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ON THE SOLUTION OF SOME OPTIMAL CONTROL PROBLEMS USING EMBEDDED CONTROL OPERATOR IN MODIFIED CONJUGATE GRADIENT METHOD

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ABSTRACT:
The conjugate gradient method, which is one of the most effective method of solving linear and unconstrained optimization problems, was modified in this paper to accommodate a control operator. We now applied the embedded control operator in the modified conjugate gradient method to solve some optimal control problems. The result obtained compares favourably with some existing results and analytical solutions.

KEYWORDS: Analytical solution, Optimal control problem, Control operator, Embed

1.0 INTRODUCTION

Optimal control is an important part of optimization which has a lot of applications in different areas, especially in engineering. It deals with the problem of finding a control law for a system in order to achieve a certain optimality criterion. Optimal control theory is a powerful mathematical tool that can be used to make decisions where the system is generally described by a state variable been acted upon by a suitable control function.

A lot of researchers had worked on different methods of solving optimal control problems. Adekunle and Olotu [1] presented an algorithm for solving optimal control of delay differential equations. Ejieji, [2] considered the gradient-based techniques for optimizing non-quadratic functionals. Ibiejugba and Onumanyi [3] constructed a control operator which was used to devise an Extended Conjugate Gradient Method (ECGM) algorithm for solving the Lagrange form of optimal control problems without delay in the state equations. Some Lagrange form of optimal control problems without delay in the state equations were solved in [5] by considering the repetition of the functional values and gradient norms as the basis of convergence. Olotu and Olorunsola [7] considered some algorithms for a discretized constrained continuous quadratic control problems. Omolehin [8] gave a comparative analysis of Extended Conjugate Gradient Method (ECGM) and Euler-Lagrange (E-L) algorithm for solving some optimal control problems. Olotu [6] considered the discretized algorithm for solving optimal control problems constrained by differential equations with real coefficients.

In this paper, we derived a method of solving optimal control problems by embedding a control operator constructed by [3] in the Modified Conjugate Gradient Method developed in [4]. The derived algorithm was now applied to solve some optimal control problems, without delay in the state equations, and the results obtained compared favourably with analytical solution and existing results

2.0 MATERIALS AND METHODS

By embedding the control operator Q constructed by [3] in the Modified Conjugate Gradient Method (MCGM) developed in [4], we have the Extended Modified Conjugate Gradient Method (EMCGM) algorithm as given below:

Step 1: *Guess the initial element,* $x_0, u_0 \in \mathcal{H}$

Step 2: *Compute the descent direction,* $p_{x,0} = -g_{x,0}$ *and*

$$p_{u,0} = -g_{u,0}$$

Step 3: *Set* $x_{i+1} = x_i + \alpha_{x,i} p_{x,i}$ *, Where* $\alpha_{x,i} = \frac{{g_{x,i}}^T g_{x,i}}{{p_{x,i}}^T Q p_{x,i}}$

Set $u_{i+1} = u_i + \alpha_{u,i} p_{u,i}$ *, Where* $\alpha_{u,i} = \frac{{g_{u,i}}^T g_{u,i}}{{p_{u,i}}^T Q p_{u,i}}$,

Step 4: *Compute* $g_{x,i+1} = g_{x,i} + \alpha_{x,i} Q p_{x,i}, \quad \forall\, i = 0, 1, 2, \ldots, n$

Compute $g_{u,i+1} = g_{u,i} + \alpha_{u,i} Q p_{u,i}, \quad \forall\, i = 0, 1, 2, \ldots, n$

Step 5: *Set* $p_{x,i+1} = -g_{x,i+1} + \beta_{x,i} p_{x,i}, \quad$ *Where* $\beta_{x,i} = \frac{-(g_{x,i+1} + g_{x,i})^T g_{x,i+1}}{{p_{x,i}}^T g_{x,i}}$

Set $p_{u,i+1} = -g_{u,i+1} + \beta_{u,i} p_{u,i}, \quad$ *Where* $\beta_{u,i} = \frac{-(g_{u,i+1} + g_{u,i})^T g_{u,i+1}}{{p_{u,i}}^T g_{u,i}}$

Step 6: *If* $g_i = 0$ *for some i then, terminate the sequence; else set*

$i = i + 1$ *and go to Step 3*

In the iterative steps 2 through 6 above, g_i denotes the gradient of the function f at x_i, p_i denotes the descent direction at i-th step of the algorithm and α_i denotes the step length of the descent sequence $\{x_i\}$. Step 3, 4, 5, and 6 of the algorithm reveal the crucial role of the linear operator Q in determining the step length of the descent sequence and also in generating a conjugate direction of search.

