Global Scientific JOURNALS

GSJ: Volume 13, Issue 6, June 2025, Online: ISSN 2320-9186
www.globalscientificjournal.com

Optimal Vaccination and Quarantine Strategies for Control of COVID –19

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Abstract

We present in this paper, a mathematical model that describes the dynamics of COVID 19 epidemic in Nigeria considering the effect of vaccine and quarantine. The model dynamics is expressed by a system of eleven nonlinear differential equation. The optimal control analysis was established via the Pontryagin maximum principle. Numerical simulation analysis highlights the effect vaccination and quarantine in eradicating COVID-19 infections in Nigeria.

Keyword: COVID-19, Optimal Control, Vaccination, Numerical Simulation and Quarantine.

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1.0 Introduction

The outbreak of COVID – 19 was one of the major events in most parts of the world in general and Nigeria in particular in the year 2020. Corona virus disease 2019 (COVID-19) is a disease caused by severe acute respiratory syndrome corona virus 2 (SARS-COV-2). The virus was first identified in December 2019 in Wuhan, China city of Hubei Province [1]. It was named by the World Health Organization (W.H.O) on January, 10 2020 after the health authorities of China and Center for Disease Control Prevention (CDC) discovered the pathogen as a new type of Corona-virus. Following the advice of the International Health Regulation Emergency Committee, the Director – General of WHO declared the COVID – 19 outbreaks a Public Health Emergency of International Concern (PHEIC) on 30 January 2020 and characterized it as a pandemic on 11 March 2020 [2]. SARS-Cov-2 commonly known as Novel Coronavirus, is a single, positive stranded, RNA virus belonging to order Nidovirales [3], responsible for the Global Pandemic COVID – 19 [4]. Some of the other viruses related to this family of Coronavirus are SARS- Cov, responsible for SARS (Severe Acute Respiratory Syndrome) [5] and MERS – Cov (Middle East Respiratory Syndrome) [6].

The main clinical manifestations of the infection are fever, fatigue, respiratory symptoms (mainly dry coughs) and emergence of dyspnea [7]. COVID – 19 is mainly transmitted through droplets generated when an infected person coughs, sneezes or exhales. These droplets are too heavy to hang in the air and quickly fall on floors or surfaces. The survival life span of COVID -19 on surfaces varies depending on several factors such as: type of surface, temperature, humidity, Ultra Violet Light Exposure,
Most of the people who are infected with COVID – 19 virus experience mild to moderate respiratory illness and recover without needing special treatment. Older people or those who have underlying medical problems, for example, diabetes, cardiovascular disease, cancer and chronic respiratory disease are more severe to develop the illness [9]. Common symptoms of COVID – 19 are dry cough, fever, tiredness, sore throat, aches and shortness of breath [9]. Some individuals who are infected with the coronavirus can spread

it even when they have no symptoms. It is also called the incubation period (time from exposure to the development of symptoms, reported between 2 – 14 days).

Nigeria's response to the pandemic has included vaccination efforts, but the rollout has faced challenges such as vaccine hesitancy, limited access, and distribution issues. To optimize vaccination strategies and control the pandemic, it is essential to develop a mathematical model tailored to Nigeria's specific context.

The preventive measures include staying at least one meter away from people as much as possible and even greater distance when you are indoors, wearing of nose mask when you are around others. Avoid places that are crowded, confined or involve having close contacts with others especially when indoors. Washing your hands regularly and properly using soap and water. You can also use alcohol-based sanitizers if you do not have access to water and soap. Getting a COVID-19 vaccine when offered. Avoiding touching of noses, eyes and mouth. Keeping indoor spaces ventilated when you are inside with other people. You can do this by opening windows and doors to let fresh air in. Covering of mouth and nose with a clean tissue when you sneeze or cough, cover your face with the inside of your elbow. Only interact with people who are part of your household when outdoors, that is, if local restrictions allow- outdoor gatherings. This is so because out door gatherings are safer. Endeavour to clean surfaces regularly.

Before the development of COVID – 19 vaccine, Nigerian government had sensitized its citizenry on the need to adopt safety measures such as wearing of disposable surgical nose masks, regular hand- washing with plenty of soap under running water, and the use of alcohol – based hand sanitizer, in the absence of water as recommended by WHO [10].

Numerous models have been designed and used to understand the mechanisms for the spread, control and mitigation of COVID-19 in a community. Khoshnaw in [12] worked on "Mathematical modeling for corona virus disease (COVID-19) in predicting future behavior and sensitivity analysis". Musa [14] studied "Assessment of the impacts of the pharmaceutical and non-pharmaceutical intervention on COVID-19 in South-Africa using Mathematical models". Anwar [15] worked on "Mathematical model for coronavirus disease 2019 (COVID-19) containing isolation class". Sarita [16] worked on "Mathematical modeling of COVID-19 transmission; the roles of intervention strategies and lockdown".

The work is an extension of the work by [10]. As at the time they carried out their research, there was no approved COVID-19 vaccine. Since COVID-19 vaccinations have been introduced; it is the intention of the present study to look at the effect of vaccination in addition to other interventions in the dynamics of COVID-19,

2 Mathematical Formulation of the Model.

First, we describe the transmission dynamics, state the assumptions before we present the model.

