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PENTANOMIAL LATTICE MODELS IN OPTION PRICING

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ABSTRACT

This paper defines options as derivatives connecting people with unique interests. It establishes how the values of the derivatives are based on the agreements about the financial assets in question and how the rights rather than commitments of buyers to purchase or sell the assets are more impactful. The deal occurs within a specific duration. Lattices are discontinuous time presentation of the evolution of the asset price, which incorporates skewness and kurtosis. The model entails splitting time intervals and constructing positive branch probabilities with emphasis on the matching procedures. Lattices are compound Poisson processes due to their limited distribution. The lattice model is used because it facilitates the estimation of the spread of the asset cost for each time step. Hence it is efficient and easy. The paper focuses on Asian options which are significantly dependent on the previous knowledge. The payments for Asian options are based on the average prices during the period preceding maturity. The amount spent on such options is a significant issue and projection of the future possible price is significantly complicated. In this paper, there is an evaluation of the utilization of pentanomial, lattice model in the pricing of Asian options.

INTRODUCTION.

In the last 30 plus years, derivatives such as options became predominant in the world of finance. The concept of option contracts is thought to have started moments earlier than 1973. In their early stages option contracts were likened to Over-the-counter (OTC) (Wilmott, 1995). An intermediary or a broker was therefore involved in this type of trading. The negotiator was commonly referred to as the option broker. Their role was to negotiate the price of the option on behalf of the seller and the buyer whenever an option was available for sale. However, the lack of proper standardization of the option contracts led to inappropriate handling of these contracts. OTC could however handle the option contracts due the few companies involved. Official exchange of option contracts started in 1973 following the replacement of OTC with the modern financial option market. The Black Scholes Model which is an options pricing model was also introduced in 1973.

By definition, assents encompass commitment derivatives that connect people with vested interests. The value of these derivatives is achieved through agreements of the seller and buyer of the financial asset options. During the commitments, the buyers pay premium prices to the seller. The classification of the options is based on the exercise dates .As an illustration, the European and American Options are utilized at the lapse date only while the utilization of the American options can take place any time before the lapse date. The classification of the options can also be based on the options of purchasing or selling an asset. Here, the former option entails a justified reason to buy the asset while there about the intention to dispose of it.

Lattice method is one of the proposed approaches to pricing options. The method was first proposed by Cox (1979) and Rendleman & Bartter (1979). Cox (1979) established a binomial model of lattice using the option evaluation principles without arbitraging his model, lattice to price Brownian motion comprising of the passion process was used. One of the most important characteristics of this representation was that the compatibility of the Brownian motion lattice with the Black Scholes formula that was utilized in the European options. Since their inception, lattices have been pivotal in the pricing process even in the American options. The versatility and coherence of the presentations pave the way for exploration of various appendages concerning the pricing options. The pricing lattices have been advanced to facilitate pricing of multiple assets and multimodal pricing of a single asset as suggested by Boyle(1998), Amin(1993 and Yamada & Primbs (2001)

The model of binomial lattice that is often referred to as CRR values various options on condition that the returns of those assets have a normal distribution. The models were developed by equating the mean and variance of discrete variables across a short time interval with those of continuous random variables. When utilizing it, the financial market agents and investors hold each of the factors constant. With the success of the approach, other multinomial lattice methods with a capacity to value increasingly complex options on numerous underlying variables have been proposed. As an illustration, the CRR binomial lattice model was extended to for a single underlying variable was developed by Boyle (1988).

The trinomial model is an extension of the binomial. The two braces (price and share rise and fall) is expanded to price and share rise, fall, or remain constant. Recombination of the trinomial model leads to the generation of trinomial lattice and imply that nodes ending up with the same price at the same time are taken as one on. The phenomenon arises from the results obtained by multiplying the nod’s stock price by any of the three factors. It is critical to note that the three factors in the form of ratios (1, <1 and >1). Besides, there are risk-neutral probabilities whose role is to tell whether the price will remain the same, increase or decrease. The approach has been applied in the European, Americana, Asian, barrier and look-back options. Another exotic option, the Russian option is rarely used in the stock markets.

The Markov tree approach for option costing was advanced by Bhat, & Kumar. The scholars utilized a non-iid procedure which encompassed alteration of the binomial model of option pricing that uses the first-order Markov behaviour. The other approach was a Markov tree and a wind-up incorporating a combination of normal distribution. The models mentioned above contents the first two moments (Variance and mean) since only the stock return was considered to be normally distributed. Then again, stock returns are assumed to have a lognormal distribution. It is vital to include the third and fourth moments (skewness and kurtosis ass suggested by Rubinstein (1994)

However, Primbs et al. (2007) have claimed that one could use four moments by developing a quadrinomial lattice (four branches). Still, the requirement of positive probabilities is limiting for the quadrinomial lattice based on the range of kurtosis and skewness which must be taken into account.

