,9)(4,6,5)(7,8,9)(4,6,5)(4
,6,5)(7,9,8)(4,5,6)(7,9,8)(4,5,6)(7,8,9
)(4,5,6)(1,2,3)(4,5,6)(7,8,9)
(1,2,3)(4,5,6)(1,2,3)(4,5,6)(7,9,8)(1,2
,3)(7,9,8)(1,2,3)(7,8,9)(1,2,3)(1,2,3)(
4,6,5)(1,2,3)(4,6,5)(7,9,8)(1,2,3)
(4,6,5)(7,8,9)(1,3,2)(4,6,5)(7,9,8)(1,3
,2)(4,6,5)(7,8,9)(1,3,2)(4,6,5)(1,3,2)(
4,5,6)(1,3,2)(4,5,6)(7,9,8)(1,3,2)
(4,5,6)(7,8,9)(1,3,2)(7,8,9)(1,3,2)(1,3
,2)(7,9,8)(1,7,4)(2,8,5)(3,9,6)(1,7,6,3
,9,5,2,8,4)(1,7,5,2,8,6,3,9,4)
(1,7,5)(2,8,6)(3,9,4)(1,7,4,3,9,6,2,8,5
)(1,7,6,2,8,4,3,9,5)(1,7,6)(2,8,4)(3,9,
5)(1,7,5,3,9,4,2,8,6)(1,7,4,2,8,5,3,9,6
)
(1,8,6)(2,9,4)(3,7,5)(1,8,5,3,7,4,2,9,6
)(1,8,4,2,9,5,3,7,6)(1,8,4)(2,9,5)(3,7,
6)(1,8,6,3,7,5,2,9,4)(1,8,5,2,9,6,3,7,4
)
(1,8,5)(2,9,6)(3,7,4)(1,8,4,3,7,6,2,9,5
)(1,8,6,2,9,4,3,7,5)(1,9,5)(2,7,6)(3,8,
4)(1,9,4,3,8,6,2,7,5)(1,9,6,2,7,4,3,8,5
)
(1,9,6)(2,7,4)(3,8,5)(1,9,5,3,8,4,2,7,6
)(1,9,4,2,7,5,3,8,6)(1,9,4)(2,7,5)(3,8,
6)(1,9,6,3,8,5,2,7,4)(1,9,5,2,7,6,3,8,4
)
(1,4,7)(2,5,8)(3,6,9)(1,4,7,3,6,9,2,5,8
)(1,4,7,2,5,8,3,6,9)(1,4,9)(2,5,7)(3,6,
8)(1,4,9,3,6,8,2,5,7)(1,4,9,2,5,7,3,6,8
)
(1,4,8)(2,5,9)(3,6,7)(1,4,8,3,6,7,2,5,9
)(1,4,8,2,5,9,3,6,7)(1,5,9)(2,6,7)(3,4,
8)(1,5,9,3,4,8,2,6,7)(1,5,9,2,6,7,3,4,8
)
(1,5,8)(2,6,9)(3,4,7)(1,5,8,3,4,7,2,6,9
)(1,5,8,2,6,9,3,4,7)(1,5,7)(2,6,8)(3,4,
9)(1,5,7,3,4,9,2,6,8)(1,5,7,2,6,8,3,4,9
)
(1,6,8)(2,4,9)(3,5,7)(1,6,8,3,5,7,2,4,9
)(1,6,8,2,4,9,3,5,7)(1,6,7)(2,4,8)(3,5,
9)(1,6,7,3,5,9,2,4,8)(1,6,7,2,4,8,3,5,9
)
(1,6,9)(2,4,7)(3,5,8)(1,6,9,3,5,8,2,4,7
)(1,6,9,2,4,7,3,5,8)
gap> IsSimple(H6);
false
```

gap> quit;

```
gap>
```

gap> quit;

3.9 Algorithms for the result in 2.6.1

```
gap> D6:=DihedralGroup(IsGroup,6);
Group([ (1,2,3), (2,3) ])
gap> N1:=SylowSubgroup(D6,2);
Group([ (2,3) ])
gap> for i in N1 do;
> Print(i,"");
> od;
()(2,3)
gap> IsSimple(N1);
true
gap> N2:=SylowSubgroup(D6,3);
Group([ (1,2,3) ])
gap> for i in N2 do;
> Print(i,"");
> od;
()(1,3,2)(1,2,3)
gap> IsSimple(N2);
true
```

gap> quit;

3.10 Algorithms for the result in 2.6.2

```
gap> D8:=DihedralGroup(IsGroup,8);
Group([ (1,2,3,4), (2,4) ])
gap> N3:=SylowSubgroup(D8,2);
Group([ (2,4), (1,2,3,4), (1,3)(2,4) ])
gap> IsSimple(N3);
false
gap> for i in N3 do;
> Print(i,"");
> od;
()(2,4)(1,3)(2,4)(1,3)(1,4,3,2)(1,4)(2,
3)(1,2,3,4)(1,2)(3,4)
```

gap> quit;

3.11 Algorithms for the result in 2.6.3

```
gap> D24:=DihedralGroup(IsGroup,24);
Group([ (1,2,3,4,5,6,7,8,9,10,11,12),
(2,12)(3,11)(4,10)(5,9)(6,8) ])
gap> N4:=SylowSubgroup(D24,2);
Group([ (2,12)(3,11)(4,10)(5,9)(6,8),
(1,10,7,4)(2,11,8,5)(3,12,9,6),
(1,7)(2,8)(3,9)(4,10)(5,11)(6,12) ])
gap> for i in N4 do;
> Print(i,"");
> od;
()( 2,12)( 3,11)( 4,10)( 5, 9)( 6, 8)(
1, 7)( 2, 6)( 3, 5)( 8,12)( 9,11)( 1,
7)( 2, 8)( 3, 9)( 4,10)( 5,11)( 6,12)
( 1, 4, 7,10)( 2, 5, 8,11)( 3, 6,
9,12)( 1, 4)( 2, 3)( 5,12)( 6,11)(
7,10)( 8, 9)( 1,10)( 2, 9)( 3, 8)( 4,
7)( 5, 6)
(11,12)( 1,10, 7, 4)( 2,11, 8, 5)(
3,12, 9, 6)
gap> IsSimple(N4);
false
gap> N5:=SylowSubgroup(D24,3);
Group([ (1,5,9)(2,6,10)(3,7,11)(4,8,12)
])
gap> for i in N5 do;
> Print(i,"");
> od;
()( 1, 9, 5)( 2,10, 6)( 3,11, 7)( 4,12,
8)( 1, 5, 9)( 2, 6,10)( 3, 7,11)( 4,
8,12)
gap> IsSimple(N5);
true
```

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