2.1 Transmission Dynamics

We consider a deterministic compartmental modeling approach to describe the COVID-19 transmission dynamics in Nigeria. At any time, t, the population was divided into ten (10) compartments depending on the health status of the individuals. These compartments are as follows:

- Susceptible (denoted by S): Individuals are recruited into the susceptible class through birth and legal immigration at a rate, Λ. The susceptible class also gains individuals through recovery at a rate κ, Quarantined individuals who test negative to COVID – 19 are returned to susceptible class at a rate χ. A fraction of the susceptible individuals are vaccinated and are in a separate class. People exit the susceptible compartment either through infection induced by the disease with a force of infection given as:

$$\theta = \frac{\tau\left(a_1 E + a_2 Q + I_A + I_s + a_3 H + a_4 H_{IC}\right)}{N} \tag{2.1}$$

  Where $N = E + Q + I_A + I_S + H + H_{IC}$, a_1, a_2, a_3, a_4 are the modification factors for the exposed, quarantined, hospitalized and hospitalized in intensive care individuals respectively.

- Vaccinated (denoted by V): Since COVID – 19 vaccine is now available, it is realistic to consider a specific vaccinated class, V. This compartment gains population from the susceptible individuals who later go for COVID – 19 vaccine. The vaccine available does not have 100% efficacy. Vaccine efficacy against symptomatic disease was assumed to be 88% on the basis of Pfizer – Bio N Tech and Oxford Astrazeneca vaccines being administered in the UK, and protection against infection were varied from 0% to 85%. As a matter of fact, some of those vaccinated can be exposed to the disease and are taken to the exposed class at a rate of ε
- Exposed (denoted by E): The individuals in this class are in the incubation period after being infected by the disease pathogen. Some of the individuals in the exposed class are quarantine, some become asymptomatically infected (i.e. infected but with mild or no visible symptoms) and some become symptomatically infected (with visible symptoms)
- Quarantined (denoted by Q): Individuals are kept in a separate compartment during the incubation period that lasts for 14 days. They are tested for the virus within this period. Those who test negative are returned to the Susceptible, S(t) compartment while those who test positive are taken to the hospital for treatment.
- Infected Asymptomatic (denoted by I_A): From the exposed class, some individuals have no visible clinical signs. They could infect other people but with a lower probability than people in the symptomatically infectious class [17]
- Infected Symptomatic (denoted by I_S): This compartment comprises of individuals who are infected and have visible symptoms of COVID – 19
- Hospitalized (denoted by H): Both individuals in the I_A, I_S and Q are taken to the hospitalized class where they can be treated by trained personnel.
- Hospitalized in intensive care (denoted by H_{IC}): Some of the individuals in the hospitalized compartment are taken to intensive care unit because of the severity of their condition.
- Recovered (denoted by R): It is assumed that when proper COVID – 19 treatments are given to infected individuals in their various compartments, most of these infected individuals recover from the disease. It is not established that the disease gives permanent immunity to those who recover from the disease, because of this, the recovered individuals from COVID – 19 disease become susceptible to the virus again after a short period of time
- Disease induced dead (denoted by D): All the individuals who died as a result of COVID – 19 disease are taken to the dead compartment.

2.2 Assumptions of the Model

(a) There is no vertical transmission of the infection from mother to unborn baby
(b) There are some individuals in the susceptible and vaccinated classes who have had sufficient interaction with the infected individuals or contaminated surfaces or objects [18]
(c) There is homogeneous mixing of individuals in the different groups. The population mixes randomly and uniformly, ignoring age, gender, and spatial structure [19]
(d) Certain intervention procedures were introduced which includes vaccination, isolation, quarantine, use of nose mask, hand sanitizers etc.
(e) There may be disease induced deaths at infectious compartments whereas natural death rate is the same in all the compartments.
(f) There are available and accessible vaccine.
(g) The recovered individuals from COVID – 19 disease become susceptible to the virus again after a short period.
(h) This model equally assumes that the population size remains constant, ignoring migration, births, or deaths [20]
(i) There is no healthcare system overload as our healthcare can handle the case load ignoring potential overload and resource constraints.
(j) The model assumes no co – infections with other diseases, ignoring potential interactions [21]
(k) It also assumes uniform transmission dynamics across Nigeria, ignoring regional differences [22]

2.3 Model Parameters and Description

Λ Recruitment rate through birth and legal immigration

α Rate at which individuals go for vaccination

χ Rate at which quarantined individuals return to the susceptible populations if they test negative after 14 days

δ Rate associated with movement from E to Q

φ Rate of progression from quarantined to hospitalized class

ε Rate of progression from vaccinated class to exposed class

ω Rate associated with movement from exposed to infected class

κ Rate at which recovered individuals migrate to the susceptible compartment.

ν The recovery rate of individuals in the symptomatically infected compartment.

ρ Rate associated with movement from H to H_{IC}

η Recovery rate of hospitalized individuals

π_A Rate at which I_A migrate to H

π_S Rate at which I_S migrate to H

σ_1 The COVID – 19 disease mortality rates for individuals in the H compartment.

σ_2 The COVID 19 disease mortality rate for individuals in the H_{IC} compartment.