1.3. A Specific Goals of Research

i. To estimate the transition probabilities,

ii. To calculate the option prices at different nodes.

iii. To compare the pentanomial method, rates with the trinomial method and Black-Scholes method.

The Model

Though the pentanomial tree method and the trinomial tree method have a slight difference, the two methods are similar in many ways. From every node, the underlying stock (share) prices branch into three new prices. The prices will either be more than, less than, or equal to the previous price. The prices are mainly depended on the probabilities considered and ratios used to multiply the current price.

Assumptions

i. Time steps are all equal

ii. The interest rate used is the risk-free rates

iii. Probabilities remain the same throughout

Model one-Lattice Model

The form of the exponential Levy process model takes the form below

$\mathbf{S_t}=S_0e^{X}{}_{t}$

One arrives at this model by developing a lattice equating to X. The exponential model gives S's lattice model. Before the creation of X's problem generation, it is critical to come up with a composition of generating a noncontiguous random variable equating to the defied set of moments.

This is first achieved by first equating the moments of random variable *X* with a discrete random value *Z*. Let ***Z*** denote a non-continuous random value

$Z=m_1 + (2l\text{-}L\text{-}1)\alpha$ *with probability* p_1, *i= 1,2,….L*

Where

A=refers to the distance between two outcomes (Jump size)

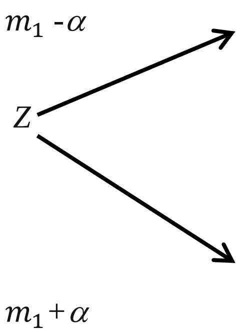
m_1= refers to the mean of X.

L=denotes the numerical lattice nodes.

α =is a positive real number

3.2 Binomial-Lattice

When we have two branches i.e. L=2, a two dimensional lattice is achieved. Hence



$Z = m_1 - \alpha$ with probability $P_{1,}$

$Z = m_1 + \alpha$ with probability $P_{2.}$

Matching the equations of Z to the first two central moments of
X the below is yields

$$(-\alpha)P_{1,} + (\alpha)P_{2,} = \mu 1,$$

$$(-\alpha)^2 P_{1,} + (\alpha)^2 P_{2,} = \mu 2,$$

and

$P_{1,} + P_{2,} = 1.$

In matrix form we have

$$\begin{bmatrix} 1 & 1 \\ -\alpha & \alpha \\ (-\alpha)^2 & (\alpha)^2 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 1 \\ \mu_1 \\ \mu_2 \end{bmatrix} \quad (3a)$$

Simplifying the above equation gives$\alpha = \sqrt{\mu_2}$ and

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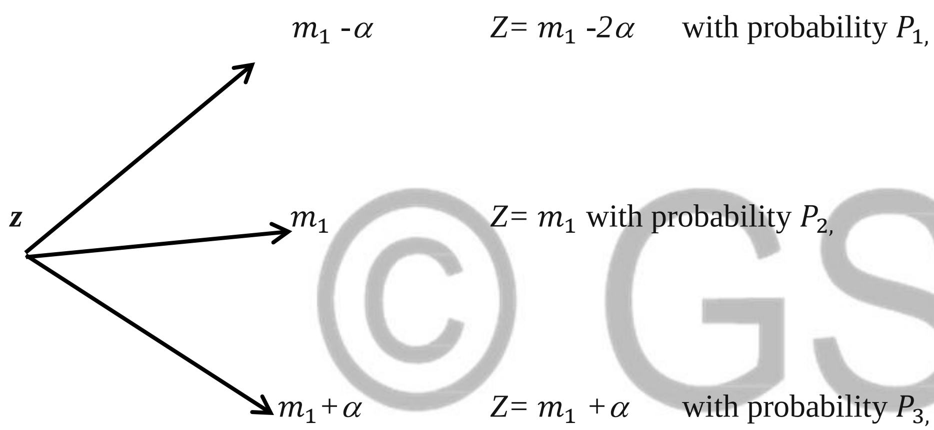
$P_1 = \left(1 - \frac{\mu_1}{\sqrt{\mu_2}}\right)$,

$P_2 = \left(1 + \frac{\mu_1}{\sqrt{\mu_2}}\right)$.