In this paper, we solved some optimal control problems, without delay in the state equations, by making use of the EMCGM algorithm and we compared our results with analytical solutions and existing results to confirm the accuracy. This makes our results to be better than the results in [5] which only considered the repetition of the function values and gradient norms as the basis of convergence without making recourse to analytical solution or any existing results.

3.0 NUMERICAL COMPUTATION

We tested the performance of our EMCGM algorithm on a number of optimal control problems, without delay in the state equations, and we compared our results with analytical solutions and the results obtained in [2]. The results are as shown below:

Problem 1

$$\min_{(x,u)} \int_0^1 \{x^2(t) + u^2(t)\}dt,$$

$$subject\ to\colon\ \dot{x}(t) = u(t);\ 0 \le t \le 1,$$

$$x(0) = 1,\ u(t) = 0.5$$

Table 1: Solution to Problem 1

ITR	X	U	FV (EMCGM)	ITR	FV (Ejieji, [2])	GRADIENT NORM (EMCGM)
0	1	0.5	1.37472500	0	1.373725	2.33426181
1	-0.31885345	-0.15942673	0.13976489	1	0.102190	1.28734450
2	-0.40012558	-0.23285936	0.24137606	3	0.284660	0.90882562
3	-0.45912946	-0.38314157	0.43083457	5	0.441401	0.10649548
4	-0.46888767	-0.45628469	0.53192021	7	0.576102	0.02069852
5	-0.40624263	-0.6340313	0.7675844	10	0.743442	0.00236367

Analytical objective function value = 0.761594, Ejieji [2]

Problem 2

$$\min_{(x,u)} \int_0^1 \{x^2(t) + u^2(t)\}dt$$

$$subject\ to\colon\ \dot{x}(t) = -\pi x(t) + u(t);\ 0 \le t \le 1$$

$$x(0) = 0.4,\ u(t) = 0.5$$

Table 2: Solution to Problem 2

ITR	X	U	FV (EMCGM)	ITR	FV (Ejieji, [2])	GRADIENT NORM (EMCGM)
0	0.4	0.5	1.1844925	0	1.1844925	2.2360679
1	-0.34825341	-0.55426743	0.72876327	1	0.02059500	0.1976736
2	-0.39821255	-0.59218592	0.69443739	5	0.127293	0.1873127
3	-0.41829461	-0.68831412	0.62008341	8	0.312082	0.09649621
4	-0.43297767	-0.79284695	0.60599202	10	0.619088	0.00298559

Analytical objective function value = 0.6082 Ejieji [2]

Problem 3

$$\min_{(x,u)} \int_0^1 \{x^2(t) + u^2(t)\}dt$$

$$subject\ to: \dot{x}(t) = -x(t) + u(t);\ 0 \le t \le 1$$

$$x(0) = 1,\ u(0) = 0.5$$

Table 3: Solution to Problem 3

ITR	X	U	FV (EMCGM)	ITR	FV (Ejieji, [2])	GRADIENT NORM (EMCGM)
0	1	0.5	1.373725	0	1.373725	2.23606798
1	-0.53355746	-0.26677873	0.39107693	1	0.089111	0.52986365e-1
2	-0.5348318	-0.26750026	0.39297005	5	0.297969	0.53382035e-1

3	**-0.44359279**	**-0.43794812**	**0.3858889**	**10**	**0.382982**	**0.01016495e-1**

Analytical objective function value = 0.385819, Ejieji [2]

4.0 DISCUSSION OF RESULTS

From table 1 above, we can easily see that the analytical function value for problem 1 is 0.761594 and the function value converges to 0.7675844 in just five iterations by using our algorithm. Table 2 gives the analytical function value for problem 2 as 0.6082 and the function value converges to 0.60599202 in just four iterations by using EMCGM algorithm. Finally, we can see from table 3 that the analytical function value for problem 3 is 0.385819 and the problem converges to 0.3858889 in just three iterations by using the new algorithm. In all the problems considered, our algorithm showed a high degree of accuracy when compared to the method used in [2].

5.0 CONCLUSION

Computationally, from the table of results for the problems considered, we can easily see that with the penalty parameter, $\mu = 10$ for all the problems, the results obtained compares favourably with the analytical solution and it showed improved convergence profile over the work of [2] by considering the analytical solutions.

6.0 REFERENCES

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