γ The recovery rate of the asymptomatically infected individuals

ϕ Recovery rate of individuals in intensive care unit

μ Natural death rate

p Fraction of susceptible individuals vaccinated

$(1-p)$ Fraction of susceptible individuals moved to the exposed class

q Fraction of exposed individuals with mild or no symptoms

$(1-q)$ Fraction of exposed individuals with visible symptoms

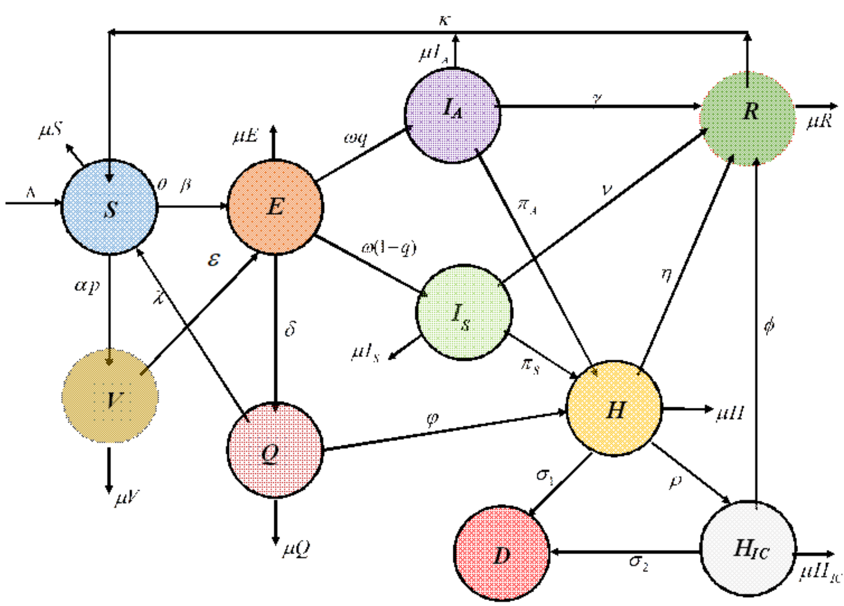
β Transmission rate from susceptible to exposed

ι Transmission rate

θ Force of infection

2.4 The Mathematical Model

The proposed model in this PAPER is displayed in the multi-compartmental diagram shown in the diagram below



2.5 The Model Equation

$$
\begin{aligned}
\frac{dS}{dt} &= \Lambda + \chi Q - \theta\beta S - (\alpha p + \mu) S \\
\frac{dV}{dt} &= \alpha p S - \mu V \\
\frac{dE}{dt} &= \theta\beta S - (\delta + \omega + \mu) E \\
\frac{dQ}{dt} &= \delta E - (\chi + \varphi + \mu) Q \\
\frac{dI_A}{dt} &= \omega q E - (\pi_A + \gamma + \mu) I_A \\
\frac{dI_S}{dt} &= \omega (1-q) E - (\pi_S + \nu + \mu) I_S \\
\frac{dH}{dt} &= \pi_S I_S + \varphi Q + \pi_A I_A - (\eta + \sigma_1 + \rho + \mu) H \\
\frac{dH_{IC}}{dt} &= \rho H - (\phi + \sigma_2 + \mu) H_{IC} \\
\frac{dR}{dt} &= \nu I_S + \gamma I_A + \eta H + \phi H_{IC} - (\kappa + \mu) R \\
\frac{dD}{dt} &= \sigma_1 H + \sigma_2 H_{IC} - \mu_0 D \qquad (2.2) \\
\frac{dN}{dt} &= \pi - \sigma_1 H - \sigma_2 H_{IC} - \mu N
\end{aligned}
$$

3.0 OPTIMAL CONTROL ANALYSIS OF COVID-19

In this section, we employ the Pontryagin maximum Fprinciple (PMP) to perform optimal control analysis on using vaccination and quarantining as control measures for COVID-19. Basically, the aim is to find the optimal state variables and the control parameters that can minimize the number of people exposed or infected by the epidemics, and at the same time, minimize the cost of implementing the various control measures. If the disease is vector-borne, the major effort would be to minimize the population of the vector that carries the disease-causing pathogen.

3.1 The Pontryagin maximum principle

The Pontryagin maximum principle converts a constrained control problem to unconstrained one by introducing additional variable to the original problem.

Definition 1: Let $[t_0, T] \subset \mathbb{R}, \bigcup$ be a bounded subset of $\mathbb{R}^m$, and u a measurable function defined by $u : [t_0, T] \to \bigcup$. Consider a fixed $u,$ and a constant x_0 then $u(t)$ is called the control applied at time t.

Definition 2: The function $y(t) : [t_0, T] \to \mathbb{R}^n$ is called the system state space or trajectory corresponding to control $u(t)$ and initial condition y_0.

Definition 3: Let $f = (f_1, f_2, f_3, ..., f_n) : \mathbb{R}^n \to \mathbb{R}$ be a function that is continuously differentiable on $\mathbb{R}^n$ and continuous on $\bigcup$. Consider the function $f : \mathbb{R}^n \times \bigcup \to \mathbb{R}$ that is continuous in $\bigcup$ and continuously differentiable on $\mathbb{R}^n$, then the cost function is a function of the form

$$J(y,u)=\int_{t_0}^{T} f\left(y(t),u(t)\right)dt \tag{2.3}$$

The aim of every optimal control problem is to find a control $u(t)$ and a corresponding state variable, $y(t)$ that minimize the cost function, $J(y,u)$. Mathematically, this means to find an admissible pair, (y^*,u^*) ϶

$$J(y^*,u^*)\le J(y,u) \tag{2.4}$$

$\forall (y,u)\in \mathrm{B}$, where B is the set of all admissible pairs. The Pontryagin maximum principle provides the condition under which (y^*,u^*) is optimal. It states that suppose y^* and u^* are optimal for the control problem

$$\max \int_{t_0}^{T} f\left(y(t),(t)\right)dt$$

subject to

$$\frac{dy(t)}{dt}=g\left(y(t),u(t)\right),\;\; y(t_o)=y_0,$$

then, there exists a piecewise differential function $\phi(t)$ called the adjoint variable or costate such that the Hamiltonian, H satisfies the following conditions

$$\mathrm{H}(t,y,u,\phi)\le \mathrm{H}(t,y^*,u^*,\phi) \tag{2.5}$$

$$\frac{d\mathrm{H}(t,y,u,\phi)}{dt}=0 \tag{2.6}$$

$$\frac{d\phi(t)}{dt}=-\frac{d\mathrm{H}(t,y^*,u^*,\phi)}{dy} \tag{2.7}$$

$\forall u(t)$ at each time, t. The adjoint variable, $\phi(t)$ satisfies the transversality condition, $\phi(T)=0$.