3.3 Trinomial- Lattice

A trinomial lattice model is when the branches are three i.e. L= 3 we have the trinomial lattice. Therefore



Matching the first three moments we have

(-2α) P_1 + 0P_2 + (2α)P_3 =μ_1,

(-2α)$^2 P_1$ + $0^2 P_2$ + (2α)$^2 P_3$ =μ_2,

(-2α)$^3 P_1$ + $0^2 P_2$ + (2α)$^3 P_3$ =μ_3.

And

P_1+P_2+P_3=1

A contrast of the above methodology with (20.10) in a matrix form yields

$$\begin{bmatrix} 1 & 1 & 1 \\ (-2\alpha) & 0 & (2\alpha) \\ (-2\alpha)^2 & 0 & (2\alpha)^2 \\ (-2\alpha)^3 & 0 & (2\alpha)^3 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix} \quad (3b)$$

Deducing the equations yields to

$$\alpha = \frac{1}{2}\sqrt{\frac{\mu_3}{\mu_1}}\ , \quad (3c)$$

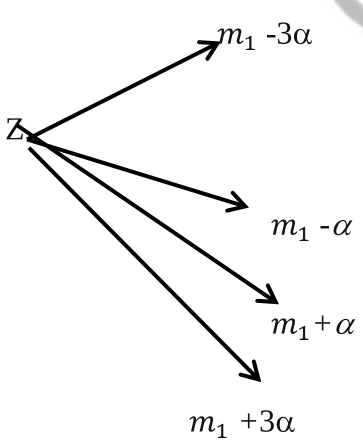
$$P_1 = \frac{1}{2}\left(\frac{\mu_1\mu_2}{\mu_3} - \sqrt{\frac{\mu_1^3}{\mu_3}}\right) \quad (3d)$$

$$P_2 = 1 - \frac{\mu_1\mu_2}{\mu_3}\ , \quad (3e)$$

$$P_3 = \frac{1}{2}\left(\frac{\mu_1\mu_2}{\mu_3} + \sqrt{\frac{\mu_1^3}{\mu_3}}\right) \quad (3f)$$

3.4 The Quadrinomial-Lattice

When the branches are four, i.e. L = 4, we have a quadrinomial lattice. Therefore



$Z = m_1 - 3\alpha$ with probability $P_{1,}$

$Z = m_1 - \alpha$ with probability $P_{2,}$

$Z = m_1 + \alpha$ with probability $P_{3,}$

$Z = m_1 + 3\alpha$ with probability $P_{4.}$

Matching the first ***L*** moments we have

$(-3\alpha)P_1 + (-\alpha)P_2 + (\alpha)P_3 + (3\alpha)P_4 = \mu_1$,

$(-3\alpha)^2P_1 + (-\alpha)2\ P_2 + (\alpha)^2P_3 + (3\alpha)^2P_4 = \mu_2$,

$(-3\alpha)^3P_1 + (-\alpha)3\ P_2 + (\alpha)^3P_3 + (3\alpha)^3P_4 = \mu_3$,

$(-3\alpha)^4P_1 + (-\alpha)^4P_2 + (\alpha)^4P_3 + (3\alpha)^4P_4 = \mu_4$,

and

$p_1 + p_2 + p_3 + p_4 = 1$.

In matrix form we have.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ (-3\alpha) & (-\alpha) & (\alpha) & (3\alpha) \\ (-3\alpha)^2 & (-\alpha)^2 & (\alpha)^2 & (3\alpha)^2 \\ (-3\alpha)^3 & (-\alpha)^3 & (\alpha)^3 & (3\alpha)^3 \\ (-3\alpha)^4 & (-\alpha)^4 & (\alpha)^4 & (3\alpha)^4 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} 1 \\ \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{bmatrix} \qquad (3g)$$

It is easy to find the expression for *afunction probabilities by ignoring* the last row of the matrix,

$$P_1 = \frac{1}{16}\left(-1\frac{\mu_1}{3\alpha} + \frac{\mu_2}{\alpha^2} - \frac{\mu_3}{3\alpha^3}\right),$$

$$P_2 = \frac{1}{16}\left(3 - \frac{3\mu_1}{3\alpha} + \frac{\mu_2}{3\alpha^2} + \frac{\mu_3}{3\alpha^3}\right),$$

$$P_3 = \frac{1}{16}\left(3 + \frac{3\mu_1}{\alpha} - \frac{\mu_2}{\alpha^2} - \frac{\mu_3}{3\alpha^3}\right),$$

$$P_4 = \frac{1}{16}\left(-1\frac{\mu_1}{3\alpha} + \frac{\mu_2}{\alpha^2} + \frac{\mu_3}{3\alpha^3}\right),$$

To equate this with the last row to make the premise true, the following condition must be satisfied byα

$$\mu_4 = 81\alpha^4P_1 + \alpha^4P_2 + \alpha^4P_3 + 81\alpha^4P_4$$

$= -9\alpha^4+10\mu_2\alpha^2$ (3h)

Determining the above equation of this expression for αyields to four- roots

$$\alpha_1=\frac{1}{3}\left(5\mu_2+\sqrt{25\mu_2^2}-9\mu_4\right)$$

$$\alpha_2=\frac{1}{3}\left(5\mu_2+\sqrt{25\mu_2^2}-9\mu_4\right)$$

$$\alpha_3=\frac{1}{3}\sqrt{5\mu_2-\sqrt{\sqrt{25\mu_2^2}-9\mu_4}}$$

$$\alpha_3=-\frac{1}{3}\sqrt{5\mu_2-\sqrt{\sqrt{25\mu_2^2}-9\mu_4}}$$

It is possible to make another access without necessarily finding α by ignoring first row. A set of different probability equations emerge:

$$P_1=\frac{1}{16}\left(\frac{3\mu_1}{3\alpha}-\frac{\mu_2}{9\alpha^2}-\frac{\mu_3}{3\alpha^3}+\frac{\mu_4}{9\alpha^4}\right),$$

$$P_2=\frac{1}{16}\left(-\frac{9\mu_1}{\alpha}+\frac{9\mu_2}{\alpha^2}+\frac{\mu_3}{\alpha^3}-\frac{\mu_4}{\alpha^4}\right),$$

$$P_3=\frac{1}{16}\left(\frac{9\mu_1}{\alpha}+\frac{9\mu_2}{\alpha^2}-\frac{\mu_3}{\alpha^3}-\frac{\mu_4}{\alpha^4}\right),$$

$$P_4=\frac{1}{16}\left(\frac{\mu_1}{3\alpha}+\frac{\mu_2}{9\alpha^2}+\frac{\mu_3}{3\alpha^3}-\frac{\mu_4}{9\alpha^4}\right),$$

The imposed condition is given by in the first row as

$1= p_1+ p_2+ p_3+ p_4$

$= \frac{10}{9}\frac{\mu_2}{\alpha_2} - \frac{\mu_4}{9\alpha^4}$ (3j)

This can be taken as:

$\alpha^4 - \frac{10}{9}\mu_2\alpha^2 + \frac{1}{9}\mu_4 = 0,$

Which gives the same solution for α as (3h).

3.5 Pentanomial lattice model

3.5(a) A Non-continuous Moment-generating Random Variable

Firstly, we generate the matching moment set up for r.v *X* that has a discontinuous random variable *Z.* Let's take into account an r.v *X*. Let M_J indicate its *k-th* raw-moment, its *k*-th central moment, and *c3* its *kth* row. A non-continuous r.v *Z* is constructed for random variable X that matches its moments.

Let *Z* be a variable that is not continuous as indicated below

Z $= m_1 + (2l - L - l)$ x, *l*=1….*L* that has with probability density functions *pi*
Where x is a framework and $m_1$is the mean of *X*.

Therefore, ***Z*** is a random variable which is non-continuous that may take on *L* range.

Theorem

Moment of Equations of Z.

The moments of Z must match with the moments of X. X moments are considered, and the below identifications should be kept constant:

$$\sum_{l=1}^{L}\big((2l - L - 1)X\big)j\ pl = \mu j$$

(3.5(a))

Definitions μ_j=refers to the *j*-th central moments of X, and $\boldsymbol{\mu_1}$=0.

Matching the four moments of the Model. To continue further, four matching moments are accounted for. The connotation L=5 implies that the lattice comprises of five branches. Four moments are considered in the problems that are faced in finance. Given this, taking kurtosis and skewedness of the yield-distributions of the asset into consideration is ideal for this case. We can achieve this by incorporating the ideas of the first four moments. The quadranomial lattice facilitates solving the pricing problem, but there combination condition and accounting for the condition requirement of positive likelihoods is associated with problems in regards to the captured kurtosis and skewedness range. In light of this, a pentanomial lattice proves to be increasingly accommodative and informational with its lower number of problems or complexity. Since it helps to achieve our main objective:

Solving (1) for *pl*, with *l*=1,….,5 yields.