3.2 Optimal Control Problem

In order to control the spread of COVID-19, we need to increase the number of susceptible persons that are successfully vaccinated, and increase the number of exposed persons that are quarantined. Therefore, the optimal control problem is stated as follows: Find the optimal state vector, $y^*=\left(S^*,V^*,E^*,Q^*,I_A^*,I_S^*,H^*,H_{IC}^*,R^*,D^*,N^*\right)$ and the optimal control vector $u^*=\left(\alpha^*,\delta^*\right)$, which minimizes the objective or cost function:

$$J\left(y;(\alpha,\delta)\right)=\int_0^T\left(A_1E(t)+A_2I_A(t)+A_3I_S(t)+\frac{1}{2}A_4\alpha^2(t)+\frac{1}{2}A_5\delta^2(t)\right)dt \tag{2.8}$$

Where A_1, A_2, A_3, A_4 and A_5 are positive constants.

Therefore, the Hamiltonian of the optimal control problem is given by

$$
\begin{aligned}
&\mathrm{H}(t,y,\alpha,\delta,\lambda)\\
&=A_1E(t)+A_2I_A(t)+A_3I_S(t)+\frac{1}{2}A_4\alpha^2(t)+\frac{1}{2}A_5\delta^2(t)+\lambda_1\frac{dS}{dt}+\lambda_2\frac{dV}{dt}+\lambda_3\frac{dE}{dt}\\
&+\lambda_4\frac{dQ}{dt}+\lambda_5\frac{dI_A}{dt}+\lambda_6\frac{dI_S}{dt}+\lambda_7\frac{dH}{dt}+\lambda_8\frac{dH_{IC}}{dt}+\lambda_9\frac{dR}{dt}+\lambda_{10}\frac{dD}{dt}+\lambda_{11}\frac{dN}{dt} \qquad (2.9)
\end{aligned}
$$

Where y is the vector of the state variables and λ is the vector of the adjoint variables.

The partial derivatives of H with respect to the state variables are

$$
\begin{aligned}
&\frac{\partial \mathrm{H}}{\partial S}=\frac{\theta\beta(N-S)}{N}(\lambda_3-\lambda_1)-(\alpha p+\mu)\lambda_1+\alpha p\lambda_2\\
&\frac{\partial \mathrm{H}}{\partial V}=\frac{\theta\beta}{N}S(\lambda_1-\lambda_3)-\mu\lambda_2\\
&\frac{\partial \mathrm{H}}{\partial E}=\beta a_1\tau\frac{(N-E)S}{N^2}(\lambda_3-\lambda_1)-\mathrm{K}_1\lambda_3+\delta\lambda_4+\omega q\lambda_5+\omega(1-q)\lambda_6+A_1\\
&\frac{\partial \mathrm{H}}{\partial Q}=\beta a_2\tau\frac{(N-Q)S}{N^2}(\lambda_3-\lambda_1)+\chi\lambda_1-\mathrm{K}_2\lambda_4+\varphi\lambda_7\\
&\frac{\partial \mathrm{H}}{\partial I_A}=\beta\tau\frac{(N-I_A)S}{N^2}(\lambda_3-\lambda_1)-\kappa_3\lambda_5+\pi_A\lambda_7+\gamma\lambda_9+A_2\\
&\frac{\partial \mathrm{H}}{\partial I_S}=\beta\tau\frac{(N-I_S)S}{N^2}(\lambda_3-\lambda_1)-\mathrm{K}_4\lambda_6+\pi_S\lambda_7+\nu\lambda_9+A_3\\
&\frac{\partial \mathrm{H}}{\partial H}=\beta a_3\tau\frac{(N-H)S}{N^2}(\lambda_3-\lambda_1)-\mathrm{K}_5\lambda_7+\rho\lambda_8+\eta\lambda_9+\sigma_1\lambda_{10}-\sigma_1\lambda_{11}\\
&\frac{\partial \mathrm{H}}{\partial H_{IC}}=\beta a_4\tau\frac{(N-H_{IC})S}{N^2}(\lambda_3-\lambda_1)-\mathrm{K}_6\lambda_8+\phi\lambda_9+\sigma_2\lambda_{10}-\sigma_2\lambda_{11}\\
&\frac{\partial \mathrm{H}}{\partial R}=\frac{\theta\beta}{N}S(\lambda_1-\lambda_3)-\mathrm{K}_7\lambda_9+\kappa\lambda_1\\
&\frac{\partial \mathrm{H}}{\partial D}=-\mu_0\lambda_{10}\\
&\frac{\partial \mathrm{H}}{\partial N}=\frac{\theta\beta}{N}S(\lambda_1-\lambda_3)-\mu\lambda_{11} \qquad (2.10)
\end{aligned}
$$

Hence, using (2.7), we get that the adjoint variables $\lambda_i i=1,2,3,\ldots,11$ satisfy the following system of ordinary differential equations

GSJ: Volume 13, Issue 6, June 2025
ISSN 2320-9186

$$\frac{d\lambda_1}{dt} = \frac{\theta\beta(N-S)}{N}(\lambda_1-\lambda_3)+\alpha p(\lambda_1-\lambda_3)+\mu\lambda_1$$
$$\frac{d\lambda_2}{dt} = \frac{\theta\beta}{N}S(\lambda_3-\lambda_1)+\mu\lambda_2$$
$$\frac{d\lambda_3}{dt} = \beta a_1\tau\frac{(N-E)S}{N^2}(\lambda_1-\lambda_3)+\mathrm{K}_1\lambda_3-\delta\lambda_4+\omega q(\lambda_6-\lambda_5)-\omega\lambda_6-A_1$$
$$\frac{d\lambda_4}{dt} = \beta a_2\tau\frac{(N-Q)S}{N^2}(\lambda_1-\lambda_3)-\chi\lambda_1+\mathrm{K}_2\lambda_4-\varphi\lambda_7$$
$$\frac{d\lambda_5}{dt} = \beta\tau\frac{(N-I_A)S}{N^2}(\lambda_1-\lambda_3)+\mathrm{K}_3\lambda_5-\pi_A\lambda_7-\gamma\lambda_9-A_2$$
$$\frac{d\lambda_6}{dt} = \beta\tau\frac{(N-I_S)S}{N^2}(\lambda_1-\lambda_3)+\mathrm{K}_4\lambda_6-\pi_S\lambda_7-\nu\lambda_9-A_3$$
$$\frac{d\lambda_7}{dt} = \beta a_3\tau\frac{(N-H)}{N^2}(\lambda_1-\lambda_3)+\mathrm{K}_5\lambda_7-\rho\lambda_8-\eta\lambda_9-\sigma_1(\lambda_{11}-\lambda_{10})$$
$$\frac{d\lambda_8}{dt} = \beta a_4\tau\frac{(N-H_{IC})S}{N^2}(\lambda_1-\lambda_3)+\mathrm{K}_6\lambda_8-\phi\lambda_9-\sigma_2(\lambda_{11}-\lambda_{10})$$
$$\frac{d\lambda_9}{dt} = \frac{\theta\beta}{N}S(\lambda_3-\lambda_1)+\mathrm{K}_7\lambda_9-\kappa\lambda_1$$
$$\frac{\partial\lambda_{10}}{dt} = \mu_0\lambda_{10}$$
$$\frac{\partial\lambda_{11}}{\partial t} = \frac{\theta\beta}{N}S(\lambda_3-\lambda_1)+\mu\lambda_{11} \tag{2.11}$$

The partial derivatives of H with respect to the control variables are

$$\frac{\partial\mathrm{H}}{\partial\alpha^*} = A_4\alpha^*-\lambda_1 pS+\lambda_2 pS \tag{2.12}$$

$$\frac{\partial\mathrm{H}}{\partial\delta^*} = A_5\delta^*-\lambda_3 E(t)+\lambda_4 E(t) \tag{2.13}$$

The optimality condition requires that $\frac{\partial\mathrm{H}}{\partial\alpha^*} = \frac{\partial\mathrm{H}}{\partial\delta^*} - 0$. This gives

$$A_4\alpha^*-\lambda_1 pS(t)+\lambda_2 pS(t)=0$$
$$A_5\delta^*-\lambda_3 E(t)+\lambda_4 E(t)=0$$

Solving for each of the control variables, we get the optimal control variables that minimizes the objective function as

$$\alpha^*(t) = \frac{(\lambda_1(t)-\lambda_2(t))pS^*(t)}{A_4} \tag{2.14}$$

$$\delta^*(t) = \frac{(\lambda_3(t) - \lambda_4(t))E^*(t)}{A_5} \tag{2.15}$$

Hence, the characterizations of the optimal control variables are

$$\alpha^*(t) = \max\{0, \min(\alpha, \Theta_1)\}$$
$$\delta^*(t) = \max\{0, \min(\delta, \Theta_2)\}$$

Where

$$\Theta_1 = \frac{(\lambda_1(t) - \lambda_2(t))\, pS^*(t)}{A_4}$$
$$\Theta_2 = \frac{(\lambda_3(t) - \lambda_4(t))E(t)}{A_5}$$

4.0 Numerical Simulation

In this section, we present some numerical results to demonstrate our optimal analysis on the dynamics of the COVID-19 model. we conduct numerical experiment using the parameters in Table 4.1 given below to visualize the effect of optimal use of vaccination and quarantining for effective control of COVID – 19. That is, we see the effect of indirect time – dependent control parameters, $\alpha(t)$ and $\delta(t)$ on the relevant compartments. The values of the constants A_1, A_2, A_3, A_4 and A_5 used are 100,100, 50, 1, and 1, respectively. The result of the implementation is presented graphically in figure 4.1-figure 4.7. The figures show the effect of optimally applying the stated control measures in controlling the spread of covid-19. Specifically, we show the effect of optimal control on the number of persons that are exposed to covid-19, the number of persons that are infectious with symptoms, the number of persons that are hospitalized due to covid-19 infection, the number of persons that are hospitalized in the intensive care unit of hospital, and the number of covid-19 casualties. Each figure shows the number of persons in these classes when there is control and when there is optimal control.