$$Z_5 = \begin{cases} m_1 - 4xp_1 = \frac{(\boldsymbol{\mu_4} - 4x^2\boldsymbol{\mu_2} - \mathbf{4x}\boldsymbol{\mu_3})}{384x^4} \\ m_1 - 2xp_2 = \frac{(-\boldsymbol{\mu_4} + 16x^2\boldsymbol{\mu_2} - \mathbf{2}x\boldsymbol{\mu_3})}{96x^4} \\ m_1 p_3 = 1 + \frac{(-20x^2\mu_2 + \mu_4)}{64x^4} \quad (2) \\ m_1 + 2xp_4 = \frac{-2x\mu_3 - \mu_4 + 16x^2\mu_2}{96x^4} \\ m_1 + 4xp_5 = \frac{(\boldsymbol{\mu_4} - 4x^2\boldsymbol{\mu_2} + \mathbf{4x}\boldsymbol{\mu_3}}{384x^4} \end{cases}$$

Therefore, when we assume$x > 0$, a question arises about the range of X's (the pdf of X) with positive probabilities?

The inquisition below addresses the problem.

*Theorem 1.*Given that $_2\boldsymbol{\mu}_4 \geq 3\mu_3^2$and $25\mu_2^2 \geq 16\mu_4$(or equal to k$\geq 3s^2$-3 and k$\geq 3s^2 - \frac{23}{16}$

Where $s = \frac{\mu3}{\mu_2^{3/2}}$refers to skewness and $\boldsymbol{K} = \frac{\mu4}{\mu_2^2}$ *-3* is kurtosis), there prevails an area of values of $x$derived by

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$\frac{1}{16\mu_2}(_{3+(} \mu_3^2 {}_{+} 16\boldsymbol{\mu}_2 \boldsymbol{\mu}_{4)} \frac{1}{2}) \leq x \leq \frac{1}{4\mu_2}(-2\boldsymbol{\mu}_{3+2\,(}\ \mu_{3+}^2 \boldsymbol{\mu}_2 \boldsymbol{\mu}_4)^{\ 1/2})$ (3.5(b))

This exclusively includes

$x = \sqrt{\frac{\mu_4}{12\mu_2}} = \sigma\sqrt{\frac{3+k}{12}}$(3.5©)

For which all the probabilities *Pt*. ***1=*** 1,….,*5*, are not negative.

Theorem 1 gives us a strong case for the level of positivity of the probabilities. Furthermore, it illustrates that *a* yields a non-negative probability for figures of $\mu_{2,}\mu_{3,}$ *and* $\mu_{4,}$clarifying that $2\boldsymbol{\mu}_4 \geq 3\mu_3^2$ and $3\mu_2^2 \geq 2\mu_{4,}$ (or equal to$k \geq 3s^2$-3 and $k \geq -\frac{3}{2}$.), which prove to bemore accommodative than the other conditions. That said, (3.5(b)) fails to keep up with these added conditions which seek for an all-inclusive model. For values whose kurtosis is lower than $-\frac{23}{16}$, compositepositivity conditions are established. Therequirements are not included as p. The excess positive kurtosis (*K>0*) is a significant issue in finance and remains unexplored.

We have seen cases where the tails are more massive and picks higher. This proposition not only describes but also determines a variation of kurtosis and skewness figures that are adaptable with a pentanomial. It also identifies a large area of Kurtosis and skewness that is likely to be more compatible with a lattice-model. The spacing restrictions between results of ***Z*** are the random variable inclined by the described parameter. Ultimately, this analysis introduces and describes a unique formula for determining that spacing. Consequently, this simple proposition acts as a foundation for demonstrating a lattice model coupled with a detailed understanding of its characteristics and disadvantages.

One can upgrade these findings with similar studies as in theorem 1 other than for more than one-branch lattice which utilize more than five branches. Then again, the logic used to establish the theoretical conditions is very long and exhausting. That is to say, an analysis of more than five branches would be tedious. However, if kurtosis and skewedness are the primary factors that are seized, the pentanomial lattice facilitates a grasp of most interest of its parameters. It allows for more straight-forward understandable definition of characteristics. Precisely, pentanomial is preferred because it captures not only the complexity but also the practicability of the options.

Concerning the creation of a lattice model, *w*e deduce how the equation of the moments of an r.v X with a non-continuous r.v ***Z can be crucial.*** This is then reduced to a lattice- model, with an assumption that*A'1*is a Levy process. Ideally, for any time ***t,*** the obtained answers from the existing part show that it is possible to equate the moments of X with a non-continuous r.v ***Z (t).*** Since *A'1* is a Levy process, the scale of its cumulants is in line with time t. We define its cumulants at any time ***t*** by defining its yearly- cumulants. Practically,

Let C_j be the *j*th cumulants of X_1, then the *j*th cumulants of X_1, is $C_j t$.
Let τbe an increment in *t* that gives the step size of the lattice. To generate the lattice model, each increase Xπ with the discrete r.v ***Z(τ)*** that equals its moments is determined. This leads to the below model which explains the lattice.

3.5(b). The Lattice-Model

let $s_0$be the cost of the mentioned underlying asset. Then a lattice model estimating S, = *Soeχt* is given by

$$Sn\ (\pi) = s_0 \exp(\sum_{k=1}^{n} z^k(^{t})$$

Given *n* refers to the count of time step size **t** and the $\boldsymbol{Z_K(\ t)}$ are iid r.v distributed as

Z (τ) =

$$\begin{cases} c_1\tau\text{-}4xp_1(\tau)\dfrac{\left(\left(c_4\tau+3c_2^2(\tau)^2\right)\text{-}\ 4x^2c_2\tau\ \text{-}\ 4xc_3\tau\right)}{384x^4} \\ c_1\tau - 2xp_2(\tau) = \dfrac{(-\left(c_4\tau+3c_2^2(\tau)^2\right)+16x^2c_2\tau+2xc_3\tau)}{96x^4} \\ c_1\tau p_3(\tau) = 1 + \dfrac{(-20x^2c_2\tau+\left(c_4\tau+3c_2^2(\tau)^2\right))}{64x^4}(5 \\ c_1\tau + 2xp_4(\tau) = \dfrac{-2xc_3\tau\left(c_4\tau+3c_2^2(\tau)^2\right)+16x^2c_2\tau}{96x^4} \\ c_1\tau + 4xp_5(\tau) = \dfrac{\left(\left(c_4\tau+3c_2^2(\tau)^2\right)\text{-}\ 4x^2c_2\tau+4xc_3\tau\right)}{384x^4} \end{cases}$$

In the above diagram, a one-step distanceτ of the lattice model at an ith time is establsihed.
Given that the lattice model is linked with the cumulants of X_t, positivity-condition of theorem 1 in relations to of cumulants is also rephrased.

Values **Probabilities:**

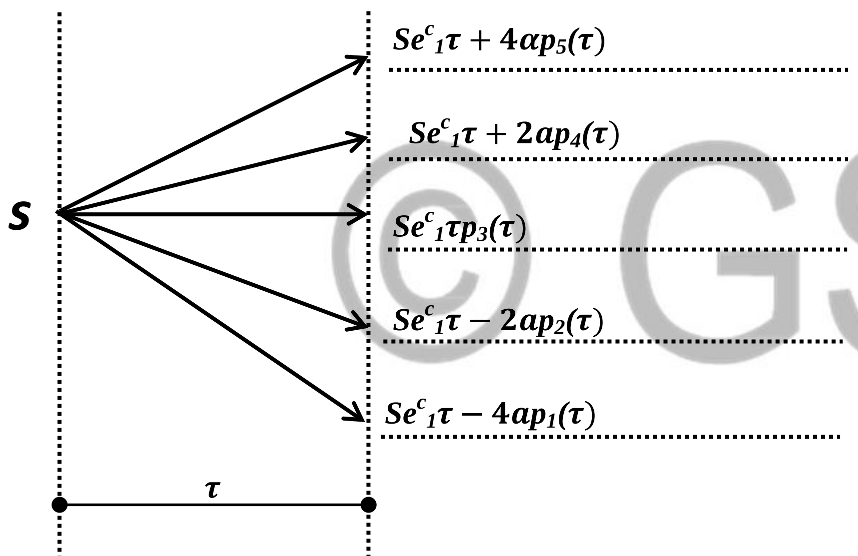

Figure 1.pentanomial lattice model with one step

*Proposition 2.*Provided

$c_4c_2 \geq 3c_3^2 - 3c_3^2\tau$and $c_4 \geq -\frac{23}{16}c_2^2\,\tau$ (3.5(d))

the assessment results in an extensive magnitude of α values that are deduced to;

$$\frac{1}{16c_2\tau}\left(c_3\tau + (c_3^2\tau^2 + 16c_2\tau(c_4\tau + 3c_2^2\tau^2))^{1/2}\right) \leq \chi \leq \frac{1}{4c_2\tau}(-2c_3\tau + 2(c_3^2\tau^2 + c_2\tau(c_4\tau + 3c_2^2\tau^2))^{1/2} \quad (7)$$

This also includes

$$\propto = \frac{1}{2}\sqrt{c_2\tau} + \frac{c_4}{3c_2} \quad (3.5(e))$$

For which the probabilities i. p_l, *1*=1. .... *5.* Are positive.