The baseline parameter values

Parameter	Sensitivity Indices	Parameter Value
β	+1	0.02
τ	+1	0.4
a_1	+0.3628	0.2
a_2	+0.3553	0.32
a_3	+0.0096	0.13
a_4	+0.0232	0.233
q	-0.1687	0.7

α	-0.9959	0.04
p	-0.9959	0.6
δ	-0.7240	0.36
ω	0.0344	0.13
φ	-0.2001	0.95
χ	-0.1596	0.72
π_A	-0.2117	0.9
γ	-0.0340	0.1398
π_S	-0.0450	0.8
ν	-0.0300	0.4
η	-0.0105	0.25
σ_1	-0.0063	0.151
ρ	-0.0160	0.38
ϕ	-0.0142	0.45
σ_2	-0.0091	0.288

Table 4.1: Sensitivity indices of the parameters

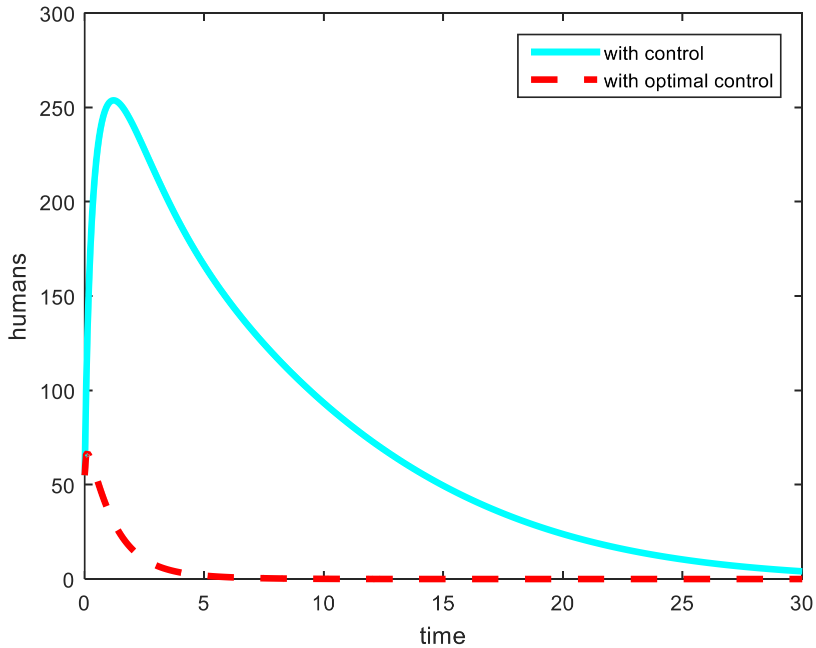


Figure1: Graph showing the number of people that are exposed to the disease.