Comparing the results in provision with the cumulants of the process X. this explains why the circumstances scale concerning the time-step- τ. Particularly, when the limit as $\tau \rightarrow 0$, which is our area of concern.

Limitations of the Lattice-Model.

In the below segment the restrictions of the lattice model in continuous time as $\tau \rightarrow 0$ are taken into account. Besides, there is an assumption that the third and fourth cumulants $C_3$and $C_4$are real positive numbers. If the cumulants were zero then it would be valid to deduce that the lattice will merge to a geometric Brownian motion because of the condition of positivity.

Concerning the discreet model having a continuous time limit, the condition of positivity must be workable or seen as$\tau \rightarrow 0$. In the limit, (6) deduces to

$$c_4 c_{2\geq} 3c_3^2 \text{and} c_{4\geq} 0 \quad (3.5(f))$$

The other assumption is that (3.5(f)) holds. Note that the requirement of $c_4 \geq O$ corresponds to the expectation of a positive excess of kurtosis.

We aspire to have the lattice containing an explained maximum as the size-step approaches zero. It follows that we assume that the model has a limit as$\tau \rightarrow 0$. Given this, let us presuppose that the limit is given by α=$lim_{t\rightarrow 0}$α where it is within a specified spread.

For continuation purposes, new probabilities, (q_1, q_2, *q4*,q_5) are included to refer to the branch probabilities:

$$\lim_{\tau} \to 0(1/\tau p_1(\tau) = \frac{-4x_0^2\, c_2 - 4x_0 c_3 + c_4}{384x_0^4} = \lambda q_1$$

$$\lim_{\tau} \to 0(1/\tau p_2(\tau) = \frac{16x_0^2\, c_2 - 2x_0 c_3 - c_4}{96x_0^4} = \lambda q_2$$

$$\lim_{\tau} \to 0(1/\tau p_3(\tau) - 1 = \frac{(-20x_0^2 c_2 + c_4)}{64x_0^4} = -\lambda$$

$$\lim_{\tau} \to 0(1/\tau p_4(\tau) = \frac{16x_0^2\, c_2 - 2x_0 c_3 - c_4}{96x_0^4} = \lambda q_4$$

$$\lim_{\tau} \to 0(1/\tau p_5(\tau) = \frac{-4x_0^2\, c_2 + 4x_0 c_3 + c_4}{384x_0^4} = \lambda q_5$$

Therefore,

$q_1 + q_2 + q_4 + q_5 = 1$

If we use α to simplify to

$\lambda = \frac{3x_2^2}{2C_4}$

$q_1 = \frac{1}{6}\left(1 - c_3\sqrt{\frac{3}{C_2 C_4}}\right)$

$q_2 = \frac{1}{3}\left(1 + c_3\sqrt{\frac{3}{C_2 C_4}}\right)$

$q_4 = \frac{1}{3}\left(1 - c_3\sqrt{\frac{3}{C_2 C_4}}\right.$

$q_5 = \frac{1}{6}\left(1 + c_3\sqrt{\frac{3}{C_2 C_4}}\right)$

In the limit, $Z(\tau)$ approaches an increasing compound Poisson process with the following description:

$$C_1 t + \sum_{k=0}^{N_t} W_k$$

N_t, = Poisson process with force λ and the W_k which are iid r.v with the below given dispersal.

$$W_k = \begin{cases} -4x_0 \text{ with probability } q_1 \\ -4x_0 \text{ with probability } q_2 \\ 2x_0 \text{ with probability } q_4 \\ 4x_0 \text{ with probability } q_5 \end{cases} \quad (3.5(h))$$

Theorem 3.

Reflect a permanent range given by; [0, *T]* of time where the count of steps *n* is expanding. Then the size of the steps,$\tau = T/n$. Our area of interest is in the r.v at time *T* described below as:

$$X_n = \sum_{k=1}^{n} Z_k(\tau)$$

Implying as n -∞, $\boldsymbol{X_n}$**,** yields to the below distribution

$$\boldsymbol{C_1 T} + \sum_{k=0}^{N_T} W_k \quad (3.5(i))$$

Given that N_T is a Poisson distribution with an average ofλT and the W_k, which are iid r.v described in (3.5(h)).