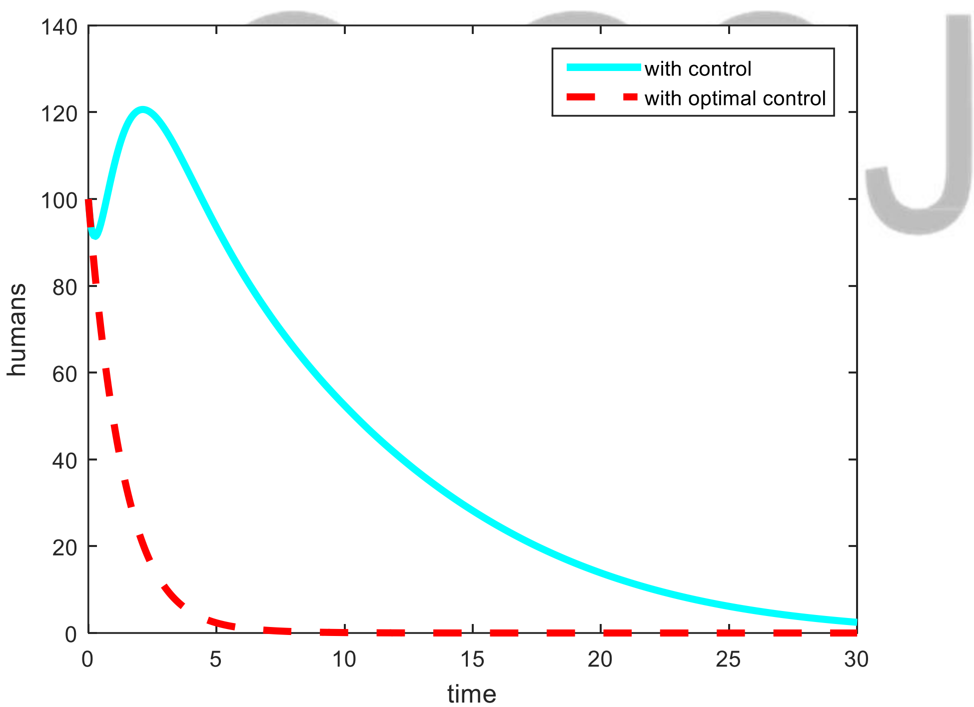


Figure 2:Graph showing the number of people that are symptomatically infected

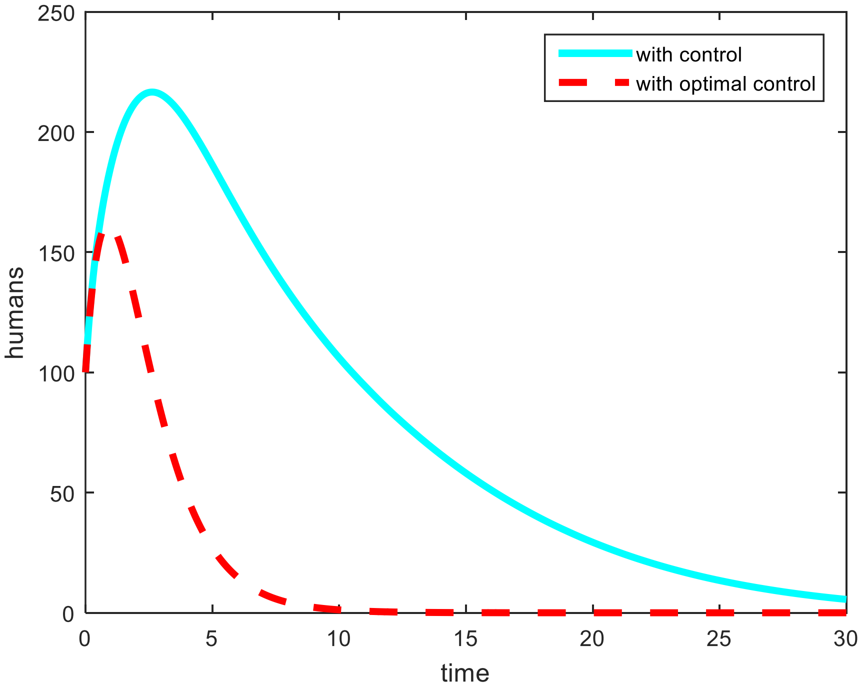


Figure 3: Graph showing the number of those that are hospitalized

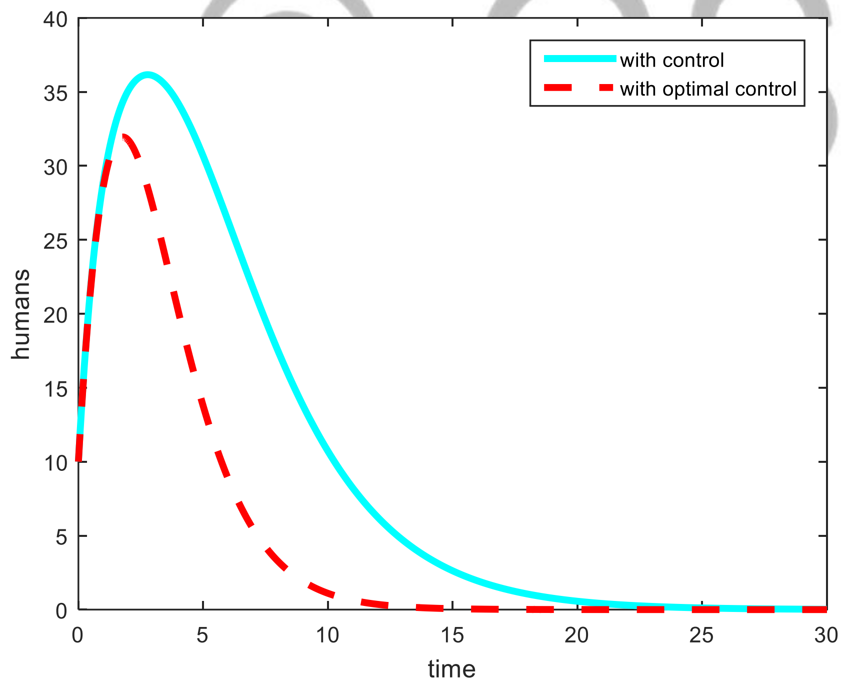


Figure 4: Graph showing the number of those hospitalized in intensive care

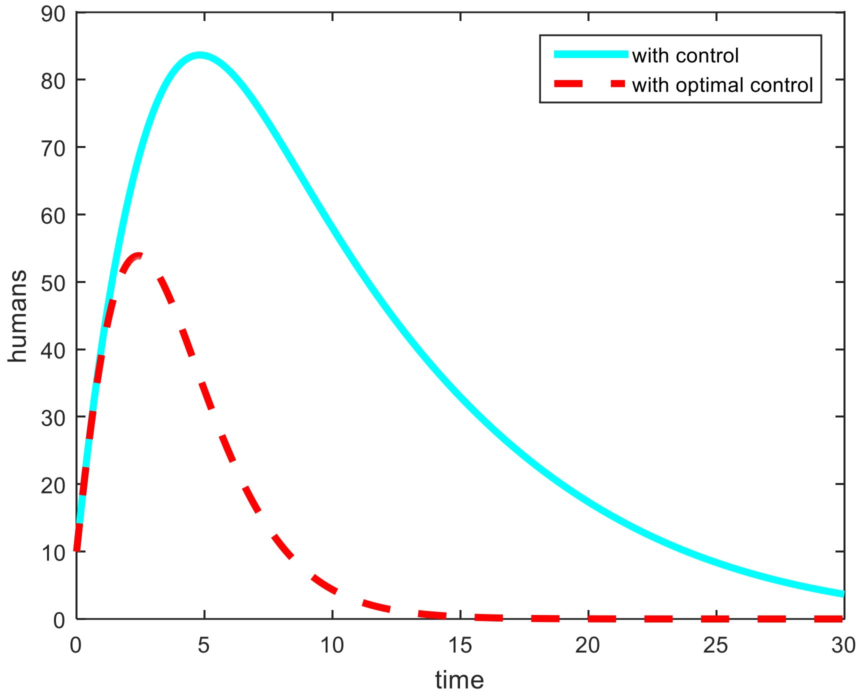


Figure 5: Graph showing the number of deceased persons

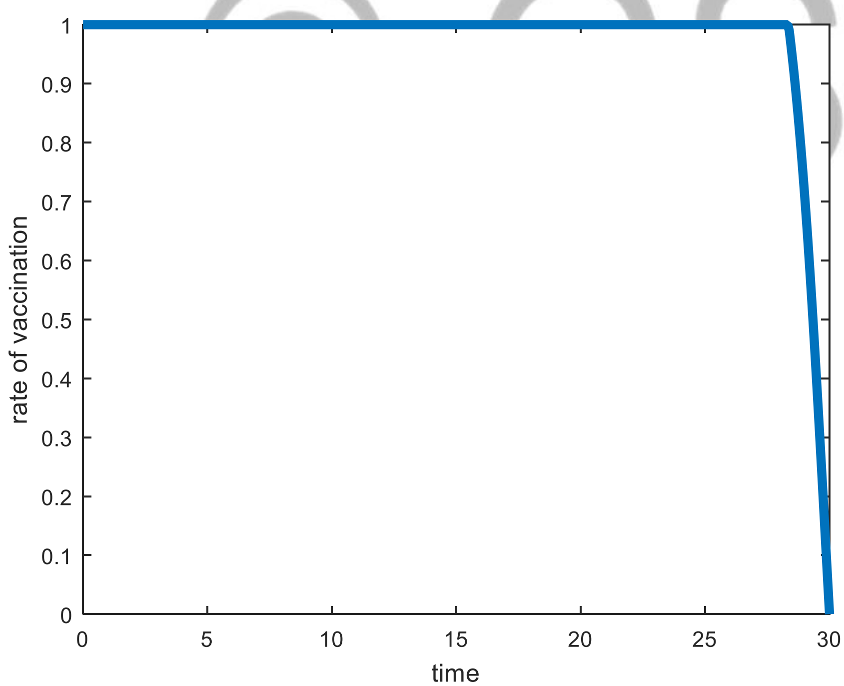


Figure 6: Graph showing rate at which people are optimally vaccinated

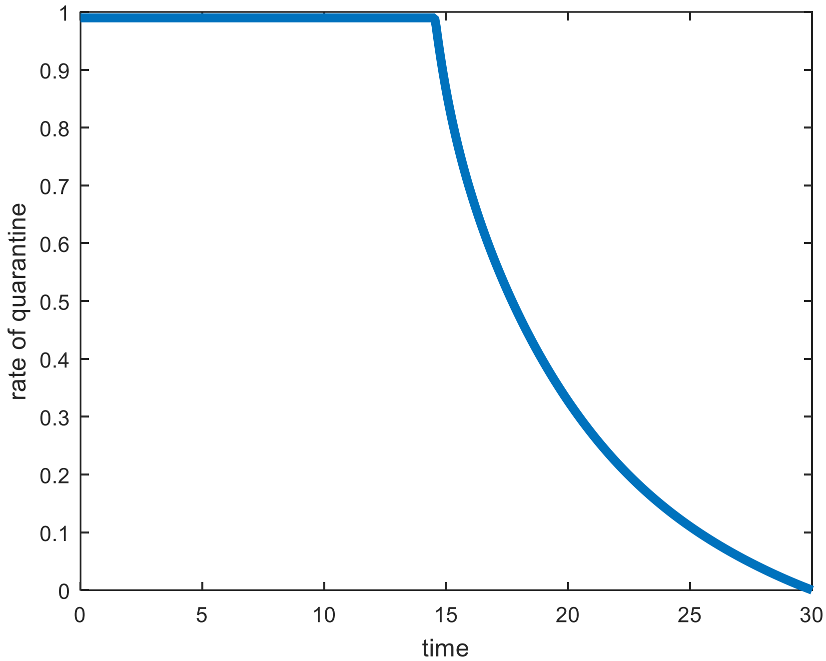


Figure 7: Graph showing rate at which people are optimally quarantined

5.0 Discussion

In figure 1, the effect of optimal control on the number of persons exposed to covid-19 is presented. The graph shows that the number of persons exposed to covid-19 is reduced when optimal control is applied. However, there is sharp increase in the number of exposed persons when optimal control is applied. This sharp and sudden increase could be attributed to the inability of some persons in the protection class to continue to observe adequate protection measures Figure 2, shows that the number of persons that are symptomatically infectious are reduced when optimal control is applied. This is because the number of persons that are exposed to covid-19 have been reduced. The same trend is noticed in the number of persons that are hospitalized in general hospital ward and in the intensive care unit (figure 3 and figure 4). In figure 5, the number of casualties can be seen to decrease substantially when optimal control is applied. This is because the number of persons admitted in the hospitals have reduced due to the application of various control measures. In figure 6 and figure 7, the optimal rates of vaccination and quarantining to be applied in order to effectively control the spread of covid-19 are respectively displaced. The figures show that, in order to effectively control the spread of the disease, large number of people need to be vaccinated initially. The rate of vaccination decreases as time increases until all the persons in the susceptible class have been successfully vaccinated. Similarly, the rate of quarantining those that are exposed to Covid-19 should be high initial but decreases as time increases.

6.0 Conclusion

This paper presents the spread of COVID-19 using differential equation approach made up of 11 equations to describe the dynamics of the disease transmission. Adopting the Pontryagin maximum principle, the optimal control analysis was established. Numerical experiment is implemented in MATLAB to generate the numerical solution. The result shows that COVID-19 transmission could be optimal control whenever optimal rates of vaccination and quarantine are applied efficiently.

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