The above hypothesis leads us to a lattice model that is applied in the European pricing options. Secondly, we utilize Fourier transform methods as in Carr &Madan (1999) to generate the costing of European options. Note that the characteristic it is vital to know the function of the distribution here. For the r.v described by (12), the attribute result of Fourier transform (Breiman, 1992) is deduced to:

$$\emptyset_T(\mu) = e^{iuc_1 T} \exp\left(\lambda T \sum_{l \in \{1,2,4,5\}} q_1 (e^{iu(2l-6)x} - 1)\right) (3.5(j))$$

The above will be applied when it comes to evaluating European in the below sections

Applying the Fourier Transform to price European Options (Calls & Puts)

This sample engenders reference to the pioneer assignment of Carr & Madan (1999) on cost of options that utilize the Fourier transform. It is evident that when it is a significantly useful model, especially when the user knows the risk-neutral probabilities for Fourier Transform. The case scenario is a slight reversal if what Can and Madan (1999) proposed since a non-continuous spread is utilized and therefore anon-continuous

Fourier transform.

Let q_T(n) be the discrete risk-neutral probability distribution of the r.v described in (12) above. Employing the limiting distribution in (12) as the risk-neutral probabilities, a call options amount is deduced as an allowable payoff expectation as stated below:

$$CT\,(\check{k}, K) = e^{-rt} \sum_{n=\check{k}}^{\infty} \left(e^{C_{1T+2x_0}n} - K\right) q_T\,(n) \qquad (3.5(k))$$

Where $\check{k} > \left(\frac{\ln\left(\frac{K}{S}\right) - C_{1T}}{2x_0}\right)$

As said earlier, the model corresponds to Carr & Madan (1994).

Fourier Transform pricing Equation & Formula

The cost of a European call option that does not pay money from its dividends on the repressed holding represented as $S_{0e}{}^{x_t}$ with the original costS_0, the dissemination of X_t at the lapse of time T of (12), and selling cost $\boldsymbol{K}$ is given or derived as follows:

$$C_T\,(\check{k}, K) = \frac{e^{-\beta\check{k}}}{2\pi} \int_{-\pi}^{\pi} \Psi\,(u, K) e^{-iu\check{k}}\, du \qquad (3.5(g))$$

where β>0 is a specification to describe Fourier Transform.,

$$\Psi(u, K) e^{-rT} \left(\frac{1}{1-e^{-(\beta+iu)}}\right) \left[S_0 e^{C_1 T} \emptyset\left(-i\left(\frac{\beta+2\alpha_0+iu}{2\alpha_0}\right) - K\emptyset\left(-i\left(\frac{\beta+iu}{2\alpha_0}\right)\right)\right] \qquad (3.5(h))$$

And

$$\emptyset(\boldsymbol{\mu}) = E\left[e^{iuX}\right] = \sum_{n=-\infty}^{\infty} e^{iu2\alpha_0 n} qT\,(n) = exp\left(\lambda T \sum_{l\in\{1,2,4,5\}} q_1 \left(e^{(2l-6)\alpha_0} - 1\right)\right) \qquad (3.5(i))$$

Is the mgf of the limiting distribution in (3.5(i)).

After factoring in the formula described above, a Fast-Fourier transform algorithm that is widely applied in the comprehensive computation of the asset price yields. This utilization of fast Fourier transform was established by Madan (1999). In a given procedure aloft, incorporation of the rate of return*(c1)* provided by

Levy process is applied in risk-neutral options to establish the value of the underlying non-dividend paying asset. **NOTE** The*c1(drift)*should be inconsiderate with the below risk-neutral condition.

e^{r^T}=[e^{X_T}]= ∅(-i) = exp (C_1T)exp$\left(\lambda T \sum_{l=\{1,2,4,5\}} q_l \left(e^{(2l-6)\alpha_0} - 1\right)\right)$

Solving for $C_i$yields

C_i= r-λ$\sum_{l\in\{1,2,4,5\}} q_1 \left(e^{(2l-6)\propto_0} - 1\right)$

Conclusion

We have examined a pentanomial lattice representation that assimilates kurtosis and skewness .We have also controlled the states on kurtosis and skewness under continuous time. We came up with the restricting distribution which is ideal compound Poisson distribution. In the end, we came up with a formula involving Fourier transform techniques that systematically employed to compute European option prices. Thus, this explains a compatible representation for estimating American and European option prices under kurtosis and skewness.

This project analyzed through a recombining pentanomial tree for coasting European options with unchanging volatilities. The above was attained by letting the time steps regular and risk neutral probabilities remaining the same for the entire contract time. This recombining trinomial tree is more flexible/ easy to use than just a tree with a lot of nodes which are more the same and thus suitable for predicting prices of options.

I recommend the use of Pentanomial lattice model in option pricing

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